1.1 EXERCISES

CONCEPTS

1. Give an example of each of the following:
   (a) A natural number
   (b) An integer that is not a natural number
   (c) A rational number that is not an integer
   (d) An irrational number

2. Complete each statement and name the property of real numbers you have used.
   (a) $ab =$ ______; ______ Property
   (b) $a + (b + c) =$ ______; ______ Property
   (c) $a(b + c) =$ ______; ______ Property

3. Express the set of real numbers between but not including 2 and 7 as follows.
   (a) In set-builder notation: ______
   (b) In interval notation: ______

4. The symbol $|x|$ stands for the ______ of the number x. If x is not 0, then the sign of $|x|$ is always ______.

5. The distance between a and b on the real line is $d(a, b) =$ ______. So the distance between -5 and 2 is ______.

6–8 ■ *Yes or No?* If *No*, give a reason. Assume that a and b are nonzero real numbers.

6. (a) Is the sum of two rational numbers always a rational number?
   (b) Is the sum of two irrational numbers always an irrational number?

7. (a) Is $a - b$ equal to $b - a$?
   (b) Is $-2(a - 5)$ equal to $-2a - 10$?

8. (a) Is the distance between any two different real numbers always positive?
   (b) Is the distance between a and b the same as the distance between b and a?

SKILLS

9–10 ■ **Real Numbers** List the elements of the given set that are
(a) natural numbers
(b) integers
(c) rational numbers
(d) irrational numbers

9. $\{-1.5, 0, \frac{5}{2}, \sqrt{7}, 2.71, -\pi, 3.1\overline{4}, 100, -8\}$

10. $\{1.3, 1.3333\ldots, \sqrt{5}, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}\}$

11–18 ■ **Properties of Real Numbers** State the property of real numbers being used.

11. $3 + 7 = 7 + 3$

12. $4(2 + 3) = (2 + 3)4$

13. $(x + 2y) + 3z = x + (2y + 3z)$

14. $2(A + B) = 2A + 2B$

15. $(5x + 1)3 = 15x + 3$

16. $(x + a)(x + b) = (x + a)x + (x + a)b$

17. $2x(3 + y) = (3 + y)2x$

18. $7(a + b + c) = 7(a + b) + 7c$

19–22 ■ **Properties of Real Numbers** Rewrite the expression using the given property of real numbers.

19. Commutative Property of Addition, $x + 3 =$

20. Associative Property of Multiplication, $7(3x) =$

21. Distributive Property, $4(A + B) =$

22. Distributive Property, $5x + 5y =$

23–28 ■ **Properties of Real Numbers** Use properties of real numbers to write the expression without parentheses.

23. $3(x + y)$
24. $(a - b)8$
25. $4(2m)$
26. $\frac{4}{3}(-6y)$
27. $-\frac{5}{2}(2x - 4y)$
28. $(3a)(b + c - 2d)$

29–32 ■ **Arithmetic Operations** Perform the indicated operations.

29. (a) $\frac{3}{10} + \frac{4}{15}$ (b) $\frac{1}{4} + \frac{1}{5}$

30. (a) $\frac{2}{3} - \frac{3}{5}$ (b) $1 + \frac{5}{8} - \frac{1}{6}$

31. (a) $\frac{2}{3}(6 - \frac{3}{2})$ (b) $(3 + \frac{1}{4})(1 - \frac{4}{5})$

32. (a) $\dfrac{2}{\frac{2}{3}} - \dfrac{\frac{2}{3}}{2}$ (b) $\dfrac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}}$

33–34 ■ **Inequalities** Place the correct symbol ($<$, $>$, or $=$) in the space.

33. (a) $3 \quad \frac{7}{2}$ (b) $-3 \quad -\frac{7}{2}$ (c) $3.5 \quad \frac{7}{2}$

34. (a) $\frac{2}{3} \quad 0.67$ (b) $\frac{2}{3} \quad -0.67$
    (c) $|0.67| \quad |-0.67|$

35–38 ■ **Inequalities** State whether each inequality is true or false.

35. (a) $-3 < -4$ (b) $3 < 4$

36. (a) $\sqrt{3} > 1.7325$ (b) $1.732 \geq \sqrt{3}$

37. (a) $\frac{10}{2} \geq 5$ (b) $\frac{6}{10} \geq \frac{5}{6}$

38. (a) $\frac{7}{11} \geq \frac{8}{13}$ (b) $-\frac{3}{5} > -\frac{3}{4}$

39–40 ■ **Inequalities** Write each statement in terms of inequalities.

39. (a) x is positive.
    (b) t is less than 4.
    (c) a is greater than or equal to π.
    (d) x is less than $\frac{1}{3}$ and is greater than -5.
    (e) The distance from p to 3 is at most 5.

40. **(a)** y is negative.
(b) z is greater than 1.
(c) b is at most 8.
(d) w is positive and is less than or equal to 17.
(e) y is at least 2 units from π.

