homework3

February 12, 2019

1 Homework 3 - Berkeley STAT 157

Handout 2/5/2019, due 2/12/2019 by 4pm in Git by committing to your repository.

Formatting: please include both a .ipynb and .pdf file in your homework submission, named homework3.ipynb and homework3.pdf. You can export your notebook to a pdf either by File -> Download as -> PDF via Latex (you may need Latex installed), or by simply printing to a pdf from your browser (you may want to do File -> Print Preview in jupyter first). Please don't change the filename.

```
In [1]: from mxnet import nd, autograd, gluon
        import matplotlib.pyplot as plt
```

2 1. Logistic Regression for Binary Classification

In multiclass classification we typically use the exponential model

$$p(y|\mathbf{o}) = \mathrm{softmax}(\mathbf{o})_y = \frac{\exp(o_y)}{\sum_{y'} \exp(o_{y'})}$$

1.1. Show that this parametrization has a spurious degree of freedom. That is, show that both $\mathbf{o}$ and $\mathbf{o} + c$ with $c \in \mathbb{R}$ lead to the same probability estimate. 1.2. For binary classification, i.e. whenever we have only two classes $\{-1, 1\}$, we can arbitrarily set $o_{-1} = 0$. Using the shorthand $o = o_1$ show that this is equivalent to

$$p(y = 1|o) = \frac{1}{1 + \exp(-o)}$$

1.3. Show that the log-likelihood loss (often called logistic loss) for labels $y \in \{-1, 1\}$ is thus given by

$$-\log p(y|o) = \log(1 + \exp(-y \cdot o))$$

1.4. Show that for $y = 1$ the logistic loss asymptotes to o for $o \to \infty$ and to $\exp(o)$ for $o \to -\infty$.

2.1 part 1

\$ p (y | **o** + c) = softmax(**o** + c)_y\$
$= \frac{exp(o_y+c)}{\sum_{y'} exp(o_y'+c)}$

$= \frac{exp(o_y+c)}{\sum_{y'} exp(o_{y'}+c)}$
$= \frac{exp(o_y)exp(c)}{\sum_{y'} exp(o_{y'})exp(c)}$

2.1.1 e^c *cancels in numerator & denominator*

$= \frac{exp(o_y)}{\sum_{y'} exp(o_{y'})}$
$= p(y|\mathbf{o})$
$\blacksquare$

2.2 part 2

$p(y=1|o) = \frac{exp(o_1)}{\sum_{y'} exp(o_{y'})}$
$= \frac{exp(o_1)}{exp(o_{-1})+exp(o_1)}$
$= \frac{exp(o)}{exp(\mathbf{0})+exp(o)}$
$= \frac{exp(o)}{1+exp(o)}$
$= \frac{1}{\frac{1}{exp(o)}+\frac{exp(o)}{exp(o)}}$
$= \frac{1}{exp(o)^{-1}+1}$
$= \frac{1}{(e^o)^{-1}+1}$
$= \frac{1}{(e^-o)+1}$
$= \frac{1}{1+exp(-o)}$
$\blacksquare$

2.3 part 3

$-logp(y|o) = -\sum_i y_i log(\hat{y}_i)$
$= -ylog(\hat{y}_i)$
$= -ylog(softmax(o))$
$= -ylog\left(\frac{exp(o)}{\sum_i exp(o_i}\right)$
$= ylog\left[\left(\frac{exp(o)}{\sum_i exp(o_i)}\right)^{-1}\right]$
$= ylog\left(\frac{\sum_i exp(o_i}{exp(o)}\right)$
$= ylog\left(\frac{exp(o_{-1})+exp(o_1)}{exp(o)}\right)$
$= ylog\left(\frac{exp(\mathbf{0})+exp(o)}{exp(o)}\right)$
$= ylog\left(\frac{1+exp(o)}{exp(o)}\right)$
$= ylog\left(\frac{1}{exp(o)}+1\right)$
$= ylog\left(exp(o)^{-1}+1\right)$
$= ylog\left(exp(-o)+1\right)$
$= log\left[(1+exp(-o))^y\right]$
$= log\left[1^y+exp(-o)^y\right]$
$= log\left[1+exp(-o\cdot y)\right]$

