MATH 1336: Calculus III

Section 8.3, Part 2: Comparison Tests

Section 8.3 - Tests for Series with POSITIVE Terms:

The tests that we will develop in this section can only be applied to series with POSITIVE terms: $a_n > 0$
$\Rightarrow$ verifying and stating that $a_n > 0$ is an important part of the argument when using these tests!

Comparison Test:

Given that $\sum a_n$ and $\sum b_n$ are series with **positive** terms, then we have the following:

(i) If $\sum b_n$ is convergent and $a_n < b_n$ for all n,
then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $b_n < a_n$ for all n,
then $\sum a_n$ is also divergent.

Limit Comparison Test:

Given that $\sum a_n$ and $\sum b_n$ are series with **positive** terms, if:

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c,$$

where $c > 0$ is **finite**, then either the series **both converge** or they **both diverge**.

Note 1:

Both tests rely on comparison with series for which we already know the convergence behavior:

- Geometric Series
- Harmonic Series
- p-Series
- Series we can test another way...

Note 2:

We don't *always* have to start with $n = 1$. Convergence is really about the tails, or end behavior, of the sequence.
If we can find a comparison that only holds for $n > 500$, for example, the tests still work!

Examples:

We will work through the following examples together.

Do the series listed below converge, diverge, or are we unable to determine the convergence behavior given the tools that we have?

Example 2: $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$

Example 3: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

Example 4: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n-1}$

Problems for Group Work:

Be sure to fully justify your reasoning as a part of your solutions.
The answers are upside-down on the bottom of this page.

For Problems 1-4, use either the Comparison Test or the Limit Comparison Test to determine whether the series is convergent or divergent. If neither of the tests can be used, explain why.

Problem 1: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\ 3^n}$

Problem 2: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+1}$

Problem 3: $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

Problem 4: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n+300}$

Answers:

Problem 1: Converge, **Problem 2:** Diverge, **Problem 3:** Cannot Determine, **Problem 4:** Converge