

# MATH 1336: Calculus III

## Section 6.3, Part 2: More Practice with Taylor & Maclaurin Series

### Taylor/Maclaurin Series Summary:

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**Key Idea:** When  $x$  is close to  $a$  and  $n$  is large: the  $n^{\text{th}}$  degree Taylor/Maclaurin polynomial should approximate  $f(x)$  very well!

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DEFINITION FOR TAYLOR SERIES FOR  $f(x)$  CENTERED AT  $a$ :

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n &= f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2!} (x-a)^2 \\ &+ \frac{f^{(3)}(a)}{3!} (x-a)^3 + \frac{f^{(4)}(a)}{4!} (x-a)^4 + \dots \end{aligned}$$

DEFINITION FOR MACLAURIN SERIES FOR  $f(x)$ :

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f^{(1)}(0)x + \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 + \frac{f^{(4)}(0)}{4!} x^4 + \dots$$

TAYLOR'S FORMULA / LAGRANGE'S FORM OF THE REMAINDER:

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!} (x-a)^{n+1},$$

ALTERNATING SERIES ESTIMATION THEOREM FORMULA:

$$|R_n| = |S - S_n| \leq b_{n+1}$$

SELECTED MACLAURIN SERIES WITH RADIUS OF CONVERGENCE,  $R$ :

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots, \quad R = \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots, \quad R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots, \quad R = \infty$$

Geometric Series:  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \dots, \quad R = 1$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots, \quad R = 1$$

## Taylor & Maclaurin Series Practice Problems

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1. Find a Taylor series for  $f(x) = e^{2x}$  at  $a = 3$ .

Give your answer in the following formats:

- (a) The first four non-zero terms of the series, followed by a “+ ...”
- (b) Using summation notation

2. The graphs of  $f$ ,  $g$ , and  $h$  are shown below. Explain why the series shown in Figure figure1 cannot be the Maclaurin series for  $f$ ,  $g$ , or  $h$ .

$$s(x) = -1 + 0.3x - 0.1x^2 + 0.08x^3 + \dots$$

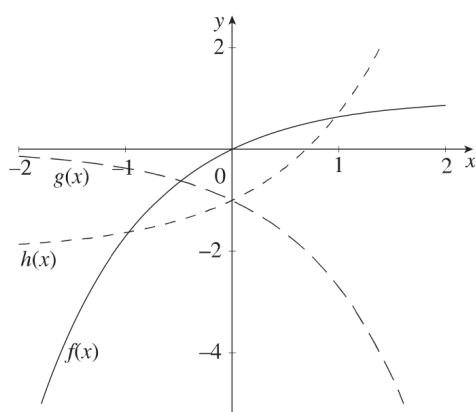


Figure 1: Graph for use with Problem like section 2.

3. Find the series representations for the following functions by any means possible:

(a)  $f(x) = \frac{1}{1+x^4}$

(b)  $f(x) = \frac{1}{1+4x^2}$

(c)  $f(x) = \ln \left| \frac{1+x}{1-x} \right|$

(d)  $f(x) = x^3 \sin x^2$