$= log\left[1 + exp(-y \cdot o)\right]$
■

2.4 part 4a

as $o \to \infty$,
log$(1 + e^{-o})$ $\to$ **log(1)** $\to$ **0 =** o_{-1}

2.5 part 4b

$$-log[p(y = 1|o)] = log[1 + exp(-o)]$$

2.5.1 so, as $o \to \infty$,

$$\to log(1 + e^{\infty})$$

$$= log(1 + \infty)$$

$$= \infty$$

2.5.2 converges to ∞

3 2. Logistic Regression and Autograd

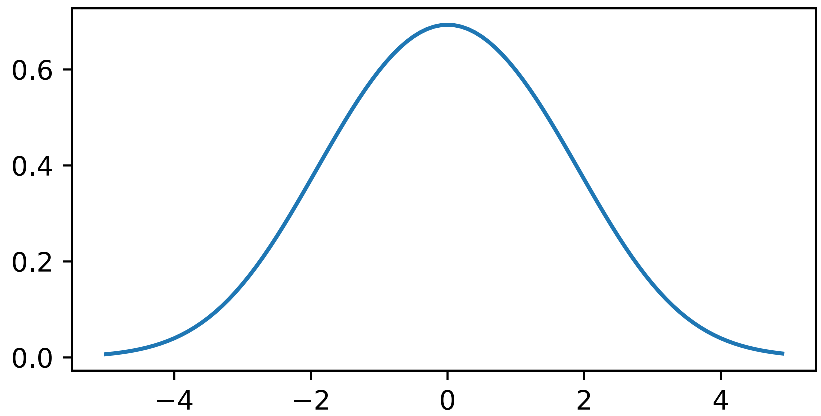
1. Implement the binary logistic loss $l(y, o) = \log(1 + \exp(-y \cdot o))$ in Gluon
2. Plot its values for $y \in \{-1, 1\}$ over the range of $o \in [-5, 5]$.
3. Plot its derivative with respect to o for $o \in [-5, 5]$ using 'autograd'.

```
In [84]: # 2.1

         def loss(y,o):
             ## add your loss function here
             return nd.log(1 + nd.exp(-y * o))
```

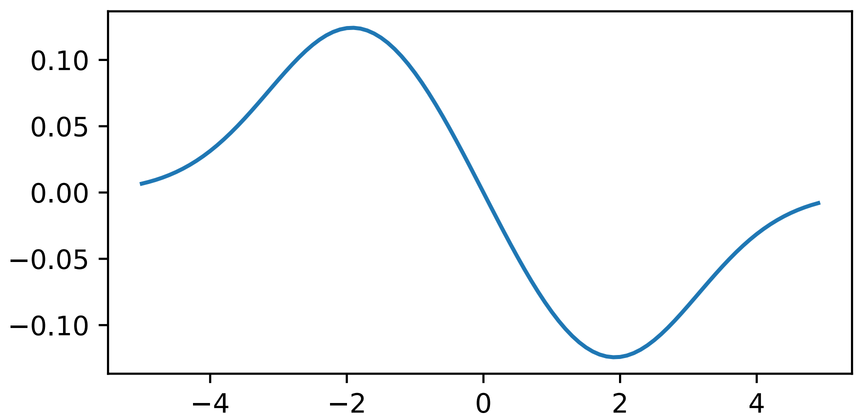
```
In [85]: o = nd.arange(-5, 5, 0.1)
         y = nd.arange(-1, 1, .02)
         o.attach_grad()
         with autograd.record():
             l = loss(y, o)
         l.backward()
```

```
In [86]: # 2.2
         from d2l import set_figsize

         set_figsize(figsize=(5, 2.5))
         plt.plot(o.asnumpy(), l.asnumpy())
```

```
Out[86]: [<matplotlib.lines.Line2D at 0x115b44cc0>]
```




```
In [87]: # 2.3

         set_figsize(figsize=(5, 2.5))
         plt.plot(o.asnumpy(), o.grad.asnumpy())

Out[87]: [<matplotlib.lines.Line2D at 0x115bde630>]
```



4 3. Ohm's Law

Imagine that you're a young physicist, maybe named Georg Simon Ohm, trying to figure out how current and voltage depend on each other for resistors. You have some idea but you aren't quite sure yet whether the dependence is linear or quadratic. So you take some measurements, conveniently given to you as 'ndarrays' in Python. They are indicated by 'current' and 'voltage'.

