# Section 8.5, Part 2: Power Series Practice & Section 8.6: Fun with Power Series!

## Section 8.6: Representing Functions with Power Series, AKA: Fun with Power Series!

The theorem below tells us that we can treat Power Series just like more familiar functions, as long as we stay within the radius of convergence. This means that if we know a given power series represents a particular function, we can manipulate it to find a power series representation for a new function!

### Power Series Differentiation & Integration Theorem:

If the power series, $\sum_{n=0}^{\infty} c_n(x-a)^n$ has a radius of convergence $R > 0$, then the function

$$f(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \ldots = \sum_{n=0}^{\infty} c_n(x-a)^n$$

is differentiable (and therefore continuous!) on the interval $I = (a-R, a+R)$ and

(i) $f'(x) = c_1 + 2c_2(x-a) + 3c_3(x-a)^2 + \ldots = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$

(ii) $\int f(x)\,dx = C + c_0(x-a) + c_1\dfrac{(x-a)^2}{2} + c_2\dfrac{(x-a)^3}{3} + \ldots = C + \sum_{n=0}^{\infty} c_n \dfrac{(x-a)^{n+1}}{n+1}$

The radius of convergence for the series in (i) and (ii) is also $R$.

Our first building block will be the Geometric Series, because we know how to find the sum of that series. A surprising number of new series can be found using that building block, but the Geometric Series can only get us so far. We will need a more systematic method for finding new series, which we will learn in the next section.

Geometric Series: $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \ldots, \quad R = 1$

### Other Moves:

- Add or Subtract Series
- Multiply Series by a Scalar
- Plug something in for $x$
  - may change the radius of convergence

## Examples:

Example 4: Discover a power series representation for $\ln|1+x|$ using the geometric series as a building block.

Example 5: Discover a power series representation for $\arctan(x)$ using the geometric series as a building block.

# Section 8.5, Part 2: Power Series Practice:

Problem 1: Find the radius and interval of convergence for the following power series:

(a) $$\sum_{n=0}^{\infty} \frac{(-5)^n (x-10)^n}{n!}$$

(b) $$\sum_{n=0}^{\infty} e^n (x-2)^n$$

(c) $$\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n^2}$$

(d) $$\sum_{n=0}^{\infty} x^n$$