MATH 1331 (Cole) **Test 1 Review Problems** Fall 2023

Content Covered: Sections 1.1-1.3, 2.2-2.3*

Note that detailed lists of objectives from each section covered can be found on Canvas.
***Specific content from 2.3 that is not included: Squeeze Theorem**

Concept Check Questions:

1. **Function Definitions:**
   (a) What is a function? What are its domain and range?
   (b) What is the graph of a function?
   (c) How can you tell whether a given curve is a graph of a function?
2. Discuss four ways of representing a function. Illustrate your discussion with examples.
3. Without using a graphing calculator/other technology, sketch the graphs of the following functions on the same axes.
   (a) $f(x) = x$
   (b) $g(x) = x^2$
   (c) $h(x) = x^3$
   (d) $j(x) = x^4$
4. Without using a graphing calculator/other technology, draw a rough sketch of each of the following functions.
   (a) $f(x = \sin(x)$
   (b) $g(x) = \tan(x)$
   (c) $h(x) = \frac{1}{x}$
   (d) $j(x) = |x|$
   (e) $k(x) = \sqrt{x}$
5. How is the function $f \circ g$ defined?
6. Suppose the graph of f is given. Write an equation for each of the graphs that are obtained by the following transformations.
   (a) Shift 3 units up
   (b) Shift 3 units down
   (c) Shift 3 units left
   (d) Shift 3 units right
   (e) Reflect across the $x-$axis
   (f) Reflect across the $y-$axis
   (g) Stretch vertically by a factor of 3
   (h) Shrink vertically by a factor of 3
   (i) Stretch horizontally by a factor of 3
   (j) Shrink vertically by a factor of 3
7. Explain what each of the following means and illustrate with a sketch.
   (a) $\lim_{x \to a} f(x) = L$

(b) $\lim_{x \to a^+} f(x) = L$

(c) $\lim_{x \to a^-} f(x) = L$

(d) $\lim_{x \to a} f(x) = \infty$

8. Describe several ways in which a limit can fail to exist. Illustrate with sketches.
9. State the following Limit Laws:
   (a) Sum Law
   (b) Difference Law
   (c) Constant Multiple Law
   (d) Product Law
   (e) Quotient Law
   (f) Power Law

(T/F)+E:

Answer the following questions **TRUE** or **FALSE**. *You must justify your answer with a complete sentence* **explaining** *why the answer is either* **TRUE** *or* **FALSE**
Note: **(T/F) + E**: represents a choice of either (**T**rue or **F**alse) plus an **E**xplanation for your choice.

1. If f is a function, then $f(a+b) = f(a) + f(b)$.
2. If $f(a) = f(b)$, then $a = b$.
3. If f is a function, then $f(2x) = 2f(x)$.
4. A vertical line intersects the graph of a function at most once.
5. If f and g are functions, then $f \circ g = g \circ f$.
6. $\lim_{x \to 4} \left(\dfrac{2x}{x-4} - \dfrac{8}{x-4} \right) = \lim_{x \to 4} \dfrac{2x}{x-4} - \lim_{x \to 4} \dfrac{8}{x-4}$
7. $\lim_{x \to 1} \dfrac{x-3}{x^2+2x-4} = \dfrac{\lim_{x \to 1}(x-3)}{\lim_{x \to 1}(x^2+2x-4)}$
8. If $\lim_{x \to 5}[f(x)g(x)]$ exists, then the limit must be $f(5)g(5)$.
9. If p is a polynomial, then $\lim_{x \to c} p(x) = p(c)$.

Selected Review Problems:

Here are some additional review problems from the material covered by Test 1. **This does not represent a practice test!** There may be some types of problems on the test that are not listed below. The actual test will be shorter than this list!

1. Let f be the function whose graph is given in Figure 1.
   (a) Estimate the value of $f(2)$.

(b) Estimate the value(s) of x such that $f(x) = 3$.

(c) State the domain of f.

(d) State the range of f.

(e) On what interval is f increasing?

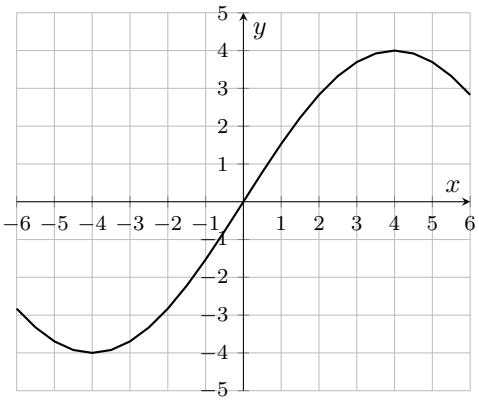

Figure 1: Graph for Review Problem 1.