Your goal is to use least mean squares regression to identify the coefficients for the following three models using automatic differentiation and least mean squares regression. The three models are:

1. Quadratic model where voltage $= c + r \cdot$ current $+ q \cdot$ current^2.
2. Linear model where voltage $= c + r \cdot$ current.
3. Ohm's law where voltage $= r \cdot$ current.

```
In [2]: current = nd.array([1.5420291, 1.8935232, 2.1603365, 2.5381863, 2.893443, \
                            3.838855, 3.925425, 4.2233696, 4.235571, 4.273397, \
                            4.9332876, 6.4704757, 6.517571, 6.87826, 7.0009003, \
                            7.035741, 7.278681, 7.7561755, 9.121138, 9.728281])
        voltage = nd.array([63.802246, 80.036026, 91.4903, 108.28776, 122.781975, \
                            161.36314, 166.50816, 176.16772, 180.29395, 179.09758, \
                            206.21027, 272.71857, 272.24033, 289.54745, 293.8488, \
                            295.2281, 306.62274, 327.93243, 383.16296, 408.65967])
```

4.1 part 1: quadratic model *(from scratch)*

```
In [92]: from d2l import data_iter, squared_loss, sgd, linreg

         second_var = current**2
         var_1 = current.reshape(20, 1)
         var_2 = second_var.reshape(20, 1)

         features  = nd.concat(var_1, var_2, dim = 1)
         labels = voltage

         w = nd.random.normal(scale = 0.01, shape = (2,1))
         b = nd.zeros(shape = (1,))

         w.attach_grad()
         b.attach_grad()
```

```
In [93]: net = linreg
         loss = squared_loss

         batch_size = 5
         lr = 0.0001
         num_epochs = 100
```

4.2 i ran this 30 times, so 3000 epochs total, for the estimates to converge. didn't want the PDF to be 30+ pages long.

```
In [126]: for epoch in range(num_epochs):
              for X, y in data_iter(batch_size, features, labels):
                  with autograd.record():
                      l = loss(net(X, w, b), y)
                  l.backward()
                  sgd([w, b], lr, batch_size)
              train_l = loss(net(features, w, b), labels)
              print("epoch %d, loss %f" % (epoch + 1, train_l.mean().asnumpy()))
```

```
epoch 1, loss 8.057441
epoch 2, loss 8.046292
epoch 3, loss 8.010896
epoch 4, loss 8.007100
epoch 5, loss 7.994840
epoch 6, loss 7.985097
epoch 7, loss 7.984781
epoch 8, loss 7.987340
epoch 9, loss 7.973763
epoch 10, loss 7.962390
epoch 11, loss 7.947775
epoch 12, loss 7.953340
epoch 13, loss 7.987454
epoch 14, loss 7.922862
epoch 15, loss 7.919008
epoch 16, loss 7.902259
epoch 17, loss 7.905784
epoch 18, loss 7.889100
epoch 19, loss 7.877214
epoch 20, loss 7.867025
epoch 21, loss 7.860958
epoch 22, loss 7.858366
epoch 23, loss 7.947656
epoch 24, loss 7.834297
epoch 25, loss 7.827324
epoch 26, loss 7.914720
epoch 27, loss 7.827928
epoch 28, loss 7.806587
epoch 29, loss 7.798125
epoch 30, loss 7.785196
epoch 31, loss 7.788493
epoch 32, loss 7.770140
epoch 33, loss 7.762079
epoch 34, loss 7.860945
epoch 35, loss 7.751613
epoch 36, loss 7.742499
epoch 37, loss 7.733298
epoch 38, loss 7.745149
epoch 39, loss 7.717100
epoch 40, loss 7.712887
epoch 41, loss 7.750579
epoch 42, loss 7.689883
epoch 43, loss 7.682284
epoch 44, loss 7.673859
epoch 45, loss 7.665999
epoch 46, loss 7.685040
epoch 47, loss 7.669961
epoch 48, loss 7.645238
```

```
epoch 49, loss 7.635342
epoch 50, loss 7.627083
epoch 51, loss 7.618154
epoch 52, loss 7.624625
epoch 53, loss 7.605255
epoch 54, loss 7.628491
epoch 55, loss 7.588293
epoch 56, loss 7.602652
epoch 57, loss 7.594010
epoch 58, loss 7.569783
epoch 59, loss 7.557950
epoch 60, loss 7.553037
epoch 61, loss 7.555341
epoch 62, loss 7.553990
epoch 63, loss 7.572806
epoch 64, loss 7.569339
epoch 65, loss 7.511954
epoch 66, loss 7.530900
epoch 67, loss 7.497385
epoch 68, loss 7.508664
epoch 69, loss 7.483739
epoch 70, loss 7.501910
epoch 71, loss 7.467653
epoch 72, loss 7.500606
epoch 73, loss 7.479414
epoch 74, loss 7.447894
epoch 75, loss 7.438989
epoch 76, loss 7.430577
epoch 77, loss 7.425569
epoch 78, loss 7.415796
epoch 79, loss 7.420139
epoch 80, loss 7.473880
epoch 81, loss 7.490071
epoch 82, loss 7.479179
epoch 83, loss 7.430433
epoch 84, loss 7.375667
epoch 85, loss 7.390142
epoch 86, loss 7.375174
epoch 87, loss 7.350253
epoch 88, loss 7.356307
epoch 89, loss 7.346594
epoch 90, loss 7.328736
epoch 91, loss 7.320992
epoch 92, loss 7.324968
epoch 93, loss 7.306693
epoch 94, loss 7.311127
epoch 95, loss 7.296299
epoch 96, loss 7.285527
```

```
epoch 97, loss 7.280166
epoch 98, loss 7.277467
epoch 99, loss 7.281362
epoch 100, loss 7.299457
```

```
In [127]: print(" r = ", w[0].asscalar(), "\n\n", "q =", w[1].asscalar(), "\n\n", "c = ", b[0]
```

```
 r =  35.331055

 q = 0.64558494

 c =  13.179283
```

