MATH 2330: Multivariable Calculus

4.3: Partial Derivatives

Section 4.3 - Partial Derivatives:

Definitions & Terminology:

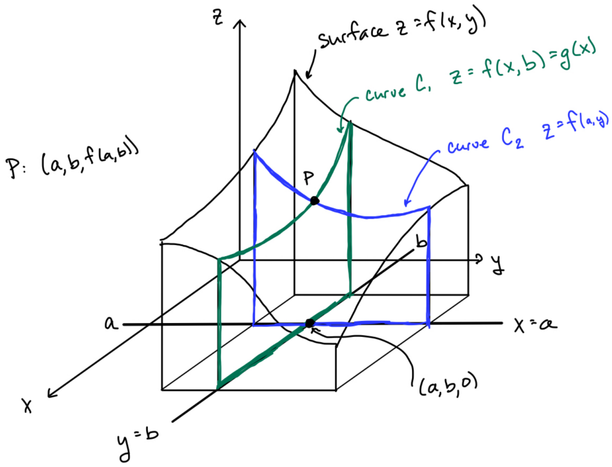

Figure 1: Partial Derivatives Geometry Diagram

The **partial derivative of f with respect to x at the point** (a, b), $f_x(a, b)$, is the slope of the line tangent to the curve $z = f(x, b)$ at the point (a, b).

- To compute f_x:
  treat y as a constant, and take the derivative w.r.t. x.

The **partial derivative of f with respect to y at the point** (a, b), $f_y(a, b)$, is the slope of the line tangent to the curve $z = f(a, y)$ at the point (a, b).

- To compute f_y:
  treat x as a constant, and take the derivative w.r.t. y.

Common types of partial derivative notation:

$$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y) = \frac{\partial z}{\partial x}$$

Clairaut's Theorem:

Suppose f is defined on a disk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are both *continuous* on D, then

$$f_{xy}(a,b) = f_{yx}(a,b)$$

Punchline: If the mixed partial derivatives are continuous, order doesn't matter!

Example 1: Consider $f(x,y) = 5x^2 - 2xy + 3y^3$. Find f_{xy} and f_{yx} and verify that the conclusion of Clairaut's Theorem holds.

Example 2: Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$:

$$x^2 - y^2 + z^2 - 2z = 4$$

Section 4.3 Group Work:

Clarifying Clairaut's Theorem

Consider $f\ !x"y"z\#!\ \ x^2 \cos {}^{!} y^3\ "\ \ z^2 {}^{"}$.

1. Why do we know that f_{zyyxxx} ! 0 without doing any computation?

2. Do we also know, without doing any computation, that f_{xyzzz} ! 0? Why or why not?

3. Suppose that f_x ! $3x$ " ay^2, f_y ! bxy " $2y$, f_y !1"1#! 3, and f has continuous mixed second partial derivatives f_{xy} and f_{yx}.

(a) Find values for a and b and thus equations for f_x and f_y. **HINT** What does Clairaut's Theorem say about the mixed partial derivatives of a function? When does the theorem apply?

(b) Can you f nd a function F !x"y#such that $\frac{\$F}{\$x}$! f_x in part (a)?

(c) Can you f nd a function G !x"y#! F !x"y#" k !y#such that $\frac{\$G}{\$y}$! f_y in part (a)? What is k !y#?

(d) What is $\frac{\$G}{\$x}$? Can you now f nd f !x"y#?

Figure 2: Clarifying Clairaut Group Work Activity