MATH 2330: Multivariable Calculus

4.5: Chain Rule

Section 4.5: Chain Rule

Chain Rule, Case 1:

Let $z = f(x, y)$ where $x = g(t), \quad y = h(t)$. Then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Example 1: (Revisited) Consider $f(x, y) = x^2 + y^2, \quad x = 3t, \quad y = e^{2t}$. Compare the result from calculating $\frac{df}{dt}$ using the Chain Rule with the result that you get from first rewriting f as a function of t, then taking the derivative.

Chain Rule, Case 2:

Let $z = f(x, y)$ where $x = g(s, t), \quad y = h(s, t)$. Then

$$\frac{\partial z}{\partial t} =$$

$$\frac{\partial z}{\partial s} =$$

Example 2: Consider $g(x, y) = x^2 + xy + y^2, \quad x = 3(t + s), \quad y = e^{2st}$. Find $\frac{\partial g}{\partial s}$ and $\frac{\partial g}{\partial t}$ at $(s, t) = (1, 0)$.

Example 3: Draw a diagram to help you write out the Chain Rule for $w = f(x, y, z)$ where $x = g(s, t), \quad y = h(s, t), \quad z = k(s, t)$.