MATH 2330: Multivariable Calculus

Section 5.6: Applications of Double Integrals

Key Concept:

Given a **density function** $\rho(x,y)$ that tells how much $\dfrac{\text{"stuff"}}{\text{unit area}}$ there is at a given point in a region D, then
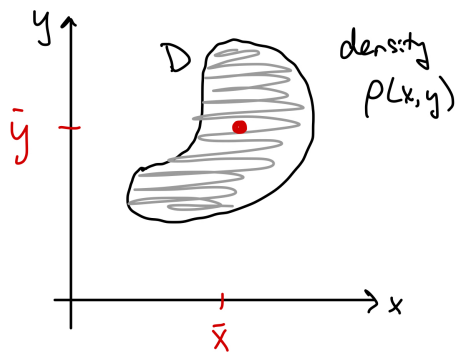
$$\iint_D \rho(x,y) \; dA$$

calculates the "total amount of stuff" within region D.

Example: If $u(x,y)$ represents the population density in region D, then $\iint_D u(x,y) \; dA$ gives the total population in region D.

Center of Mass of a Lamina:

mass density function: $\rho(x,y)$ has units $\dfrac{\text{mass}}{\text{area}}$

total mass of the lamina: $m = \iint_D \rho(x,y) \; dA$

Center of mass: $(\bar{x}, \bar{y})$

$\bar{x} = \dfrac{1}{m} \iint_D \rho(x,y)x \; dA$

$\bar{y} = \dfrac{1}{m} \iint_D \rho(x,y)y \; dA$

Example:

Find the center of mass of the lamina D that has density function $\rho(x, y) = x^2$, where D is the triangular region bounded by $x = 0, \quad y = x, \quad 2x + y = 6.$