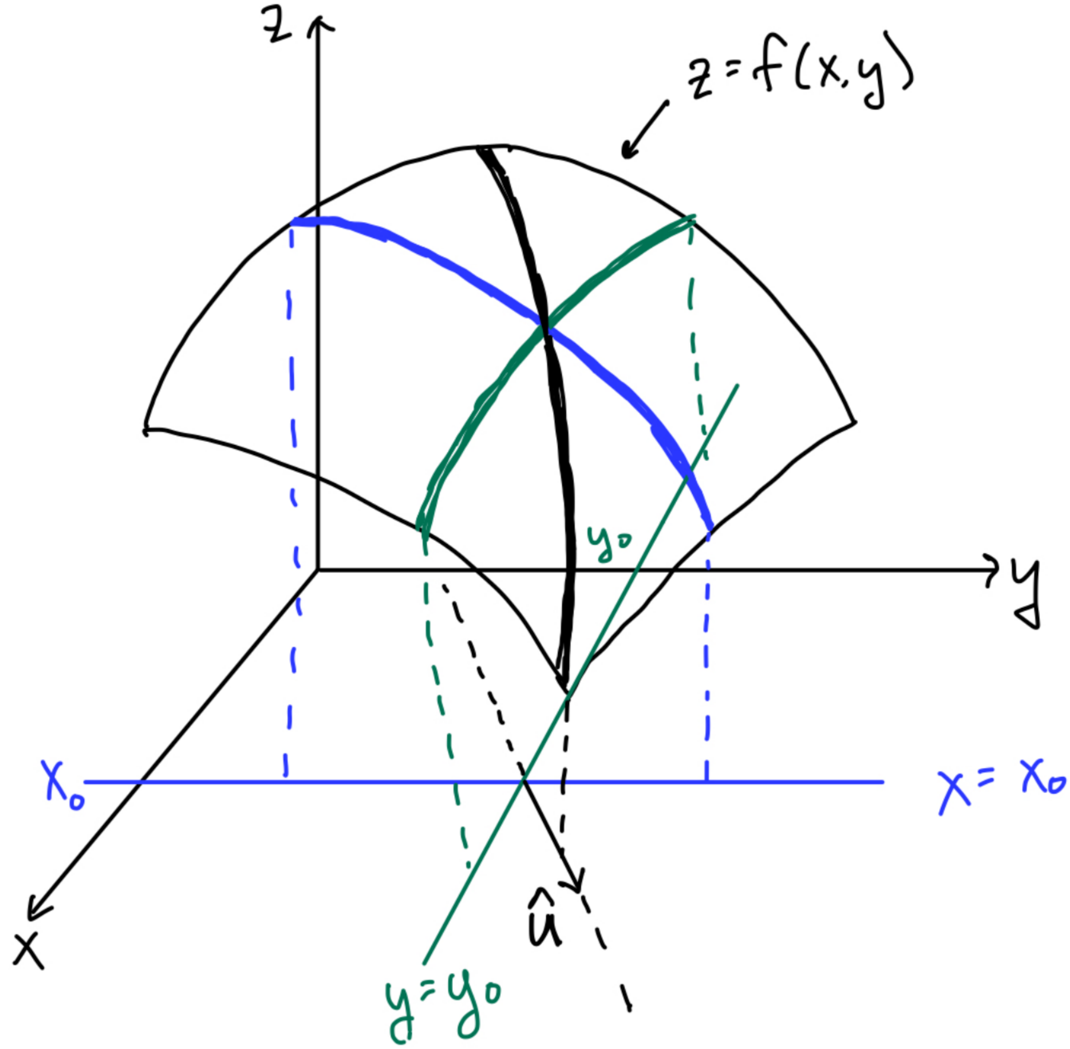
Section 4.6 - Directional Derivatives & The Gradient:

Definitions & Terminology:



Directional Derivative:

The **Directional Derivative** in the **û** direction calculates the slope of the surface in the direction of the vector **û**.

If f is a differentiable function of x and y, then f has a directional derivative in the direction of any **unit vector** $\hat{\mathbf{u}} = \langle a, b \rangle$ that can be calculated using:

$$D_u f(x,y) = f_x(x,y)a + f_y(x,y)b = \vec{\nabla} f \cdot \hat{\mathbf{u}}$$

The Gradient:

$$\vec{\nabla} f = \langle f_x, f_y \rangle$$

Discussion: Geometric Implications of $D_u f(x, y) = \vec{\nabla} f \cdot \hat{\mathbf{u}}$.