4.3 part 2: linear model (gluon)

```
In [128]: from mxnet.gluon import data as gdata, loss as gloss
          from d2l import nn
          from mxnet import init

          net = nn.Sequential()
          net.add(nn.Dense(1))
          net.initialize(init.Normal(sigma = 0.01))

          loss = gloss.L2Loss()
          trainer = gluon.Trainer(net.collect_params(), 'sgd', {'learning_rate': 0.03})
```

```
In [129]: batch_size = 5
          num_epochs = 100

          dataset = gdata.ArrayDataset(current, voltage)
          data_iter = gdata.DataLoader(dataset, batch_size, shuffle=True)
```

```
In [130]: for epoch in range(1, num_epochs + 1):
              for X, y in data_iter:
                  with autograd.record():
                      l = loss(net(X), y)
                  l.backward()
                  trainer.step(batch_size)
              l = loss(net(current), voltage)
              print('epoch %d, loss: %f' % (epoch, l.mean().asnumpy()))
```

```
epoch 1, loss: 4.818239
epoch 2, loss: 4.384062
epoch 3, loss: 4.817986
epoch 4, loss: 6.278380
epoch 5, loss: 4.811965
epoch 6, loss: 3.666365
```

```
epoch 7, loss: 3.571980
epoch 8, loss: 3.644460
epoch 9, loss: 3.334803
epoch 10, loss: 3.437017
epoch 11, loss: 3.141636
epoch 12, loss: 3.386814
epoch 13, loss: 2.906808
epoch 14, loss: 2.825375
epoch 15, loss: 3.068982
epoch 16, loss: 4.162078
epoch 17, loss: 2.649559
epoch 18, loss: 3.038372
epoch 19, loss: 2.617471
epoch 20, loss: 5.743158
epoch 21, loss: 2.493365
epoch 22, loss: 2.178969
epoch 23, loss: 2.124710
epoch 24, loss: 2.058398
epoch 25, loss: 2.022227
epoch 26, loss: 1.946573
epoch 27, loss: 2.057502
epoch 28, loss: 1.921635
epoch 29, loss: 1.771373
epoch 30, loss: 2.572592
epoch 31, loss: 1.849800
epoch 32, loss: 1.706146
epoch 33, loss: 1.721513
epoch 34, loss: 1.798129
epoch 35, loss: 1.735896
epoch 36, loss: 1.520482
epoch 37, loss: 1.653411
epoch 38, loss: 1.671662
epoch 39, loss: 1.365107
epoch 40, loss: 1.347510
epoch 41, loss: 1.311854
epoch 42, loss: 1.603087
epoch 43, loss: 1.273804
epoch 44, loss: 1.239572
epoch 45, loss: 1.196464
epoch 46, loss: 1.318221
epoch 47, loss: 1.176726
epoch 48, loss: 1.134976
epoch 49, loss: 1.103437
epoch 50, loss: 1.185562
epoch 51, loss: 1.151699
epoch 52, loss: 1.068920
epoch 53, loss: 1.389291
epoch 54, loss: 1.310020
```

```
epoch 55, loss: 1.014290
epoch 56, loss: 0.978618
epoch 57, loss: 0.965184
epoch 58, loss: 1.008923
epoch 59, loss: 2.196826
epoch 60, loss: 0.912990
epoch 61, loss: 0.992285
epoch 62, loss: 0.942468
epoch 63, loss: 0.912484
epoch 64, loss: 0.863083
epoch 65, loss: 2.400800
epoch 66, loss: 1.136953
epoch 67, loss: 0.840361
epoch 68, loss: 0.830781
epoch 69, loss: 0.913175
epoch 70, loss: 0.850288
epoch 71, loss: 0.799243
epoch 72, loss: 0.877570
epoch 73, loss: 0.783139
epoch 74, loss: 0.859367
epoch 75, loss: 0.922959
epoch 76, loss: 0.916956
epoch 77, loss: 0.941355
epoch 78, loss: 0.823137
epoch 79, loss: 0.953434
epoch 80, loss: 0.736253
epoch 81, loss: 0.782805
epoch 82, loss: 0.757767
epoch 83, loss: 0.915393
epoch 84, loss: 0.900943
epoch 85, loss: 0.819785
epoch 86, loss: 0.879930
epoch 87, loss: 0.762810
epoch 88, loss: 0.716770
epoch 89, loss: 1.054702
epoch 90, loss: 0.891248
epoch 91, loss: 0.728869
epoch 92, loss: 0.779002
epoch 93, loss: 0.772572
epoch 94, loss: 0.771436
epoch 95, loss: 0.669027
epoch 96, loss: 0.666202
epoch 97, loss: 0.679064
epoch 98, loss: 1.944078
epoch 99, loss: 0.654189
epoch 100, loss: 0.671245
```

```
In [45]: params = net.collect_params()
         values = []
         for param in params.values():
             values += [param.data()[0].asscalar()]

         print(" r =", values[0], "\n\n", "c =", values[1])
```

```
 r = 41.79493

 c = 1.410167
```

