Content Covered: Sections 1.1-1.4 & 2.1-2.4

Note that detailed lists of objectives from each section covered can be found on Canvas.

Concept Check Questions:

1. **Parametric Curves:**
   (a) What is a parametric curve?
   (b) How do you sketch a parametric curve?
   (c) How do you find the slope of a line tangent to a parametric curve?
   (d) How do you find the area under a parametric curve?
   (e) How do you find the length of a parametric curve?
2. **Polar Coordinates:**
   (a) Sketch a diagram to explain the meaning of the polar coordinates (r, θ) of a point.
   (b) Write out the equations that express the Cartesian coordinates (x, y) of a point in terms of the polar coordinates.
3. **Polar Curves:**
   (a) How do you find the slope of a line tangent to a polar curve?
   (b) How do you find the area of a region bounded by a polar curve?
   (c) How do you find the arclength of a polar curve?
4. **Intro to Vectors:**
   (a) What is the difference between a vector and a scalar?
   (b) How do you add two vectors geometrically? How do you add them algebraically?
   (c) If $\vec{\boldsymbol{a}}$ is a vector and c is a scalar, how is $c\vec{\boldsymbol{a}}$ related to $\vec{\boldsymbol{a}}$ geometrically? How do you find $c\vec{\boldsymbol{a}}$ algebraically?
5. **Dot Product:**
   (a) How do you find the dot product $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}}$ if you know their lengths and the angle between them?
   (b) How do you find the dot product $\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}}$ if you know their components?
6. Write expressions for the scalar and vector projections of $\vec{\boldsymbol{b}}$ onto $\vec{\boldsymbol{a}}$. Illustrate with a diagram.
7. **Cross Product:**
   (a) How do you find the cross product $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}$ if you know their lengths and the angle between them?
   (b) How do you find the cross product $\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}$ if you know their components?
   (c) How are cross products useful?
8. How do you find the area of a parallelogram determined by $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$?

(T/F)+E:

Answer the following questions **TRUE** or **FALSE**. *You must justify your answer with a complete sentence* **explaining** *why the answer is either* **TRUE** *or* **FALSE**
Note: **(T/F) + E**: represents a choice of either (**T**rue or **F**alse) plus an **E**xplanation for your choice.

1. If the parametric curve $x = f(t), \quad y = g(t)$ satisfies $g'(1) = 0$, and $f'(1) \neq 0$, then it has a horizontal tangent when $t = 1$.

2. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then
$$\frac{d^2y}{dx^2} = \frac{y''(t)}{x''(t)}.$$

3. The length of the curve $x = f(t), \quad y = g(t), \quad a \leq t \leq b$ is
$$\int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt.$$

4. If a point is represented by (x, y) in Cartesian coordinates (where $x \neq 0$) and (r, θ) in polar coordinates, then $\tan\theta = \frac{y}{x}$.

5. The polar curves $r = 1 - \sin 2\theta$ and $r = \sin 2\theta - 1$ have the same graph.

6. The equations $r = 2$, $x^2 + y^2 = 4$, and $x = 2\sin(3t), \quad y = 2\cos(3t), \quad 0 \leq t \leq 2\pi$ all have the same graph.

7. The parametric equations $x = t^2, \quad y = t^4$ have the same graph as $x = t^3, \quad y = t^6$.

8. The set of all points $\{(x, y, z)|x^2 + y^2 = 1\}$ is a circle.

9. If $\vec{\boldsymbol{u}} = \langle u_1, u_2\rangle$ and $\vec{\boldsymbol{v}} = \langle v_1, v_2\rangle$ then $\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = \langle u_1v_1, u_2v_2\rangle$.

10. For any three-dimensional vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$, $|\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}}| \leq ||\vec{\boldsymbol{u}}||\ ||\vec{\boldsymbol{v}}||$.

11. For any three-dimensional vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$, $(\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}}) \cdot \vec{\boldsymbol{u}} = 0$.

12. For any three-dimensional vectors $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$, $(\vec{\boldsymbol{u}} + \vec{\boldsymbol{v}}) \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}}$.

13. If $\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = 0$, then $\vec{\boldsymbol{u}} = \vec{\boldsymbol{0}}$ or $\vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$.

14. If $\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$, then $\vec{\boldsymbol{u}} = \vec{\boldsymbol{0}}$ or $\vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$.