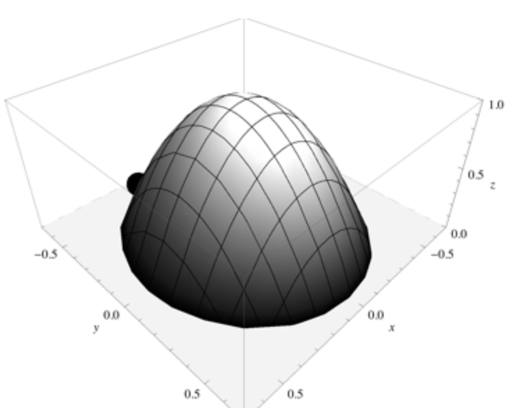
Example 1: Find the directional derivative of $f(x,y) = \ln(x^2 + y^3)$ at $(1,-3)$ in the direction of $\vec{v} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$.

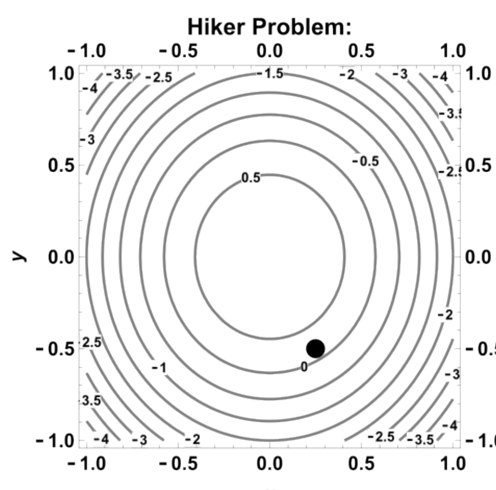
<u>Questions about the solution from the Video?</u>

$$\begin{aligned}
D_v f &= \vec{\nabla} f(1,-3) \cdot \hat{\mathbf{v}} \\
&= \left\langle \frac{2x}{x^2+y^3}, \frac{3y^2}{x^2+y^3} \right\rangle \Big|_{(1,-3)} \cdot \left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle \\
&= \left\langle -\frac{2}{26}, -\frac{27}{26} \right\rangle \cdot \left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle \\
&= \left(-\frac{2}{26}\right)\left(\frac{2}{\sqrt{13}}\right) + \left(-\frac{27}{26}\right)\left(-\frac{3}{\sqrt{13}}\right) \\
&= \frac{77}{26\sqrt{13}} \quad = \frac{77\sqrt{13}}{338}
\end{aligned}$$

Example 2: In what direction is the function $f(x,y) = xe^{2y-x}$ increasing most rapidly at $(2,1)$, and what is the maximum rate of increase?

Example 3: **Hiker Problem:**
A hiker is walking on a mountain path when it begins to rain. If the height of the mountain is modeled by the equation $z = f(x,y) = 1 - 3x^2 - \frac{5}{2}y^2$, where (x,y,z) are measured in miles, and the rain begins when the hiker is at the point $\left(\frac{1}{4}, \frac{-1}{2}, \frac{3}{16}\right)$, in what direction should she head to descend the mountainside most rapidly?





Example 4: **Example from Marsden, Tromba, & Weinstein, "Basic Multivariable Calculus"**

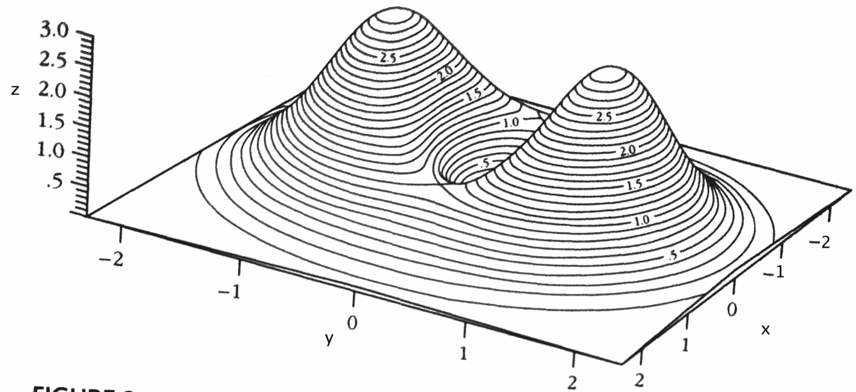


FIGURE 2.1.23. Graph of $z = (x^2 + 3y^2)e^{1-(x^2+y^2)}$.