4.4 part 3 (gluon)

```
In [3]: from mxnet.gluon import data as gdata, loss as gloss
        from d2l import nn, data_iter
        from mxnet import init

        net = nn.Sequential()
        net.add(nn.Dense(1, use_bias = False))
        net.initialize(init.Normal(sigma = 0.01))

        loss = gloss.L2Loss()
        trainer = gluon.Trainer(net.collect_params(), 'sgd', {'learning_rate': 0.03})
```

```
In [4]: features = current.reshape(20, 1)

        num_epochs = 100
        batch_size = 4
        dataset = gdata.ArrayDataset(features, voltage)
        data_iter = gdata.DataLoader(dataset, batch_size, shuffle=True)
```

```
In [5]: for epoch in range(1, num_epochs + 1):
            for X, y in data_iter:
                with autograd.record():
                    l = loss(net(X), y)
                l.backward()
                trainer.step(batch_size)
            l = loss(net(features), voltage)
            print('epoch %d, loss: %f' % (epoch, l.mean().asnumpy()))
```

```
epoch 1, loss: 0.632421
epoch 2, loss: 0.617448
epoch 3, loss: 0.646027
epoch 4, loss: 0.614750
epoch 5, loss: 0.623311
epoch 6, loss: 0.664360
epoch 7, loss: 0.670308
epoch 8, loss: 0.620153
```

```
epoch 9, loss: 0.615033
epoch 10, loss: 0.656699
epoch 11, loss: 0.610469
epoch 12, loss: 0.675199
epoch 13, loss: 0.715462
epoch 14, loss: 0.612291
epoch 15, loss: 0.703940
epoch 16, loss: 1.540874
epoch 17, loss: 0.610419
epoch 18, loss: 0.610716
epoch 19, loss: 0.879703
epoch 20, loss: 0.610583
epoch 21, loss: 0.610624
epoch 22, loss: 0.944854
epoch 23, loss: 0.682255
epoch 24, loss: 0.614452
epoch 25, loss: 0.702553
epoch 26, loss: 1.299942
epoch 27, loss: 1.396988
epoch 28, loss: 0.879210
epoch 29, loss: 0.814842
epoch 30, loss: 0.633608
epoch 31, loss: 0.610434
epoch 32, loss: 0.640284
epoch 33, loss: 0.939271
epoch 34, loss: 0.706373
epoch 35, loss: 0.884549
epoch 36, loss: 0.747642
epoch 37, loss: 0.850730
epoch 38, loss: 0.754572
epoch 39, loss: 0.825027
epoch 40, loss: 1.156675
epoch 41, loss: 0.843790
epoch 42, loss: 1.006930
epoch 43, loss: 0.620715
epoch 44, loss: 0.691962
epoch 45, loss: 0.652522
epoch 46, loss: 0.701444
epoch 47, loss: 1.267058
epoch 48, loss: 0.659544
epoch 49, loss: 1.243142
epoch 50, loss: 0.816031
epoch 51, loss: 0.613429
epoch 52, loss: 0.714893
epoch 53, loss: 0.626209
epoch 54, loss: 0.949622
epoch 55, loss: 0.684923
epoch 56, loss: 0.653516
```

```
epoch 57, loss: 0.683885
epoch 58, loss: 0.610425
epoch 59, loss: 0.616398
epoch 60, loss: 0.613604
epoch 61, loss: 0.683338
epoch 62, loss: 0.610494
epoch 63, loss: 0.651370
epoch 64, loss: 0.692909
epoch 65, loss: 0.740115
epoch 66, loss: 0.610659
epoch 67, loss: 0.610898
epoch 68, loss: 0.780961
epoch 69, loss: 0.611311
epoch 70, loss: 0.833891
epoch 71, loss: 0.615725
epoch 72, loss: 0.696121
epoch 73, loss: 0.612248
epoch 74, loss: 0.662748
epoch 75, loss: 0.637817
epoch 76, loss: 0.763421
epoch 77, loss: 0.755727
epoch 78, loss: 0.642584
epoch 79, loss: 0.638916
epoch 80, loss: 1.219784
epoch 81, loss: 0.621633
epoch 82, loss: 0.632739
epoch 83, loss: 0.662400
epoch 84, loss: 0.714247
epoch 85, loss: 1.428192
epoch 86, loss: 0.645986
epoch 87, loss: 0.616857
epoch 88, loss: 0.744783
epoch 89, loss: 0.901073
epoch 90, loss: 0.932895
epoch 91, loss: 0.610630
epoch 92, loss: 0.611691
epoch 93, loss: 0.657172
epoch 94, loss: 0.669262
epoch 95, loss: 0.641559
epoch 96, loss: 0.707285
epoch 97, loss: 0.617857
epoch 98, loss: 0.631607
epoch 99, loss: 0.655747
epoch 100, loss: 0.683684
```

```
In [6]: r = net[0].weight.data()[0]
        print("r = ", r.asscalar())
```

```
r =  42.12359
```

