

Section 8.5, Part 1: Intro to Power Series

Power Series Definition:

A series of the form:

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

is called a **Power Series centered at** $x = a$.
(Power Series in $(x-a)$, Power Series about a)

Notes:

- A Power Series **always** converges at the center: $x = a$.
- Adopt the convention that $0^0 = 1$.
- By definition: $0! = 1$.

Power Series Convergence Theorem:

For a given Power Series, $\sum_{n=0}^{\infty} c_n(x-a)^n$, there are three possibilities:

- (i) The series converges *only* when $x = a$.
- (ii) The series converges for all x .
- (iii) There is some $R > 0$ such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

Radius & Interval of Convergence:

R is the **radius of convergence**, as described above.

The **interval of convergence**, I , is the interval of all the x -values where the series converges.

**Strategy:**

1. Use the Ratio Test (or Root Test) to find the radius of convergence.
2. If you need to know the convergence behavior at the endpoints of the interval of convergence, use another series test!

Examples we will work through together:

Example 1: Motivation: For which values of x does the following series converge?

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2}$$

Find the radius and interval of convergence for the following series:

Example 2: $\sum_{n=0}^{\infty} n! x^n$

Example 3: $\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}$

Example 4: $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$

Problems for Group Work

1. Find the radius and interval of convergence for the following power series:

$$\sum_{n=0}^{\infty} \frac{n(x+5)^n}{4^{n+1}}$$

2. **Strategy Practice:**

For each of the following series, state which test you would use to determine the convergence or divergence behavior, and explain why.

(You do not have to carry out the test in detail, but follow the argument long enough to make sure your reasoning would work.)

(a) $\sum_{n=1}^{\infty} \frac{n}{2n+1}$

(b) $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 - n^2}}$

(d) $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$