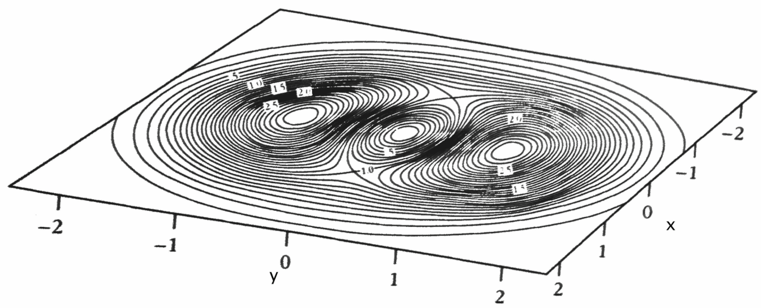
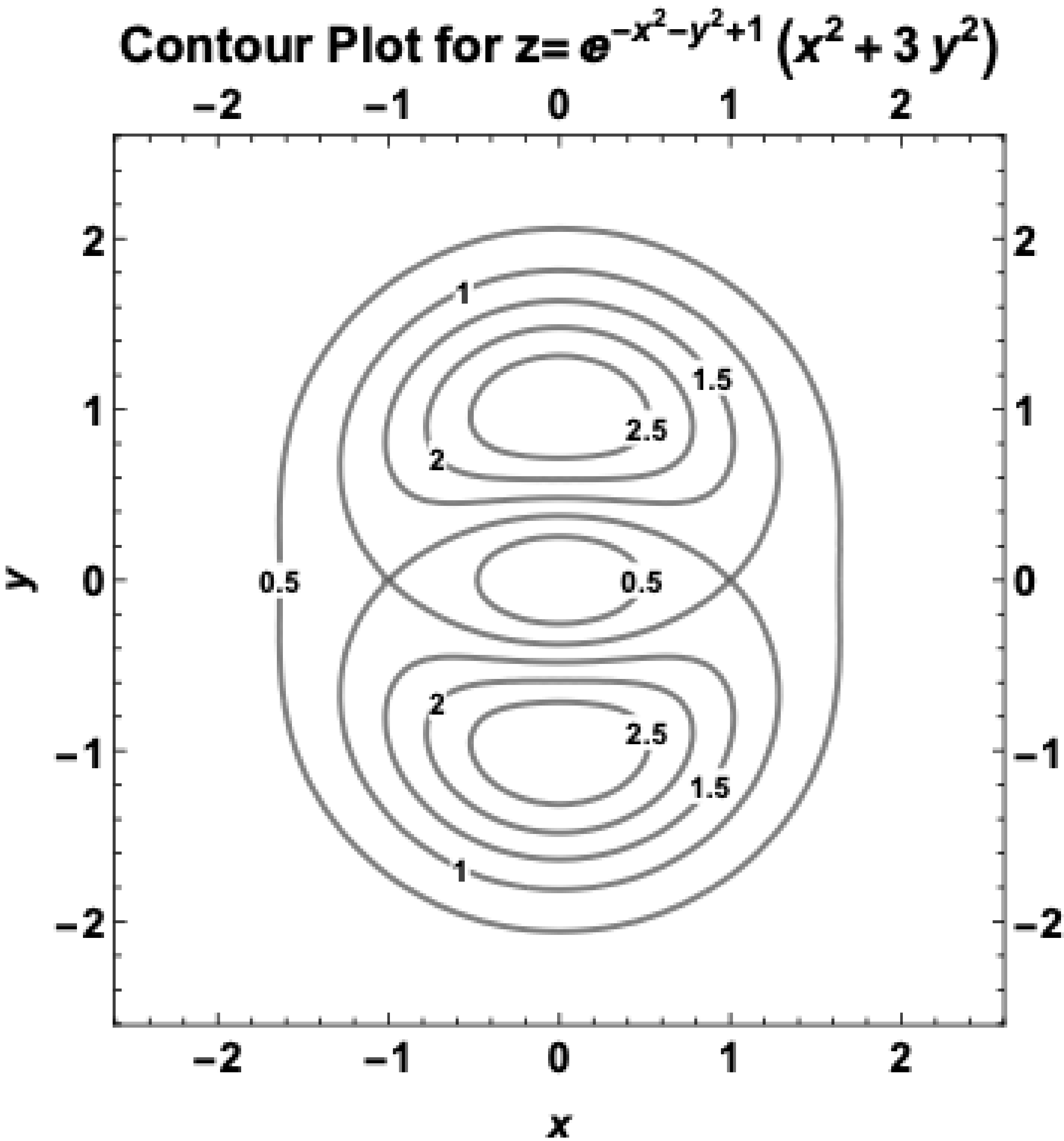


FIGURE 2.1.24. Level curves for the function $z = (x^2 + 3y^2)e^{1-(x^2+y^2)}$.



Answer the following questions about the surface $z = f(x, y)$, shown in the figures on this page, at the point $(1.2, -1)$:

- Which way should the gradient vector point?
- Is $f_x > 0$?
- In which direction would water flow?
- Is $D_u f$ positive, negative, or zero for $\vec{u} = \langle 1, 1 \rangle$?

- Find a direction in which $D_u f \approx 0$.

Now, pick some other point, and answer the questions again!