5 4. Entropy

Let's compute the *binary* entropy of a number of interesting data sources.

1. Assume that you're watching the output generated by a monkey at a typewriter. The monkey presses any of the 44 keys of the typewriter at random (you can assume that it has not discovered any special keys or the shift key yet). How many bits of randomness per character do you observe?
2. Unhappy with the monkey you replaced it by a drunk typesetter. It is able to generate words, albeit not coherently. Instead, it picks a random word out of a vocabulary of 2,000 words. Moreover, assume that the average length of a word is 4.5 letters in English. How many bits of randomness do you observe now?
3. Still unhappy with the result you replace the typesetter by a high quality language model. These can obtain perplexity numbers as low as 20 points per character. The perplexity is defined as a length normalized probability, i.e.

$$\mathrm{PPL}(x) = [p(x)]^{1/\mathrm{length}(x)}$$

In [7]:
```python
# part 1

probs = nd.ones((44,)) / 44

sum(-probs*nd.log(probs))/nd.log(nd.array([2]))
```

Out[7]:
```
[5.4594293]
<NDArray 1 @cpu(0)>
```

In [8]:
```python
# part 2 ?????

probs_2 = nd.ones((2000,))*(1/2000)#*(1/26)
sum(-probs_2 * nd.log(probs_2))/nd.log(nd.array([2])) / 4.5
```

