MATH 1336: Calculus III

Section 5.2, Part 1: Intro to Infinite Series

Infinite Series Definitions:

An **infinite series** is the sum of all of the infinitely many terms in an infinite sequence:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

The $\boldsymbol{n^{th}}$**partial sum**, s_n, is the sum of the first n terms:

$$\begin{aligned}
s_1 &= a_1 \\
s_2 &= a_1 + a_2 \\
s_3 &= a_1 + a_2 + a_3 \\
&\vdots \\
s_n &= a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n = \sum_{i=1}^{n} a_i
\end{aligned}$$

The partial sums themselves form a sequence: the sequence of partial sums $\{s_n\}$.

If $\lim_{n\to\infty} s_n = S$ exists and is finite, then we say that the series is **convergent**, and the number S is called the **sum** of the series:

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots = \lim_{n\to\infty} s_n = S$$

If $\{s_n\}$ diverges, then we say that the series is **divergent**.

Key Idea:
If we can find a formula for the n^{th} partial sum, s_n, somehow, then we can check to see if $\{s_n\}$ converges using the tools that we have already developed for sequences!

Geometric Series:

A series of the form

$$a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} + \ldots = \sum_{n=1}^{\infty} ar^{n-1}$$

is called a **geometric series**.

A geometric series converges to $S = \dfrac{a}{1-r}$ if $|r| < 1$, and diverges otherwise.

Proof of the Geometric Series Convergence Result:

(to be completed together in class)

Geometric Series Examples & Problems

Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

1. $2 + 6 + 18 + 54 + 162 + \dots$

2. $2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

3. $\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \dots$

4. $\sum_{n=1}^{\infty} \frac{3^n}{\pi^{n+1}}$