MATH 2330: Multivariable Calculus

Section 5.3: Double Integrals in Polar Coordinates

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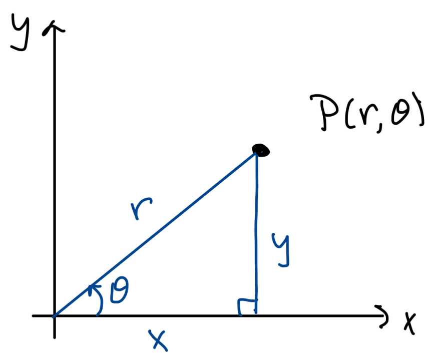
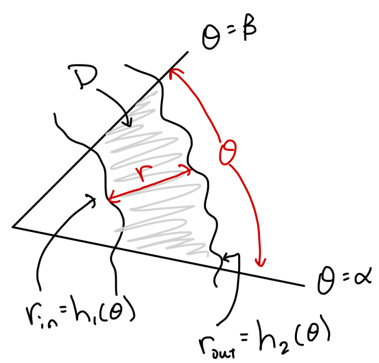
Polar Coordinates:



$x = r\cos\theta, \qquad y = r\sin\theta, \qquad x^2 + y^2 = r^2$

For the purposes of this class: $r \geq 0$

Radially Simple Regions:



$$\iint_D f(x,y)\, dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\; r\; dr\; d\theta$$

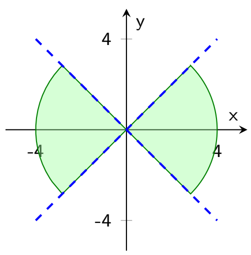
Area Element: $dA = r\; dr\; d\theta$

Make sure that you integrate in the direction of increasing θ values!

Examples:

Example 1: Evaluate $\iint_R x^2 + y^2 + 1 \; dA$ where R is the disk bounded by $x^2 + y^2 = 4$.

Example 2: Evaluate $\iint_R (x^2 + y^2)^2 \; dA$ where R is the "bow tie" region shaded below, which is bounded by $y = x$ and $y = -x$ and $x^2 + y^2 = 16$.

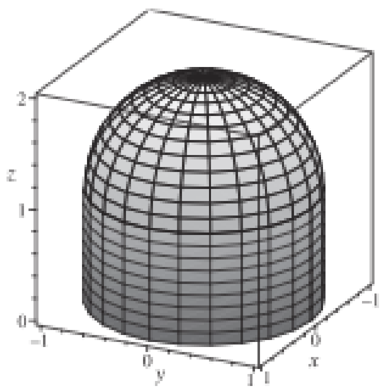


Example 3: Show that $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} \; dx = \sqrt{2\pi}$.

Section 5.3 Group Work:

Fun with Polar Volume

1. Find the volume of the region bounded above by the upper hemisphere of the sphere $x^2+y^2+(z-1)^2=1$ and bounded below by the xy-plane.



2. Find the volume of the region bounded above by the sphere $x^2+y^2+z^2=1$ and below by the sphere $x^2+y^2+(z-1)^2=1$.