MATH 2330: Multivariable Calculus

Section 4.1: Functions of Several Variables

Definitions & Terminology:

- A **function of two variables**, f, is a rule that assigns a *unique* value to each ordered pair, (x, y), in a set D, which is called the **domain** of f. The **range** is the set of all values that f takes on.
- The **graph** of f is a **surface** in $\mathbb{R}^3$ which is the set of all points (x, y, z) such that $z = f(x, y)$ for all (x, y) in the domain D.
- The **horizontal traces** of a function of two variables are cross-sectional curves that have equations $f(x, y) = k$, where k is a constant in the range of f. If you project the horizontal traces of a surface down to a contour in the $xy-$plane, you get **level curves**. If you plot a bunch of level curves, you can create a **contour map (contour plot)** of the function f.

Examples:

We will work through the following examples together.
(Note: Example 1(a) can be found in a pre-class video / lecture notes.)

Example 1: Evaluate $f(3, 2)$ and sketch the domain of f:

(b) $f(x, y) = x \ln(y^2 - x)$

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Example 2: Consider $f(x,y) = \sqrt{9 - x^2 - 4y^2}$.

(a) Evaluate $f(2,1)$ and $f(2t, t^2)$.

(b) Sketch the domain and describe the range of f.

(c) Sketch the graph of $z = f(x,y)$.

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Example 3: Consider $f(x, y) = 10 - x^2 - y^2$.

(a) Sketch a contour plot of f.

(b) Use your contour plot to sketch the graph of $z = f(x, y)$.

Group Work:

Introductions:

Introduce yourself to your neighbors. Share one unusual thing that you did over the break.

Work with your partners on the **Matching Game** activity.

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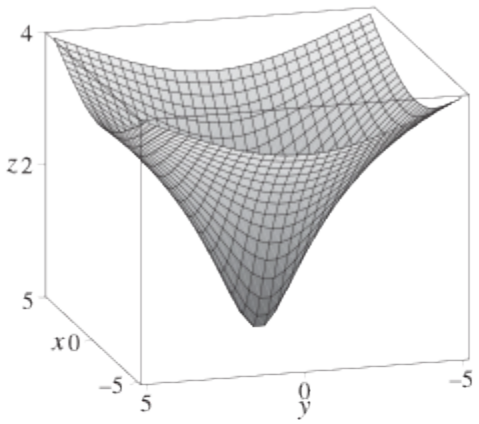
The Matching Game

Match each function with its graph. Give reasons for your choices.

1. $f(x,y) = \dfrac{1}{x^2 + y^2 + 1} - \sin y$ **2.** $f(x,y) = \sqrt{4 - x^2 - y^2}$ **3.** $f(x,y) = \cos(x - y^2)$

4. $f(x,y) = \ln(x^2 - y^2 - 1)$ **5.** $f(x,y) = x^2 y$ **6.** $f(x,y) = x^3 y$



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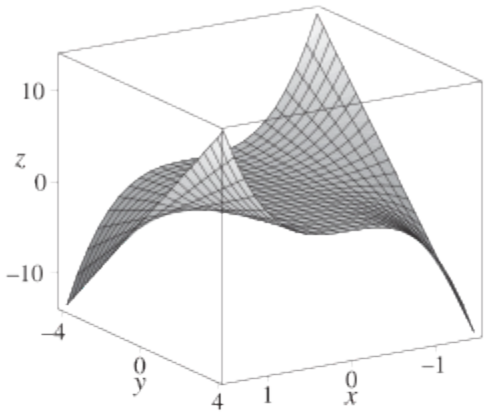

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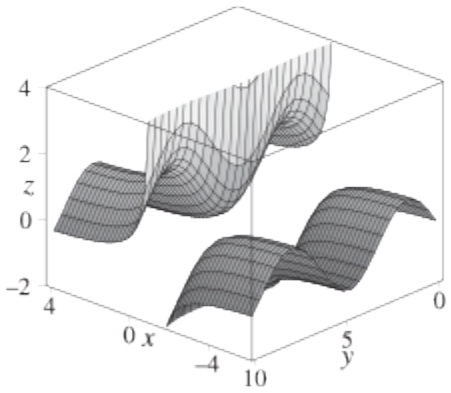

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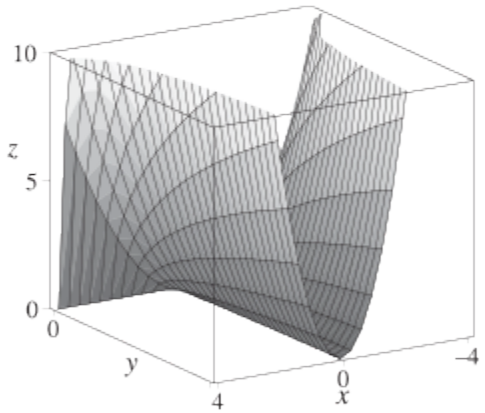

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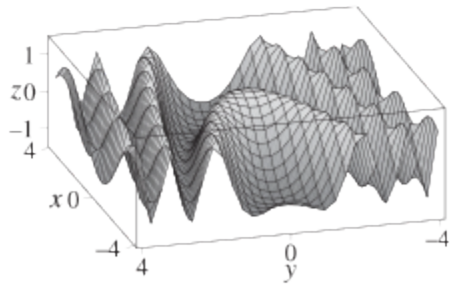

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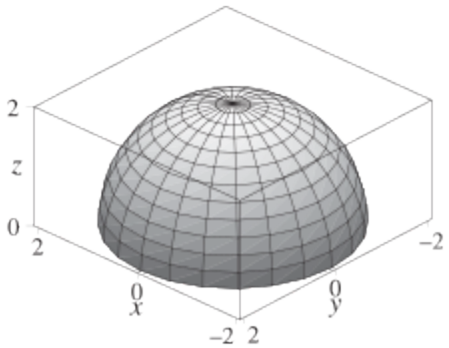

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