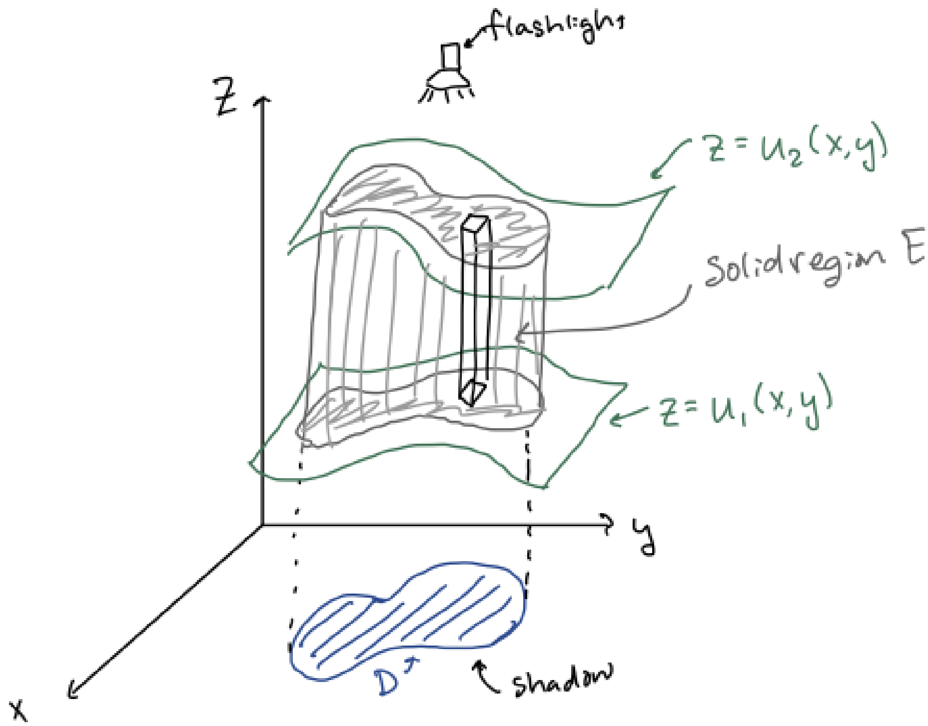
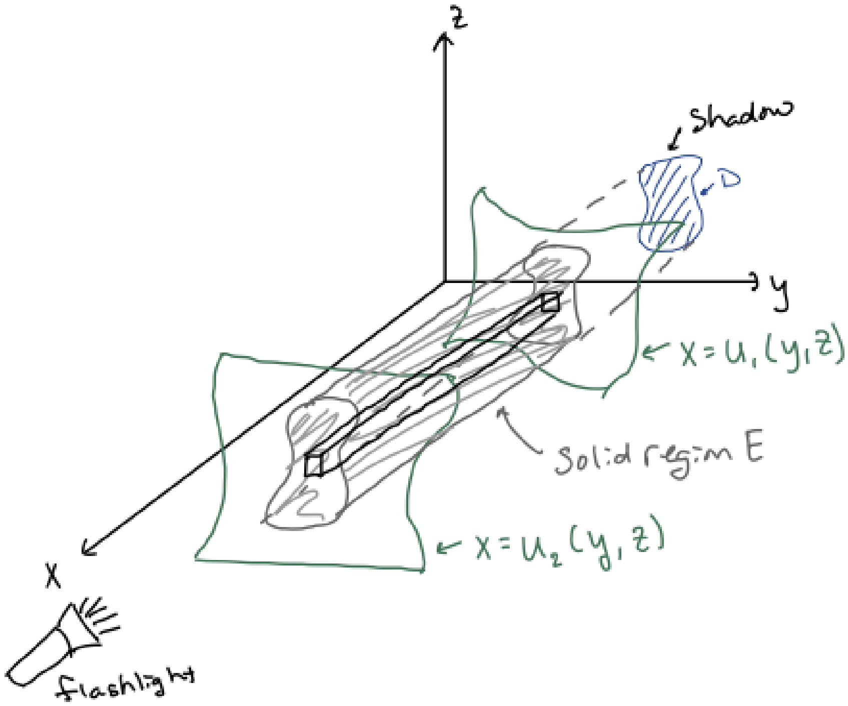
z-Simple or Type 1 Solid Regions:



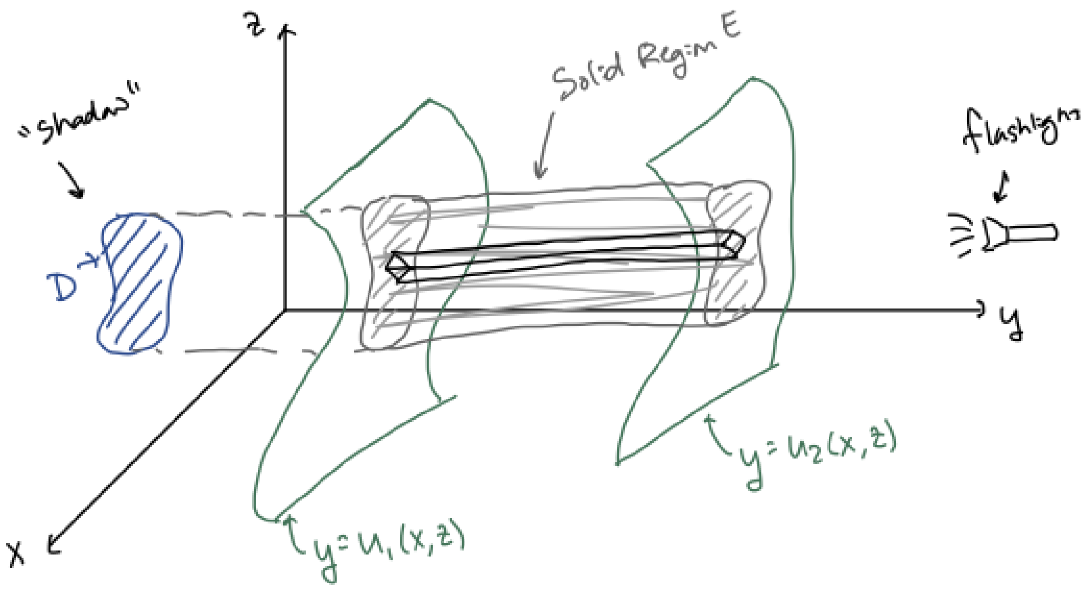
$$\iiint_E f(x,y,z)\ dV = \iint_D \left(\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\ dz \right) dA$$

x-Simple or Type 2 Solid Regions:



$$\iiint_E f(x,y,z)\ dV = \iint_D \left(\int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\ dx \right) dA$$

y-Simple or Type 3 Solid Regions:



$$\iiint_E f(x,y,z)\,dV = \iint_D \left(\int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy \right) dA$$

Triple Integral Set-up Key Ideas:

First integrate over the "height" of the solid region E.

Then set up a double integral over D, the "shadow" of the solid region in the remaining coordinate plane.

1. Sketch two things:
   Solid Region E & "Shadow" D
2. Set up the iterated integral to be as easy as possible
3. The final answer should be a *number*,
   so the iterated integral can have *at most*:
   inner limits: two variables
   middle limits: one variable
   outer limits: no variables

Example 1: Set up a triple integral to find the volume of the solid region E bounded by:
$z = 3x^2, \qquad z = 4 - x^2, \qquad y = 0, \qquad z + y = 6$

Example 2: Set up a triple integral to find the volume of the solid region enclosed by the *cylinders* $z = x^2, \qquad y = x^2$ and the *planes* $z = 0, \qquad y = 4.$

Example 3: Set up a triple integral to find the volume of the "Ice Cream Cone" solid region bounded above by $x^2 + y^2 + z^2 = 4$ and below by $x^2 + y^2 = z^2$, where $z \geq 0$.

Section 5.4 Group Work:

The Square-Root Solid

Consider the volume integral over the solid S given by $V = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} dz\, dy\, dx$.

1. Identify the solid S by drawing a picture.

2. Rewrite the volume integral as $V = \iiint_S dx\, dy\, dz$.

3. Rewrite the volume integral as $V = \iiint_S dz\, dx\, dy$.

4. Starting with the original iterated integral, compute the volume by any means at your disposal.