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# Ch 1.1 - Real Numbers

In this class, we will mainly be interested in real #s.

The **real numbers** consist of

* **Natural numbers** (counting #s) $1, 2, 3, 4$ etc

* **Integers** (positive and negative whole numbers, and zero) $-4, -3, -2, -1, 0, 1, 2, 3, 4$ etc

  $-\frac{5}{2}$ $\frac{1}{2}$ $\frac{4}{3}$ $\pi$

* **Rational #s**

  can be written as

  $$r = \frac{m}{n}, \quad m, n: \text{integers}, \ n \neq 0$$

  (decimal representation terminates or repeats)

* **Irrational #s**

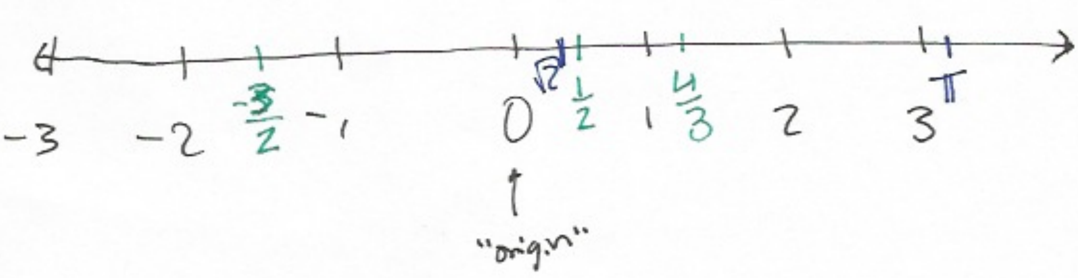
  All the real numbers that are not rational

ex: $\pi$, $\sqrt{2}$, $e$, golden ratio etc.

(decimal representations do not terminate or repeat).

Graphical POV:

We can represent the real numbers on the real number line:



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The "size" of the numbers increases from left to right:

$a < b$: a is less than b

a is left of b


$c > d$: c is greater than d

c is right of d


## Intervals on the number line:

Set of all real #s b/twn two points

if endpoints are included, it is called a closed interval

if endpoints are not included, it is called an open interval

| Interval notation | Set Notation | Graph on Number Line |
|---|---|---|
| $[a, b]$ | $\{x \mid a \le x \le b\}$ |   |
| $[a, b)$ | $\{x \mid a \le x < b\}$ |   |
| $(a, b]$ | $\{x \mid a < x \le b\}$ |   |
| $(a, b)$ | $\{x \mid a < x < b\}$ |   |

Note: $\pm\infty$ are never "included"

$[2, \infty)$ $\{x \mid 2 \le x < \infty\}$


to infinity (and beyond!)

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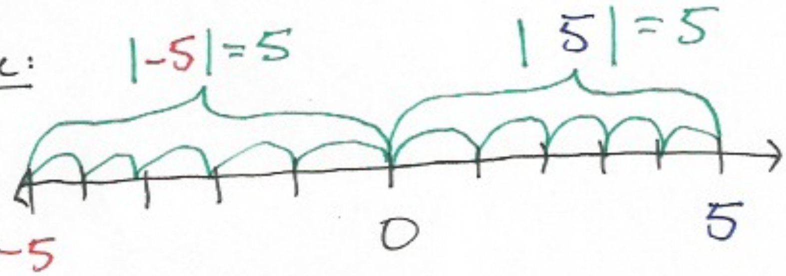
## Absolute Value

$|a|$ : The distance from a to zero on the real number line. For any real # a:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example:




$|5| = 5$

$|-5| = 5$

* see pg 9 for properties of absolute value *

Discuss:

(1) Graph the Intervals:

a) $(-8, -1)$


b) $[-1, 10)$

c) $[-5, \infty)$

(2) Fill in the blank with <, >, or = :

a) $|5|$ __<__ $|-6|$

b) $|3|\cdot|5|$ __=__ $|3(-5)|$

c) $|6-(-4)|$ __=__ $|-4-6|$

$|10|$ $\qquad$ $|-10|$

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# Properties of Real Numbers

## Commutative Properties

order doesn't matter for addition + mult.

$a+b=b+a$ , $ab=ba$

## Associative Properties

$(a+b)+c=a+(b+c)$ , $(ab)c=a(bc)$

grouping doesn't matter for addition + mult.

## Distributive Property

connects addition and mult.