41–44 ■ Sets Find the indicated set if

$$A = \{1, 2, 3, 4, 5, 6, 7\} \qquad B = \{2, 4, 6, 8\}$$
$$C = \{7, 8, 9, 10\}$$

41. (a) $A \cup B$ **(b)** $A \cap B$

42. (a) $B \cup C$ **(b)** $B \cap C$

43. (a) $A \cup C$ **(b)** $A \cap C$

44. (a) $A \cup B \cup C$ **(b)** $A \cap B \cap C$

45–46 ■ Sets Find the indicated set if

$$A = \{x \mid x \geq -2\} \qquad B = \{x \mid x < 4\}$$
$$C = \{x \mid -1 < x \leq 5\}$$

45. (a) $B \cup C$ **(b)** $B \cap C$

46. (a) $A \cap C$ **(b)** $A \cap B$

47–52 ■ Intervals Express the interval in terms of inequalities, and then graph the interval.

47. $(-3, 0)$ **48.** $(2, 8]$

49. $[2, 8)$ **50.** $\left[-6, -\frac{1}{2}\right]$

51. $[2, \infty)$ **52.** $(-\infty, 1)$

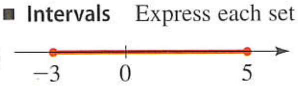
53–58 ■ Intervals Express the inequality in interval notation, and then graph the corresponding interval.

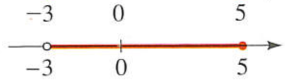
53. $x \leq 1$ **54.** $1 \leq x \leq 2$

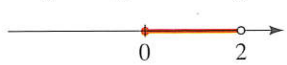
55. $-2 < x \leq 1$ **56.** $x \geq -5$

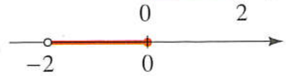
57. $x > -1$ **58.** $-5 < x < 2$

59–60 ■ Intervals Express each set in interval notation.

59. (a) 

(b) 

60. (a) 

(b) 

61–66 ■ Intervals Graph the set.

61. $(-2, 0) \cup (-1, 1)$ **62.** $(-2, 0) \cap (-1, 1)$

63. $[-4, 6] \cap [0, 8)$ **64.** $[-4, 6) \cup [0, 8)$

65. $(-\infty, -4) \cup (4, \infty)$ **66.** $(-\infty, 6] \cap (2, 10)$

67–72 ■ Absolute Value Evaluate each expression.

67. (a) $|100|$ **(b)** $|-73|$

68. (a) $|\sqrt{5} - 5|$ **(b)** $|10 - \pi|$

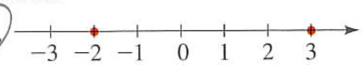
69. (a) $\big||-6| - |-4|\big|$ **(b)** $\dfrac{-1}{|-1|}$

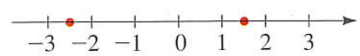
70. (a) $\big|2 - |-12|\big|$ **(b)** $-1 - \big|1 - |-1|\big|$

71. (a) $|(-2) \cdot 6|$ **(b)** $|(-\frac{1}{3})(-15)|$

72. (a) $\left|\dfrac{-6}{24}\right|$ **(b)** $\left|\dfrac{7 - 12}{12 - 7}\right|$

73–76 ■ Distance Find the distance between the given numbers.

73. 

74. 

75. (a) 2 and 17 **(b)** -3 and 21 **(c)** $\frac{11}{8}$ and $-\frac{3}{10}$

76. (a) $\frac{7}{15}$ and $-\frac{1}{21}$ **(b)** -38 and -57 **(c)** -2.6 and -1.8

SKILLS Plus

77–78 ■ Repeating Decimal Express each repeating decimal as a fraction. (See the margin note on page 3.)

77. (a) $0.\overline{7}$ **(b)** $0.2\overline{8}$ **(c)** $0.\overline{57}$

78. (a) $5.\overline{23}$ **(b)** $1.3\overline{7}$ **(c)** $2.1\overline{35}$

79–82 ■ Simplifying Absolute Value Express the quantity without using absolute value.

79. $|\pi - 3|$ **80.** $|1 - \sqrt{2}|$

81. $|a - b|$, where $a < b$

82. $a + b + |a - b|$, where $a < b$

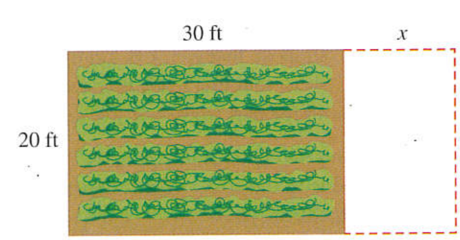
83–84 ■ Signs of Numbers Let a, b, and c be real numbers such that $a > 0$, $b < 0$, and $c < 0$. Find the sign of each expression.

