Curvature Examples:

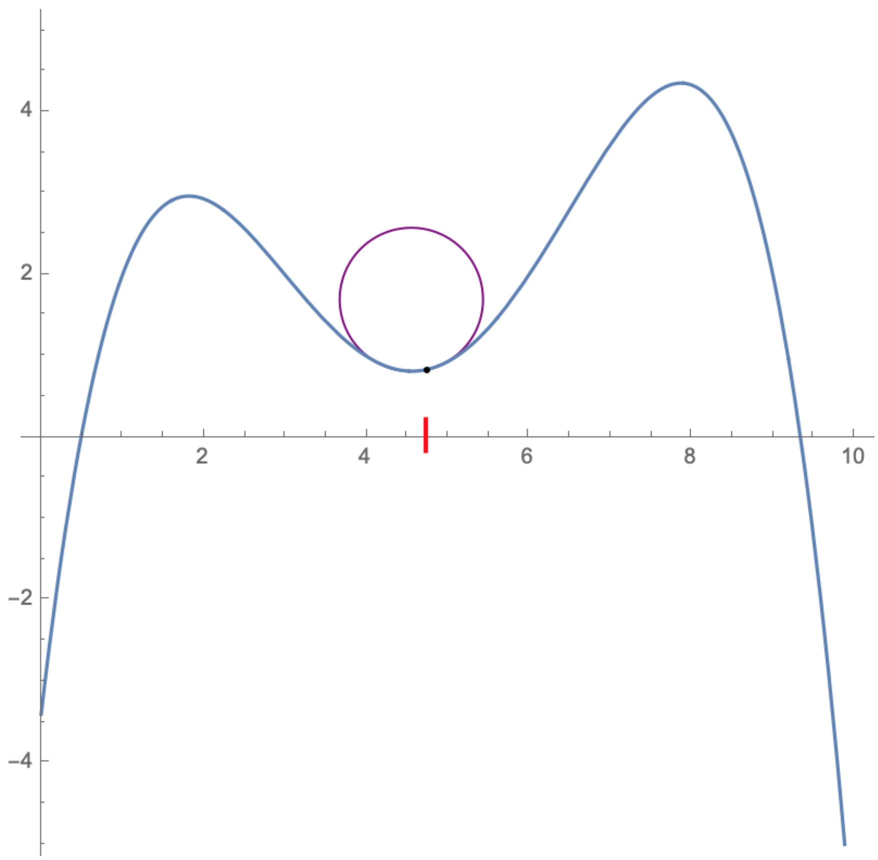

Figure 1: Diagram illustrating an osculating circle.

Curvature, denoted κ, quantifies how quickly a curve changes direction at a given point.

$$\kappa = \|\frac{d\vec{\boldsymbol{T}}}{ds}\| = \frac{\|\vec{\boldsymbol{T}}\,'(t)\|}{\|\vec{\boldsymbol{r}}\,'(t)\|} = \frac{\|\vec{\boldsymbol{r}}\,'(t) \times \vec{\boldsymbol{r}}\,''(t)\|}{\|\vec{\boldsymbol{r}}\,'(t)\|^3}$$

The **osculating circle**, shown in the diagram, is the circle that is tangent to the curve at the given point and has the same curvature as the curve at that point. It has radius $R = \dfrac{1}{\kappa}$.

Example 1: Show that the curvature of a line is zero!
Hint: the equation for a generic line is $\vec{\boldsymbol{r}}(t) =< x_0 + at, y_0 + bt, z_0 + ct >$

Example 2: Show that the curvature of the helix $\vec{\boldsymbol{r}}(t) =< R\cos t, R\sin t, \alpha t >$ is $\kappa = \dfrac{R}{R^2 + \alpha^2}$

(a) Using the formula $\kappa = \dfrac{\|\vec{\boldsymbol{T}}\,'(t)\|}{\|\vec{\boldsymbol{r}}\,'(t)\|}$

(b) Using the formula $\kappa = \dfrac{\|\vec{\boldsymbol{r}}\,'(t) \times \vec{\boldsymbol{r}}\,''(t)\|}{\|\vec{\boldsymbol{r}}\,'(t)\|^3}$

(c) Which way was easier?