Section 8.1, Part 1: Intro to Sequences

A **sequence** is an ordered list of numbers. An **infinite sequence** is a sequence that does not terminate.

$$\{a_1, a_2, a_3, \ldots, a_{n-1}, a_n, a_{n+1}, \ldots\}$$

Convergence/Divergence of Infinite Sequences:

A sequence $\{a_n\}$ has the limit L:

$$\lim_{n\to\infty} a_n = L,$$

if we can make the terms a_n "as close to L as we want" for n sufficiently large.

If $\lim\limits_{n\to\infty} a_n = L$ exists then we say that the sequence **converges** to L.

If $\lim\limits_{n\to\infty} a_n$ does not exist then we say that the sequence **diverges**.

$\epsilon - N$ **Limit Definition:**

The sequence $\{a_n\}$ has the limit L if for every $\epsilon > 0$ there is a corresponding integer N such that

$$\text{if } n > N, \ \textbf{then } |a_n - L| < \epsilon.$$

Problems for Group Work/Examples:

Write out the first four terms of the sequence to build your intuition.
Then, determine the convergence or divergence of the sequences.
Be sure to fully justify your conclusions using tools in the next section.

1. $a_n = \dfrac{n}{1+n^2}$

2. $b_n = \dfrac{\ln(1+2e^n)}{n}$

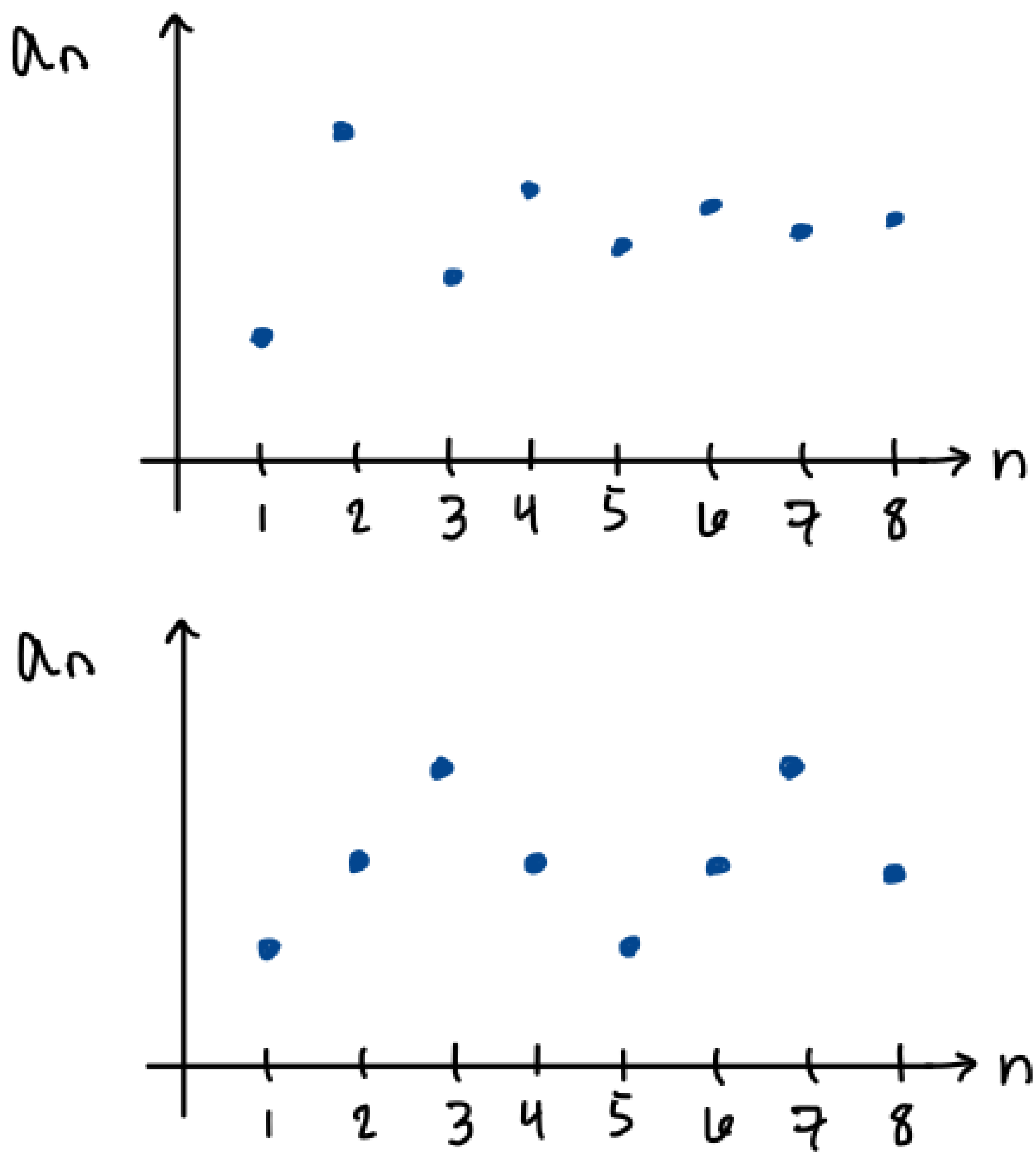

Figure 1: Sequence Illustrations: First graph shows a sequence that converges, second graph shows a sequence that diverges due to oscillation.

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3. $c_n = \sin(n\pi)$

4. $d_n = \dfrac{1 + n\cos(2\pi n)}{n}$

5. $a_n = \dfrac{e^n}{3^n}$

6. $b_n = (-1)^n \sqrt{n}$

7. $c_n = (-1)^n \dfrac{1}{\sqrt{n}}$

8. $d_n = (-1)^{2n+1}$

9. **Challenge Problem:** Note that n factorial is defined as $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$
$a_n = \dfrac{(-3)^n}{n!}$

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Tools for Finding Limits of Sequences:

The following limit laws can be used without a formal citation:

Limit Laws for Sequences:
Given

$$\lim_{n\to\infty} a_n = L_1, \qquad \lim_{n\to\infty} b_n = L_2, \qquad c: \text{ some finite constant}$$

then

$$\lim_{n\to\infty} (a_n \pm b_n) = L_1 \pm L_2$$

$$\lim_{n\to\infty} c\ a_n = c\ L_1$$

$$\lim_{n\to\infty} (a_n b_n) = L_1 L_2$$

$$\lim_{n\to\infty} c = c$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L_1}{L_2} \quad \text{if } L_2 \neq 0 \text{ and none of the } b_n = 0$$

$$\lim_{n\to\infty} (a_n)^p = (L_1)^p \qquad \text{if } p > 0 \text{ and } a_n > 0$$

Theorem 6: If $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$.

The following theorems must be formally cited when used:

Squeeze Theorem:

If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L,$$

then $\lim_{n\to\infty} b_n = L$.

Continuity & Convergence Theorem:
If $\lim_{n\to\infty} a_n = L$ and f is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(L)$.

Theorem 3: If $\lim_{x\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim_{n\to\infty} a_n = L$.

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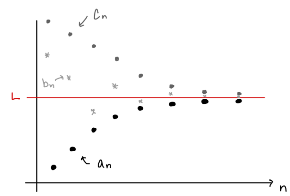

Figure 2: Figure illustrates the Squeeze Theorem for sequences.

If you want to use L'Hopital's Rule:

First switch to a function, f, of the *continuous* variable x, such that $f(n) = a_n$ when n is an integer.
Then use L'Hopital's Rule to find the limit of the *continuous* function $f(x)$ as $x \to \infty$.
Finally, cite Theorem 3.

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