$a(b+c)=ab+ac$ and $(b+c)a=ba+ca$

(See nice proof by picture on pg 3!)

**Example:** Write without parentheses:

$(3a)(b+c-2d)$

$= 3ab + 3ac - 6ad$

## Order of operations and parentheses

* work separately above and below any fraction bars
* If there are "nested" parentheses, work from the inside toward the outside!

| | |
|---|---|
| Please | Parentheses |
| Excuse | Exponents (and roots) |
| My Dear | Mult + Div from Left to Right |
| Aunt Sally | Add + Sub from Left to Right |

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Use order of operations to decode "run-on sentence" expressions like:

$$9 - 3\times 4 - (-2)*(3-5) + 2*6 \div 3 = -3$$

or perform calculations like

$$\frac{[4-(1+2(6-3)^2)]^2}{2-3} = -225$$

* Read about properties of negative #s on page 4.

and

* Properties of Fractions on page 5.

<u>Discuss</u>

a) $\frac{5}{5}\cdot\frac{1}{3} + \frac{2}{5}\cdot\frac{3}{3} = \frac{5+6}{15} = \frac{11}{15}$

b) $\frac{d}{d}\cdot\frac{a}{b} - \frac{c}{d}\cdot\frac{b}{b} = \frac{ad-bc}{bd}$

c) $24 \div \frac{2}{3} = 24\cdot\frac{3}{2} = \frac{24\cdot 3}{2} = 12\cdot 3 = 36$

# 1.2 - Exponents and Radicals

If $a$ is any real number and $n$ is a positive integer, then the nth power of a is:

$$\underset{\text{base}}{a}^{\overset{\text{exponent}}{n}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a}$$

Examples:

$3^2 = 3 \cdot 3 = 9$ , $1.8^3 = (1.8)(1.8)(1.8) = 5.832$

$(-3)^2 = (-3)(-3) = 9$

Be careful:

(9) $(-2)^4 = \underline{16}$ $\qquad -(2)^4 = \underline{-16}$

(10) $3(-2)^5 = \underline{3 \cdot (-32) = -96}$ $\qquad (\cancel{3} \cdot (-2))^5 = \underline{(-\cancel{6})^5 = -7776}$

* See handout for Laws of Exponents *

which are used to simplify expressions containing exponents.

If $n$ is any positive integer, then the principal nth root of a is defined such that

$$a^{1/n} = \sqrt[n]{a} = b \quad \text{means that} \quad b^n = a$$

If $n$ is even, we must have $a \geq 0$ and $b \geq 0$.

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Examples

14 a) $\sqrt{64} = \sqrt{8^2} = \boxed{8}$

15 b) $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = (-4)^{3/3} = \boxed{-4}$

16 c) $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = \boxed{-2}$

17 d) $\left(-\frac{27}{8}\right)^{2/3} = \left(-\frac{27^{1/3}}{8^{1/3}}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

Now work through the problems on the handout.

If you have extra time, do:

1.1 # 77

1.2 # 101

Then start on homework

# MATH 1021-04: Precalculus - Algebra

## Sections 1.1 & 1.2 Handout

### Warm-up:

Please try the following exercises:

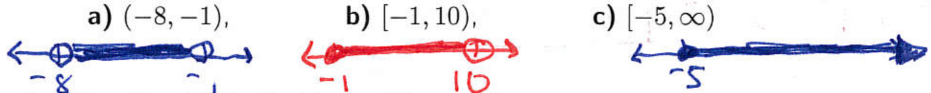
a) Expand: $6(t+4)$ = $6t + 24$ (Distribute!)

b) Evaluate the expression shown in **Problem 3** below.

### Section 1.1 - The Real Numbers:

**The Number Line and Absolute Value:**

**Problem 1.** Graph the following intervals on a number line:
**a)** $(-8,-1)$, **b)** $[-1,10)$, **c)** $[-5,\infty)$




**Problem 2.** Fill in the blank with one of <, >, or =:
**a)** $|5|$ $\underline{<}$ $|-6|$, **b)**$|3|\cdot|-5|$ $\underline{=}$ $|3(-5)|$, **b)**$|6-(-4)|$ $\underline{=}$ $|-4-6|$

