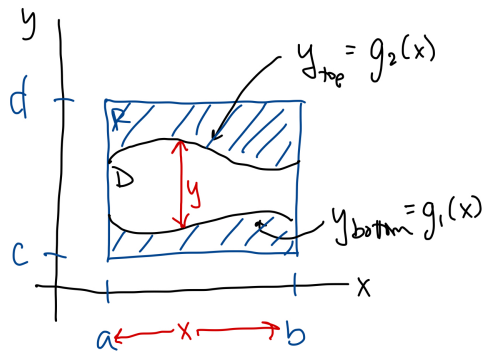
MATH 2330: Multivariable Calculus

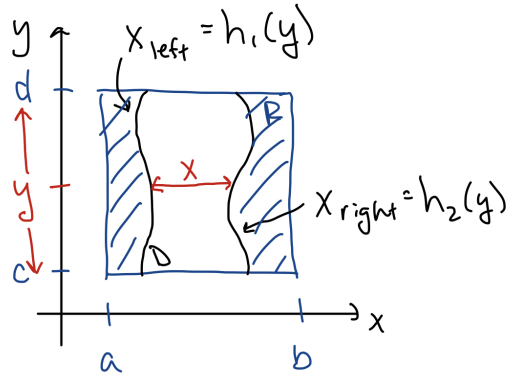
Section 5.2: Double Integrals over General Regions

Vertically Simple or Type I Regions:



$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

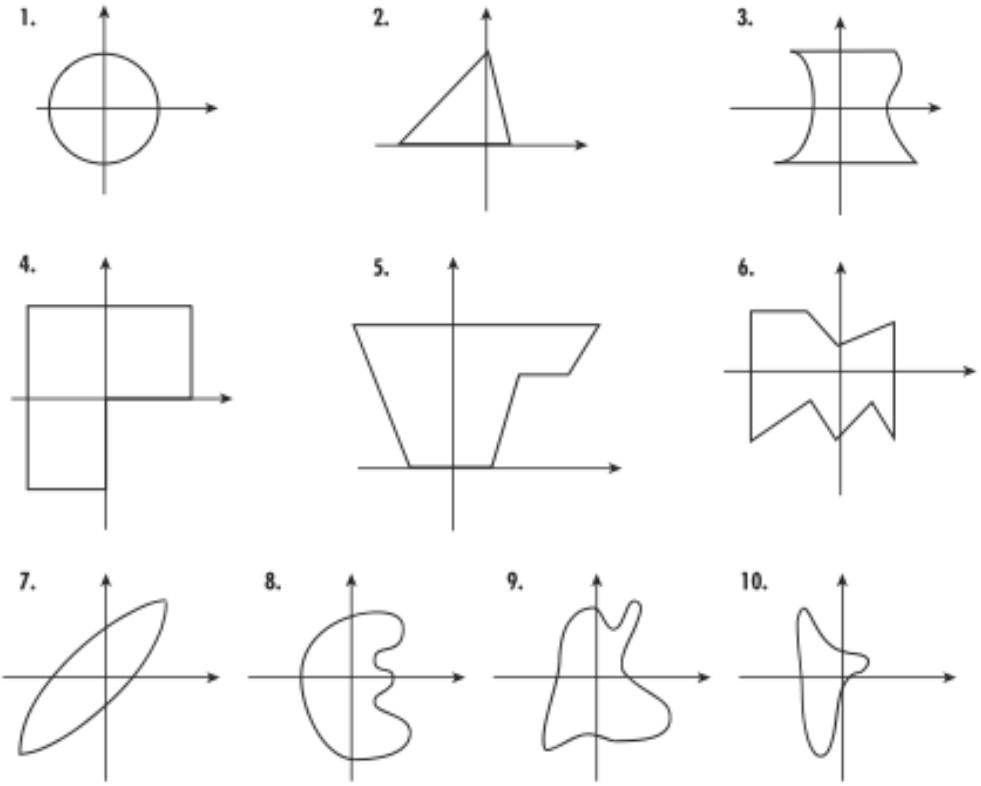
Horizontally Simple or Type II Regions:



$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

Type I or Type II?

Classify each of the following regions as type I, type II, both, or neither.



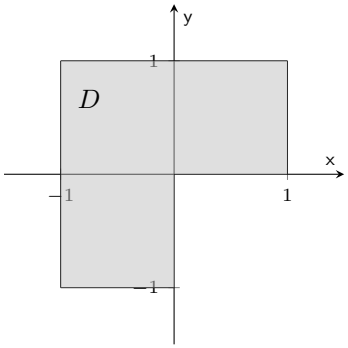
Examples:

Example 1: Find the volume of the region under $f(x,y) = 4x + 10y$ above the region D in the $xy-$plane that is bounded below by $y = x$ and above by $y = x^2$, to the left by $x = 1$ and to the right by $x = 2$.

Example 2: Evaluate $\iint_D x^2 + y^2 \, dA$ where D is the region in the $xy-$plane bounded by $x = y$ and $x = y^2$.

Example 3: Evaluate $\int_0^1 \int_x^1 e^{y^2} \, dy \, dx$.

Example 4: Evaluate $\iint_D x + y \, dA$ for the region D shaded below.

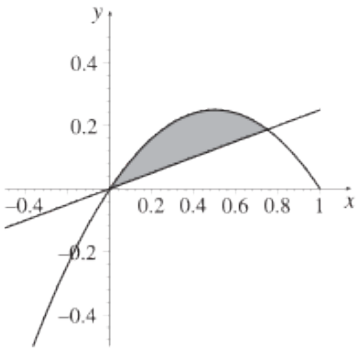


Group Work:

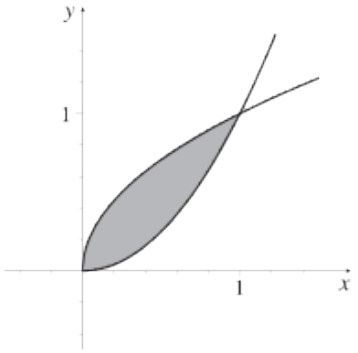
Fun with Double Integration

1. Write double integrals that represent the following areas.

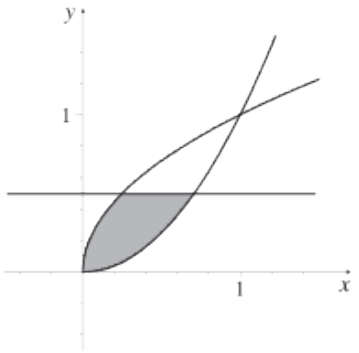
(a) The area enclosed by the curve $y = x - x^2$ and the line $y = \frac{x}{4}$



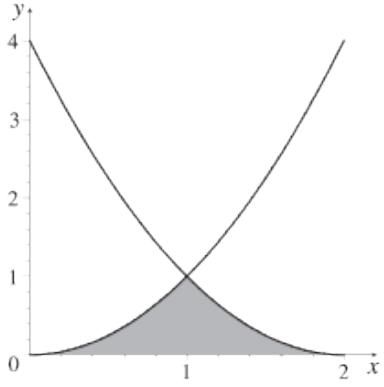
(b) The area enclosed by the curves $y = \sqrt{x}$ and $\sqrt{y} = x$



(c) The area enclosed by the curves $y = \sqrt{x}$ and $\sqrt{y} = x$, and the line $y = \frac{1}{2}$



(d) The area enclosed by the curves $y = x^2$ and $y = -x - 2$, and the line $y = 0$



2. What solid region of $\mathbb{R}^3$ do you think is represented by $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1 - x^2 - y^2}\, dy\, dx$?