

## Section 8.3, Part 2: Comparison Tests

### Section 8.3 - Tests for Series with POSITIVE Terms:

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**The tests that we will develop in this section can only be applied to series with POSITIVE terms:**  $a_n > 0$   
 $\Rightarrow$  verifying and stating that  $a_n > 0$  is an important part of the argument when using these tests!

#### Comparison Test:

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Given that  $\sum a_n$  and  $\sum b_n$  are series with **positive** terms, then we have the following:

- (i) If  $\sum b_n$  is convergent and  $a_n < b_n$  for all  $n$ ,  
then  $\sum a_n$  is also convergent.
- (ii) If  $\sum b_n$  is divergent and  $b_n < a_n$  for all  $n$ ,  
then  $\sum a_n$  is also divergent.

#### Limit Comparison Test:

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Given that  $\sum a_n$  and  $\sum b_n$  are series with **positive** terms, if:

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c,$$

where  $c > 0$  is **finite**, then either the series **both converge** or they **both diverge**.

#### Note 1:

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Both tests rely on comparison with series for which we already know the convergence behavior:

- Geometric Series
- Harmonic Series
- p-Series
- Series we can test another way...

## Note 2:

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We don't *always* have to start with  $n = 1$ . Convergence is really about the tails, or end behavior, of the sequence. If we can find a comparison that only holds for  $n > 500$ , for example, the tests still work!

## Examples:

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We will work through the following examples together.

**Do the series listed below converge, diverge, or are we unable to determine the convergence behavior given the tools that we have?**

Example 2: 
$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$$

Example 3: 
$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

Example 4: 
$$\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$$

## Problems for Group Work:

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**Be sure to fully justify your reasoning as a part of your solutions.**

The answers are upside-down on the bottom of this page.

For Problems 1-4, use either the Comparison Test or the Limit Comparison Test to determine whether the series is convergent or divergent. If neither of the tests can be used, explain why.

Problem 1: 
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} 3^n}$$

Problem 2: 
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n} + 1}$$

Problem 3: 
$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Problem 4: 
$$\sum_{n=1}^{\infty} \frac{1}{5^n + 300}$$

## Answers:

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**Problem 1:** Converge, **Problem 2:** Diverge, **Problem 3:** Cannot Determine, **Problem 4:** Converge