83. (a) $-a$ **(b)** bc **(c)** $a - b$ **(d)** $ab + ac$

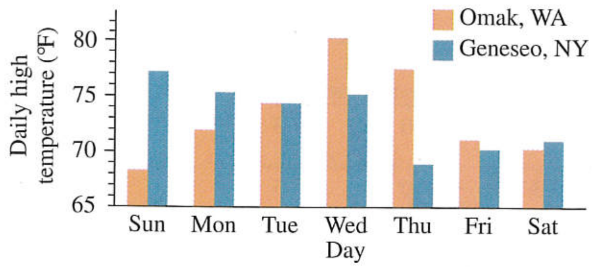
84. (a) $-b$ **(b)** $a + bc$ **(c)** $c - a$ **(d)** ab^2

APPLICATIONS

85. Area of a Garden Mary's backyard vegetable garden measures 20 ft by 30 ft, so its area is $20 \times 30 = 600$ ft^2. She decides to make it longer, as shown in the figure, so that the area increases to $A = 20(30 + x)$. Which property of real numbers tells us that the new area can also be written $A = 600 + 20x$?



86. Temperature Variation The bar graph shows the daily high temperatures for Omak, Washington, and Geneseo, New York, during a certain week in June. Let T_O represent the temperature in Omak and T_G the temperature in Geneseo. Calculate $T_O - T_G$ and $|T_O - T_G|$ for each day shown. Which of these two values gives more information?

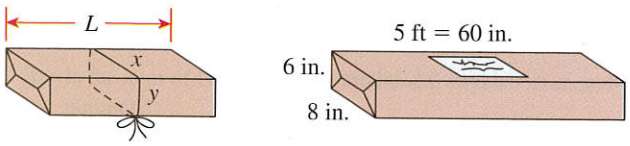


87. Mailing a Package The post office will accept only packages for which the length plus the "girth" (distance around) is no more than 108 in. Thus for the package in the figure, we must have

$$L + 2(x + y) \le 108$$

(a) Will the post office accept a package that is 6 in. wide, 8 in. deep, and 5 ft long? What about a package that measures 2 ft by 2 ft by 4 ft?

(b) What is the greatest acceptable length for a package that has a square base measuring 9 in. by 9 in.?



DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

88. DISCUSS: Sums and Products of Rational and Irrational Numbers Explain why the sum, the difference, and the product of two rational numbers are rational numbers. Is the product of two irrational numbers necessarily irrational? What about the sum?

89. DISCOVER ■ PROVE: Combining Rational and Irrational Numbers Is $\frac{1}{2} + \sqrt{2}$ rational or irrational? Is $\frac{1}{2} \cdot \sqrt{2}$ rational or irrational? Experiment with sums and products of other rational and irrational numbers. Prove the following.

(a) The sum of a rational number r and an irrational number t is irrational.

(b) The product of a rational number r and an irrational number t is irrational.

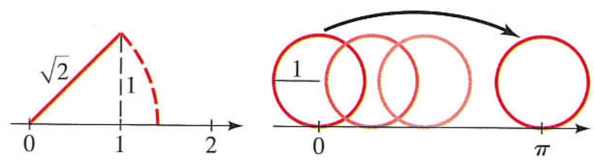
[*Hint:* For part (a), suppose that $r + t$ is a rational number q, that is, $r + t = q$. Show that this leads to a contradiction. Use similar reasoning for part (b).]

90. DISCOVER: Limiting Behavior of Reciprocals Complete the tables. What happens to the size of the fraction $1/x$ as x gets large? As x gets small?

x	$1/x$
1	
2	
10	
100	
1000	

x	$1/x$
1.0	
0.5	
0.1	
0.01	
0.001	

91. DISCOVER: Locating Irrational Numbers on the Real Line Using the figures below, explain how to locate the point $\sqrt{2}$ on a number line. Can you locate $\sqrt{5}$ by a similar method? How can the circle shown in the figure help us to locate π on a number line? List some other irrational numbers that you can locate on a number line.



92. PROVE: Maximum and Minimum Formulas Let $\max(a, b)$ denote the maximum and $\min(a, b)$ denote the minimum of the real numbers a and b. For example, $\max(2, 5) = 5$ and $\min(-1, -2) = -2$.

(a) Prove that $\max(a, b) = \dfrac{a + b + |a - b|}{2}$.

(b) Prove that $\min(a, b) = \dfrac{a + b - |a - b|}{2}$.

[*Hint:* Take cases and write these expressions without absolute values. See Exercises 81 and 82.]

93. WRITE: Real Numbers in the Real World Write a paragraph describing different real-world situations in which you would use natural numbers, integers, rational numbers, and irrational numbers. Give examples for each type of situation.

94. DISCUSS: Commutative and Noncommutative Operations We have learned that addition and multiplication are both commutative operations.

(a) Is subtraction commutative?

(b) Is division of nonzero real numbers commutative?

(c) Are the actions of putting on your socks and putting on your shoes commutative?

(d) Are the actions of putting on your hat and putting on your coat commutative?

(e) Are the actions of washing laundry and drying it commutative?

95. PROVE: Triangle Inequality We prove Property 5 of absolute values, the Triangle Inequality:

$$|x + y| \le |x| + |y|$$

(a) Verify that the Triangle Inequality holds for $x = 2$ and $y = 3$, for $x = -2$ and $y = -3$, and for $x = -2$ and $y = 3$.

(b) Prove that the Triangle Inequality is true for all real numbers x and y. [*Hint:* Take cases.]