MATH 1336: Calculus III

Sections 2.1 & 2.2: Intro to Vectors & Three Dimensional Coordinate Systems

Three Dimensional Coordinate Systems:

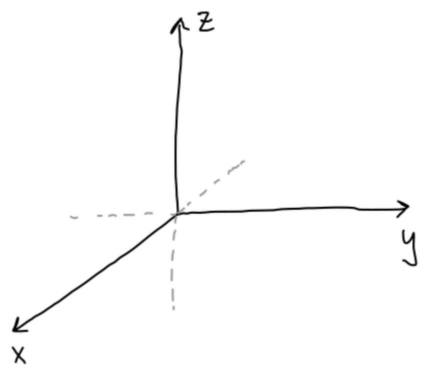

Figure 1: Drawing of a set of 3D coordinate axes in the standard orientation.

To describe the position of a point in three dimensional space, we need three coordinate axes: the $x-$axis, $y-$axis, and $z-$ axis.
A point P with coordinates $x = a,\ y = b,\ z = c$, is described using an ordered triple, $P : (a, b, c)$.

Distance Formula: To find the distance between two points, P_1 and P_2, use:

$$|P_1P_2| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

A **cylinder** is a surface that consists of all lines that are parallel to a given line and pass through a given plane curve.

Example 1:

Let's get some practice visualizing surfaces in 3D!

Draw, and describe in words, the surfaces described by the following equations.
Hint: Start by sketching cross-sections in the relevant coordinate plane.

(a) $y = 6$

(b) $y = x$

(c) $x^2 + y^2 = 1$

(d) $1 \leq x^2 + y^2 + z^2 \leq 4, \qquad z \geq 0$

Problems for Group Work:

Problem 1 Calculate the distance from $P_1 : (4, -2, 6)$ to each of the following:

(a) The origin

(b) The point $P_2 : (1, 1, 0)$

(c) The $xy-$plane

Problem 2 Draw, and describe in words, the surfaces described by the following equations.
Hint: Start by sketching cross-sections in the relevant coordinate plane.

(a) $x^2 + y^2 = 3^2$

(b) $y^2 + z^2 = 1$

(c) $z = y^2$

(d) $xy = 1$

(e) $x^2 + y^2 = z^2$

Intro to Vectors:

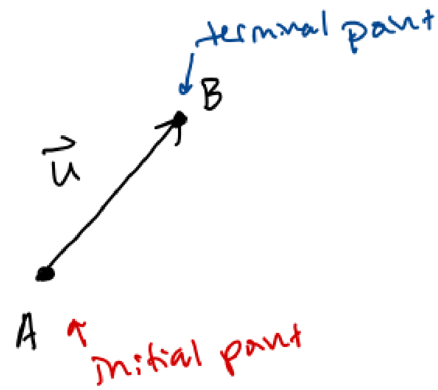

Figure 2: Illustration of a vector $\vec{u}$ with initial point A and terminal point B.

A **scalar** is a quantity that that is fully described by a magnitude (numerical value) alone.
Example: -5

A **vector** is a quantity that is fully described by both a magnitude and a direction.
Example: $\vec{v} = \langle 2, -3, 7\rangle = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

To find the **magnitude** (length) of a vector, use the same idea as the distance formula. If $\vec{a} = \langle a_1, a_2, a_3\rangle$, then the length of $\vec{a}$ is:

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

A vector with length one is called a **unit vector**.
A "hat" instead of an arrow over a vector denotes that it has unit length:

$$\hat{\mathbf{a}} = \frac{\vec{a}}{||\vec{a}||}$$

Vector Practice:

1. Some of the following quantities are vectors, and some are scalars. Classify them all by checking the appropriate box, and come up with two or three others with which to stump your classmates, and hopefully your instructor.

Quantity	**Vector?**	**Scalar?**
Speed		
Force		
Bank Account Balance		
Velocity		
Acceleration		
Energy		
Temperature		
Work		
Electrical Current		

2. Calculate the following quantities, given the vectors listed below.

$$\vec{\boldsymbol{u}} = \langle 8, 0, 0 \rangle, \qquad \vec{\boldsymbol{v}} = 5\hat{\mathbf{\imath}} + 5\hat{\mathbf{\jmath}}, \qquad \vec{\boldsymbol{a}} = \langle -1, 1, 1 \rangle, \qquad \vec{\boldsymbol{b}} = \langle 1, 2, 3 \rangle$$

(a) $3\vec{\boldsymbol{v}}$

(b) $\vec{\boldsymbol{a}} + \vec{\boldsymbol{b}}$

(c) $||\vec{\boldsymbol{b}}||$

(d) $||\vec{\boldsymbol{u}}||$

(e) $\vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}$

(f) Find a unit vector that points in the same direction as $\vec{\boldsymbol{u}}$

(g) Find a unit vector that points in the opposite direction from $\vec{\boldsymbol{a}}$