15. If $\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = 0$ and $\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$, then $\vec{\boldsymbol{u}} = \vec{\boldsymbol{0}}$ or $\vec{\boldsymbol{v}} = \vec{\boldsymbol{0}}$.

Selected Review Problems:

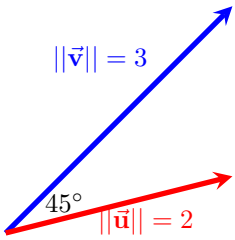
Here are some additional review problems from the material covered by Test 1. **This does not represent a practice test!** There may be some types of problems on the test that are not listed below. The actual test will be shorter than this list!

Chapter 1 Material:

1. Sketch the parametric curve and eliminate the parameter to find the Cartesian equation of the curve.
   (a) $x = t^2 + 4t, \quad y = 2 - t, \quad -4 \le t \le 1$
   (b) $x = 1 + e^{2t}, \quad y = e^t$
   (c) $x = \cos t, \quad y = \sec t, \quad 0 \le t \le \pi/2$
   (d) $x = 2\cos t, \quad y = 1 + \sin t$
2. (a) Plot the point with polar coordinates $(4, \frac{2\pi}{3})$. Then find its Cartesian coordinates.
   (b) The Cartesian coordinates of a point are $(-3, 3)$. Find two sets of polar coordinates for the point.
3. Find the slope of the tangent line to the given curve at the point corresponding to the specified value.
   (a) $x = \ln t, \quad y = 1 + t^2, \qquad t = 1$
   (b) $x = t^3 + 6t + 1, \quad y = 2t - t^2, \qquad t = -1$
   (c) $r = e^{-\theta}, \qquad \theta = \pi$
4. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
   (a) $x = t + \sin t, \quad y = t - \cos t$
   (b) $x = 1 + t^2, \quad y = t - t^3$
5. Consider the curve with parametric equations
   $$x = 2a\cos t - a\cos 2t, \qquad y = 2a\sin t - a\sin 2t$$
   (a) At what points does the curve have vertical or horizontal tangents? Use this information to help sketch the curve.
   (b) Find the area enclosed by the curve.
6. Find the points of intersection of the curves $r = 2$ and $r = 4\cos\theta$.
7. Find the length of the curve.
   (a) $x = 3t^2, \quad y = 2t^3, \quad 0 \le t \le 2$
   (b) $r = \frac{1}{\theta}, \quad \pi \le \theta \le 2\pi$ (set-up only - I would not ask you to evaluate this integral on an in-class test)

Chapter 2 Material:

8. If $\vec{u}$ and $\vec{v}$ are the vectors shown below, find*(Figure not necessarily drawn to scale)*
9. Calculate the stated quantity for the vectors
   $$\vec{a} = \langle 1, 1, -2\rangle, \qquad \vec{b} = \langle 3, -2, 1\rangle$$
   (a) $2\vec{a} + 3\vec{b}$



(a) $\vec{u} \cdot \vec{v}$

(b) $||\vec{u} \times \vec{v}||$

(c) Is $\vec{u} \times \vec{v}$ directed into the page, or out of it?

Figure 1: Figure for Problem likesubsection8.

(b) $||\vec{b}||$

(c) $\vec{a} \cdot \vec{b}$

(d) $\text{comp}_{\vec{a}}\vec{b}$

(e) $\text{proj}_{\vec{a}}\vec{b}$

(f) the angle between $\vec{a}$ and b, in radians, rounded to two decimal places

10. Calculate the stated quantity for the vectors

$$\vec{a} = \langle 1, 1, -2 \rangle, \qquad \vec{b} = \langle 3, -2, 1 \rangle, \qquad \vec{c} = \langle 0, 1, -5 \rangle$$

(a) $\vec{a} \times \vec{b}$

(b) $||\vec{b} \times \vec{c}||$

(c) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(d) $\vec{c} \times \vec{c}$

11. Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal.

12. A constant force $\vec{F} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$ moves an object along the line segment from $(1, 0, 2)$ to $(5, 3, 8)$. Find the work done if the distance is measured in meters and the force in Newtons.

13. Identify and sketch the graph of each surface.

(a) $x = 3$

(b) $x = z$

(c) $y = z^2$

(An answer key for the (T/F)+E and Selected Review Problems can now be found on the Test 1 assignment page on Canvas!)