Out[8]:
```
[2.436808]
<NDArray 1 @cpu(0)>
```

In [9]:
```python
# part 2 ?????

probs_3 = nd.ones((2000,))*(1/2000)#*(1/26)
h_word = sum(-probs_2 * nd.log(probs_2))

2**h_word
```

Out [9]:

```
[194.11934]
&lt;NDArray 1 @cpu(0)&gt;
```

$$\text{PPL}(x) = [p(x)]^{1/\text{length}(x)}$$

$$\text{PPL}(x) = 2^H[x]$$

$$4.5 * 20 \leq 2^H[x]$$

$$\frac{log(90)}{log(2)} \leq H[x]$$

then,
$\frac{H[x]}{log(2)}$ = randomness per word
$\frac{\left(\frac{H[x]}{log(2)}\right)}{4.5}$ = randomness per character

5.0.1 thus,

randomness per character = $\frac{\left(\frac{log(90)}{log(2)}\right)}{log(2)*4.5}$

In [90]:

```python
rpc = np.log2(90)/(np.log(2)*4.5)

print("randomness of character = ", rpc)
```

```
randomness of character =  2.0812809485009964
```

In [91]:

```python
# part 3

### assumptions ###
n_words = 2000
avg_word_len = 4.5

### randomness ###
```

5.0.2 a few of us on the class forum were confused about 4.3. we weren't sure what was being asked here so i wrote down my assumptions and solved for randomness per character.

https://discuss.mxnet.io/t/hw3-4-3/3168 (link to question)

5.0.3 i also used this source to find a formula relating perplexity of a word to entropy of a word.

- formula : $PPL(x) = \hat{H}$, where $\hat{H}$ = entropy of a word.
- i assumed that PPL(x) means perplexity of a word
- also found : randomness per word = randomness per character * length of word
- assumed : $\frac{\hat{H}}{log(2)}$ = randomness per word.

5.0.4 source:

http://www1.icsi.berkeley.edu/Speech/docs/HTKBook3.2/node188_mn.html

6 5. Wien's Approximation for the Temperature (bonus)

We will now abuse Gluon to estimate the temperature of a black body. The energy emanated from a black body is given by Wien's approximation.

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \exp\left(-\frac{hc}{\lambda kT}\right)$$

That is, the amount of energy depends on the fifth power of the wavelength λ and the temperature T of the body. The latter ensures a cutoff beyond a temperature-characteristic peak. Let us define this and plot it.

```
In [ ]: # Lightspeed
        c = 299792458
        # Planck's constant
        h = 6.62607004e-34
        # Boltzmann constant
        k = 1.38064852e-23
        # Wavelength scale (nanometers)
        lamscale = 1e-6
        # Pulling out all powers of 10 upfront
        p_out = 2 * h * c**2 / lamscale**5
        p_in = (h / k) * (c/lamscale)

        # Wien's law
        def wien(lam, t):
            return (p_out / lam**5) * nd.exp(-p_in / (lam * t))

        # Plot the radiance for a few different temperatures
        lam = nd.arange(0,100,0.01)
        for t in [10, 100, 150, 200, 250, 300, 350]:
            radiance = wien(lam, t)
            plt.plot(lam.asnumpy(), radiance.asnumpy(), label=('T=' + str(t) + 'K'))
        plt.legend()
        plt.show()
```

Next we assume that we are a fearless physicist measuring some data. Of course, we need to pretend that we don't really know the temperature. But we measure the radiation at a few wavelengths.

```
In [ ]: # real temperature is approximately 0C
        realtemp = 273
        # we observe at 3000nm up to 20,000nm wavelength
        wavelengths = nd.arange(3,20,2)
        # our infrared filters are pretty lousy ...
        delta = nd.random_normal(shape=(len(wavelengths))) * 1

        radiance = wien(wavelengths + delta,realtemp)
        plt.plot(wavelengths.asnumpy(), radiance.asnumpy(), label='measured')
        plt.plot(wavelengths.asnumpy(), wien(wavelengths, realtemp).asnumpy(), label='true')
        plt.legend()
        plt.show()
```

Use Gluon to estimate the real temperature based on the variables `wavelengths` and `radiance`.

- You can use Wien's law implementation `wien(lam,t)` as your forward model.
- Use the loss function $l(y, y') = (\log y - \log y')^2$ to measure accuracy.