MATH 2330: Multivariable Calculus

4.1: Functions of Several Variables, Part 2 & 4.2: Limits & Continuity

Section 4.1 - Mathematica Demonstration:

(See Figure 1)

Section 4.1 - Group Work 2:

(See Figure 2)

Section 4.2 - Limits & Continuity:

Graphing Functions of Two Variables Using Mathematica.

There are two main commands, Plot3D and ContourPlot. To find out information about these commands, type a ? followed by the name of the command. To get even more information, click on the “>>” in the description below. Feel free to play around with the inputs, and click and drag the 3D graphs to change the point of view.

```
? Plot3D
```

Plot3D[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a three–dimensional plot of f as a function of x and y.
Plot3D[{f_1, f_2, …}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several functions. >>

(Plot3D returns a plot of the surface, or graph, for the given function)

```
? ContourPlot
```

ContourPlot[f, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] generates a contour plot of f as a function of x and y.
ContourPlot[f == g, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots contour lines for which $f = g$.
ContourPlot[{f_1 == g_1, f_2 == g_2, …}, {x, x_{min}, x_{max}}, {y, y_{min}, y_{max}}] plots several contour lines. >>

(ContourPlot returns the contour map for the given function)

Graph and Contour Plot for f(x,y) = $\sqrt{9-x^2-4y^2}$

```
Plot3D[√(9 - x^2 - 4 y^2), {x, -3, 3}, {y, -1.5, 1.5}, AxesLabel → {x, y, z}]
ContourPlot[√(9 - x^2 - 4 y^2), {x, -3, 3}, {y, -1.5, 1.5}, FrameLabel → {x, y}]
```

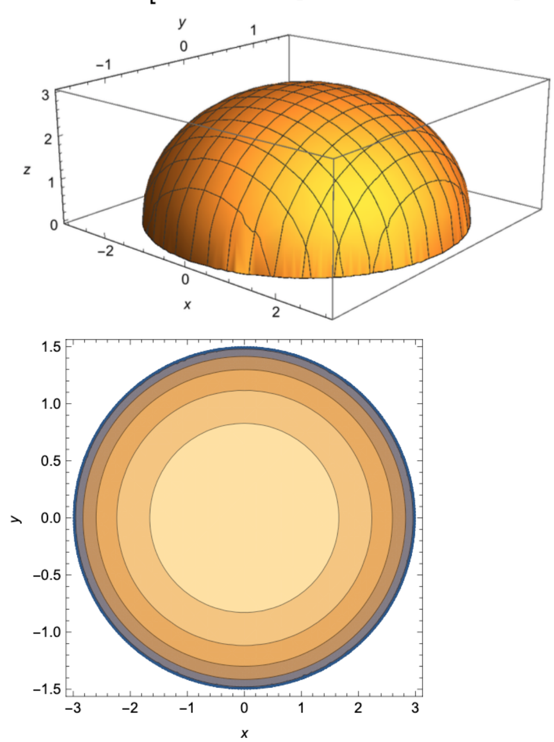
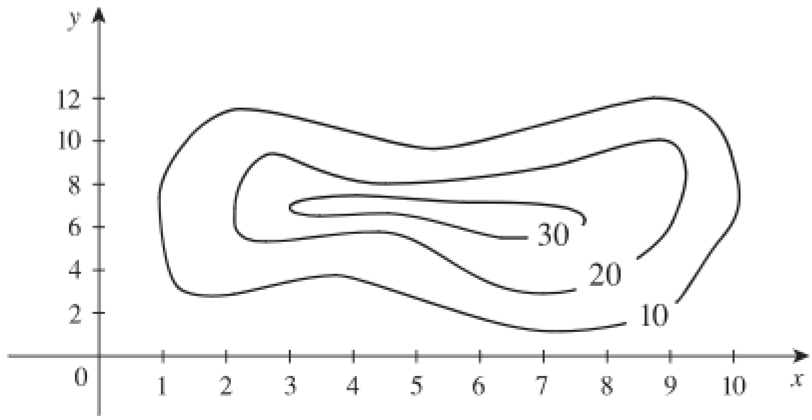