2. Determine whether each cure shown in Figure 2 is the graph of a function. If it is, state the domain and range of the function.

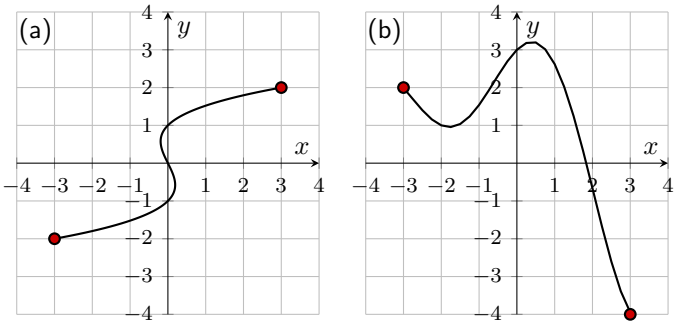

Figure 2: Graph for Review Problem 2.

3. Find the domain and range of the following functions. Write your answer in interval notation.

   (a) $g(x) = \sqrt{16 - x^2}$

   (b) $h(x) = 1 + \sin(x)$

   (c) $k(x) = \tan(2x)$

4. Suppose that the graph of f is given. Describe how the graphs of the following functions can be obtained from the graph of f.

   (a) $y = f(x) + 6$

   (b) $y = f(x + 6)$

   (c) $y = 1 + 2f(x)$

   (d) $y = f(x - 2) - 2$

5. The graph of f is shown in Figure 3. Draw graphs of the following functions.

(a) $y = f(x-1)$
(b) $y = -f(x)$
(c) $y = 1 + f(x)$
(d) $y = \frac{1}{2}f(x) - 1$

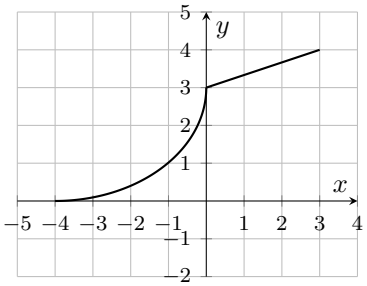

Figure 3: Graph for Review Problem 5.

6. If $f(x) = \sqrt{x}$ and $g(x) = \cos(x)$, find the following functions:
   (a) $f \circ g$
   (b) $g \circ f$
   (c) $f \circ f$
   (d) $g \circ g$
7. Express the function $F(x) = \frac{1}{\sqrt{x+\sqrt{x}}}$ as a composition of three functions.
8. The graph of f is given in Figure 4. Find each limit, or explain why it does not exist.
   (a) $\lim_{x \to 2^+} f(x)$
   (b) $\lim_{x \to -3^+} f(x)$
   (c) $\lim_{x \to -3} f(x)$
   (d) $\lim_{x \to 4} f(x)$
   (e) $\lim_{x \to 0} f(x)$
   (f) $\lim_{x \to 2^-} f(x)$
9. Find the following limits.
   (a) $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$
   (b) $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$
   (c) $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{x - 16}$
   (d) $\lim_{h \to 0} \dfrac{(h+1)^2 - 1}{h}$

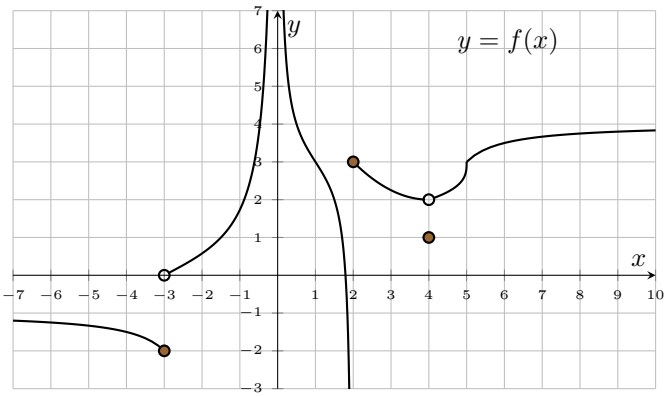

Figure 4: Graph for Review Problem 8.

10. Evaluate each limit, if it exists. If it does not exist, explain why.

$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 3 - x, & 0 \leq x < 3 \\ (x-3)^2, & x > 3 \end{cases}$$

(a) $\lim_{x \to 0^+} f(x)$

(b) $\lim_{x \to 0^-} f(x)$

(c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 3^-} f(x)$

(e) $\lim_{x \to 3^+} f(x)$

(f) $\lim_{x \to 3} f(x)$