$|10|$ $|-10|$

**Order of Operations & Parentheses:** Evaluate the following expressions:

**Problem 3.** $9-3\times 4-(-2)*(3-5)+2*6\div 3$ $= \boxed{-3}$

**Problem 4.** $3(2+4(6+1)) = 3(2+4(7)) = 3(2+28) = 3(30) = \boxed{90}$

**Problem 5.** $\dfrac{[4-(1+2(6-3)^2))]^2}{2-3} = \dfrac{[4-(1+2(3)^2)]^2}{-1} = \dfrac{[4-19]^2}{-1} = \dfrac{[-15]^2}{-1} = \boxed{-225}$

**Properties of Fractions:** Evaluate the following expressions:

**Problem 6.** $\frac{5}{5}\cdot\frac{1}{3}+\frac{2}{5}\cdot\frac{3}{3} = \frac{5}{15}+\frac{6}{15} = \boxed{\frac{11}{15}}$

**Problem 7.** $\frac{d}{d}\cdot\frac{a}{b}-\frac{c}{d}\cdot\frac{b}{b} = \boxed{\frac{ad-bc}{bd}}$

**Problem 8.** $24\div\frac{2}{3} = \cancel{24}^{12} * \frac{3}{\cancel{2}} = 12*3 = \boxed{36}$

## Section 1.2 - Exponents and Radicals:

**Definition & Properties of Exponents:** Evaluate the following Expressions:

**Problem 9.** $(-2)^4 =$ 16 $\quad -(2)^4 =$ -16

**Problem 10.** $3(-2)^5 =$ -96 $\quad (3\cdot(-2))^5 =$ -7776

$3(-32) = -96 \qquad (-6)^5 = -7776$

Write as an expression with a single exponent:

**Problem 11.** $2^5 \cdot 2^2 = 2^{5+2} = \boxed{2^7}$

**Problem 12.** $(3^2)^6 = 3^{2*6} = \boxed{3^{12}}$

**Problem 13.** $(9^{50} \cdot 9^2)^{10} = (9^{52})^{10} = \boxed{9^{520}}$

**Radicals:** Evaluate the following Expressions:

**Problem 14.** $\sqrt{64} = \sqrt{8^2} = \boxed{8}$

**Problem 15.** $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = \boxed{-4}$

**Problem 16.** $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = \boxed{-2}$

**Problem 17.** $\left(-\frac{27}{8}\right)^{2/3} = \left(-\frac{27^{1/3}}{8^{1/3}}\right)^2 = \left(-\frac{3}{2}\right)^2 = \boxed{\frac{9}{4}}$

**Work the Problems on the Laws of Exponents Handout (next page →)**

## Laws of Exponents:

$$\boxed{a^0 = 1 \quad , \quad a^{-n} = \frac{1}{a^n}}$$

① $a^m a^n = a^{m+n}$ ← to multiply powers of like bases, add exponen

② $\frac{a^m}{a^n} = a^{m-n}$ ← to divide powers of like bases, subtract

③ $(a^m)^n = a^{mn}$ ← to raise a power to a power, multiply

④ $(ab)^n = a^n b^n$ ← raise each factor to the power

⑤ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ← raise top and bottom to power

distribute the exponent!

⑥ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ ← to raise a number to a negative power, invert and change the sign of the power

⑦ $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

## Properties of $n^{th}$ roots:

① $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

② $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

③ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

④ $\sqrt[n]{a^n} = a$ if $n$ is odd

⑤ $\sqrt[n]{a^n} = |a|$ if $n$ is even

## Simplify Each Expression

a) $x^8 x^2 = x^{8+2} = \boxed{x^{10}}$

b) $x^2 x^{-6} = x^{2-6} = x^{-4} = \boxed{\frac{1}{x^4}}$

c) $\left(\frac{x^4 z^2}{4y^5}\right)\left(\frac{2x^3y^2}{z^3}\right)^2 = \left(\frac{x^4z^2}{\cancel{4}y^5}\right)\left(\frac{\cancel{4}x^6y^4}{z^6}\right)$

$= \frac{4}{4}x^{4+6}y^{4-5}z^{2-6}$

$= x^{10}y^{-1}z^{-4}$

$= \boxed{\frac{x^{10}}{yz^4}}$

d) $\left(\frac{y}{5x^{-2}}\right)^{-3} = \frac{y^{-3}}{5^{-3}x^6}$

$= \frac{5^3}{y^3x^6}$

$= \boxed{\frac{125}{y^3x^6}}$

e) $\frac{s^{3/2}}{s^{1/2}}$

$= s^{3/2-1/2} = s^{2/2} = \boxed{s}$