Figure 1: Screenshot from the Ch4s1 Mathematica Demo notebook

Dali's Target

Consider the following contour map of a continuous function f !x"y#



1. For approximately what values of y is it true that 10 !   f !5"y#!   30?

2. What can you estimate f !2"4#to be, and why?

3. Do we have any good estimates for f !5"8# Explain.

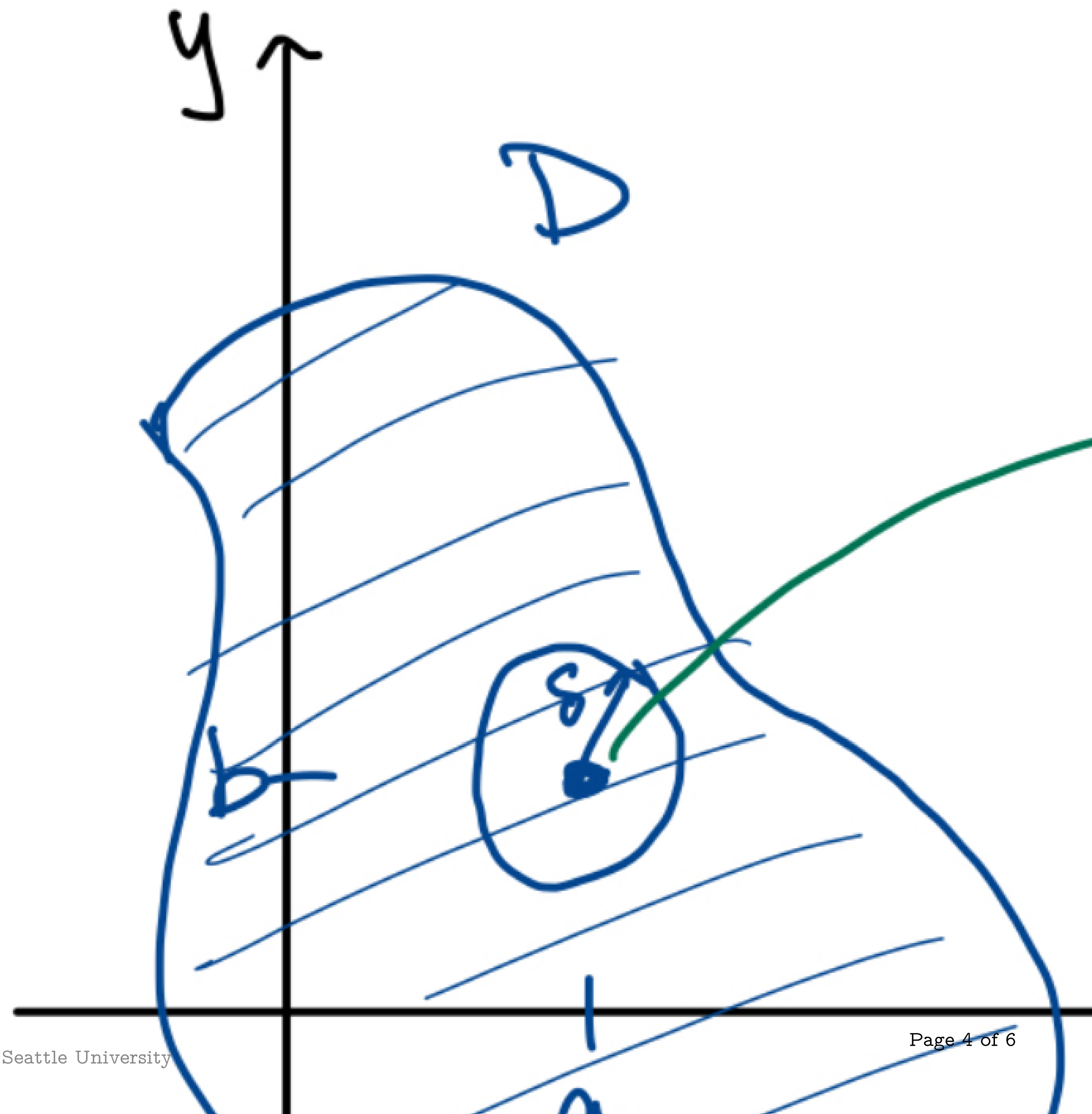
4. How many values y satisfy f !7"y#"   20?

5. How many values of x satisfy f !x"8#"   20?

Figure 2: Dali's Target Group Work Activity Screenshot

Definitions & Terminology:

Limits for functions of



Formal Limit Definition:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

If $\forall\ \varepsilon > 0$, $\exists$ a corresponding $\delta > 0$ such that

if $(x,y) \epsilon D$ and $\sqrt{(x-a)^2 + (y-b)^2} < \delta$,

then $|f(x,y) - L| < \epsilon$.

- **WARNING: L'Hopital's Rule can only be used for expressions with a single variable.**
- **Strategy for Evaluating Limits at the Origin:**
  BE SKEPTICAL! It is easier to show that a limit does not exist than to prove that it does exist.
  1. Find limits along different paths, such as:
     - $x-$axis: set $y = 0$ and find the limit as $x \to 0$
     - $y-$axis: set $x = 0$ and find the limit as $y \to 0$
     - line of slope m: set $y = mx$ and find the limit as $x \to 0$
     - parabolas, etc.

