Section 8.4, Part 1: Alternating Series & Absolute Convergence

Section 8.4 - More Series Tests!:

In this section, we'll learn tests that can be applied to series that may have some negative terms.

Alternating Series Test (AST):

If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \ldots, b_n > 0$$

satisfies both of the conditions listed below, then the series is convergent.

(i) $b_{n+1} \leq b_n$

(ii) $\lim_{n\to\infty} b_n = 0$

Alternating Series Estimation Theorem:

If $S = \sum_{n=1}^{\infty}(-1)^{n-1}b_n$ is the sum of a convergent alternating series, then

$$|R_n| = |S - S_n| \leq b_{n+1}$$

Absolute & Conditional Convergence:

A series $\sum a_n$ is called **absolutely convergent** if the series $\sum |a_n|$ is convergent.

A series $\sum a_n$ is called **conditionally convergent** if it is convergent, but *not* absolutely convergent.

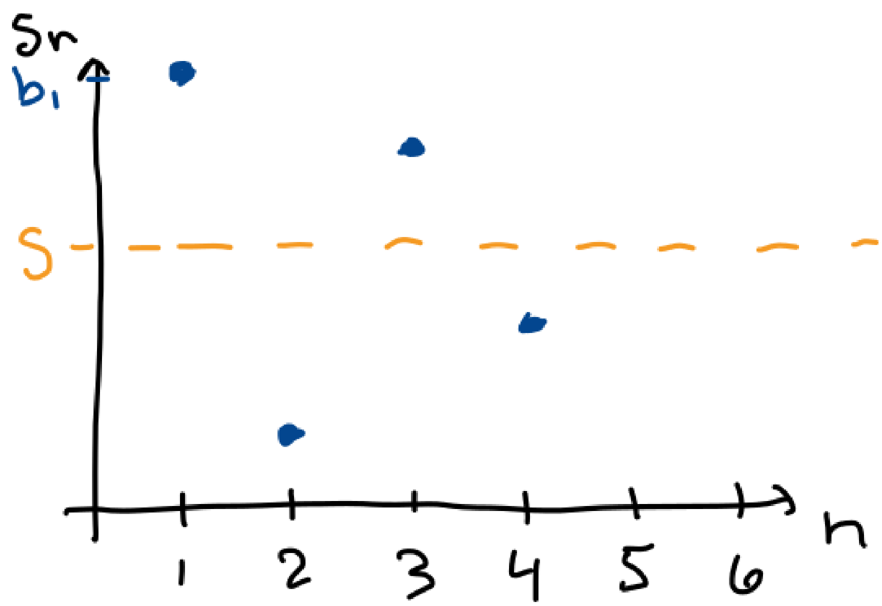

Figure 1: Figure illustrating the Alternating Series Estimation Theorem.

Theorem:
If a series $\sum a_n$ is absolutely convergent, then it is convergent.

Examples:

Example 1: Use the AST to show that the Alternating Harmonic Series converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

Follow-up Question: How many terms would we need to use to obtain an estimate for the sum of the series that is within 0.01 of the exact value?

Example 2: Determine the convergence behavior of the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5n}{6n-2}$$

Example 3: Let's determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Example 4: Apply the Alternating Series Test to the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n}$$

Example 5: Find an upper bound on the error from using only the first 5 terms of the series in Example 4 to approximate the sum of the series.

Problems for Group Work:

Be sure to fully justify your reasoning as a part of your solutions.
The answers are upside-down on the bottom of this page.

For Problems 1-4, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{2/3}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^3}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+\frac{1}{n}}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n - 3^n}$

Answers:

Problem 1: Conditionally Convergent, **Problem 2:** Absolutely Convergent,
Problem 3: Divergent, **Problem 4:** Absolutely Convergent