     If limits disagree along any two paths, or if the limit depends on the slope of the line, m, the the limit does not exist.
  2. If you have evidence that suggests that the limit does exist, **prove** it using one of the following methods (not covered in the text)
     - formal $\varepsilon - \delta$ definition of the limit
     - the "Squeeze Theorem"
     - polar coordinates:
       Set $x = r\cos\theta$, $y = r\sin\theta$. If the limit as $r \to 0$ exists and does not depend on the value of θ, then the limit exists.
- A function $f(x,y)$ is **continuous at** (a,b) if

  $$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

  We say that f is **continuous on** D if f is continuous at every point (a,b) in D.

Examples:

Example 1: (Revisited) Show that the limit does not exist by showing that the limit along $y = mx$ depends on the slope of the line.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

Example 2: Prove that the limit exists using: (a) the Squeeze Theorem, (b) Polar Coordinates.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$

Mathematica Demo:

(See Figure 3)

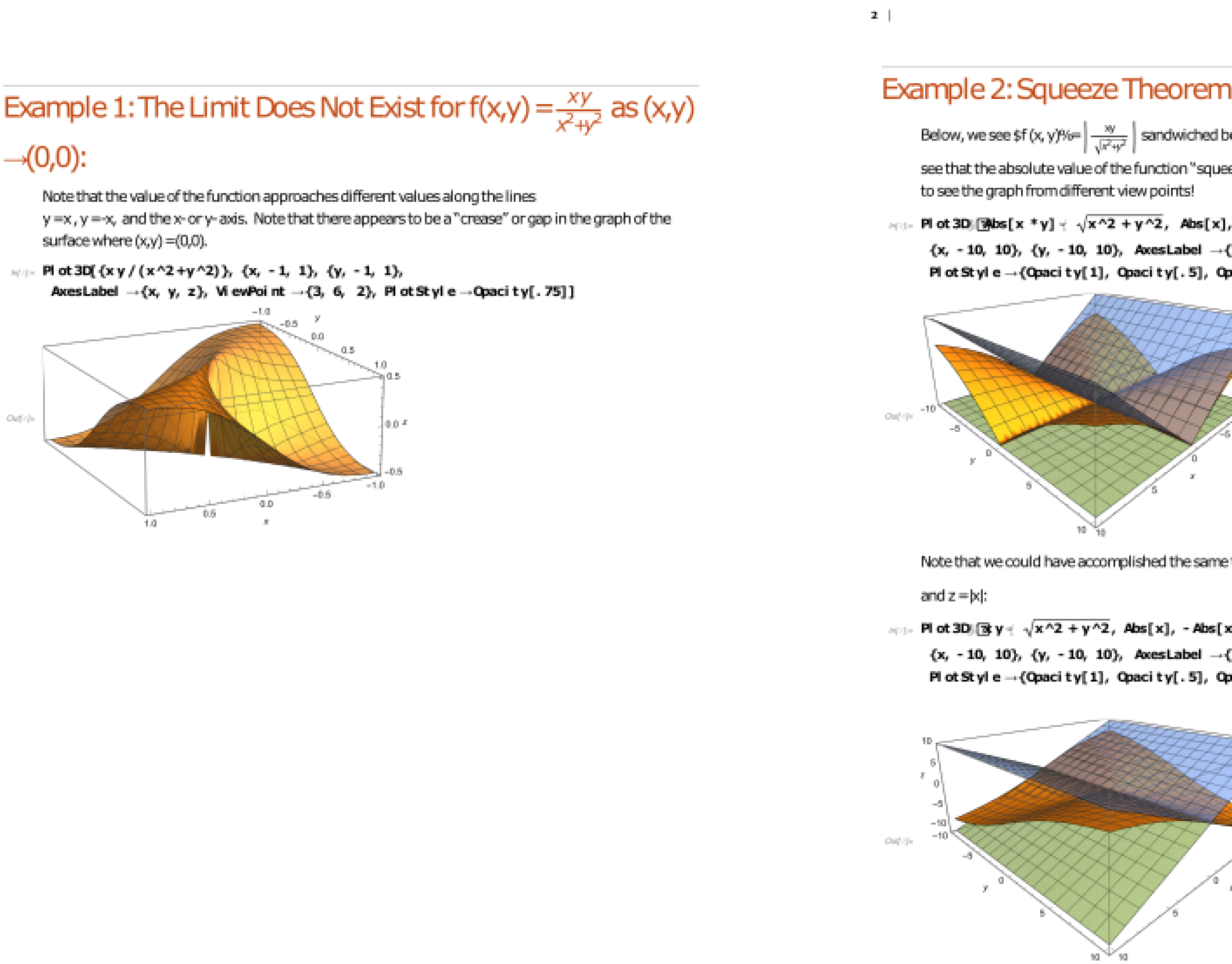


Figure 3: Screenshot from the Ch4s2 Mathematica Demo notebook