# MATH 1336: Calculus III

Sections 3.1-3.3: Vector Functions, Space Curves, & Arclength

## Vector Functions, Space Curves, & Arclength:

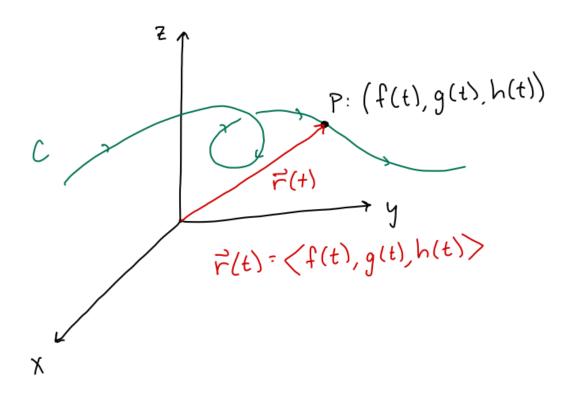


Figure 1: Diagram illustrating a general space curve in three dimensions.

A space curve in  $\mathbb{R}^3$  can be parametrized as a **vector function** whose tip traces out the curve C:

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

Tangent Vector:

$$\vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

Unit Tangent Vector:

$$\vec{\boldsymbol{T}}(t) = \frac{\vec{\boldsymbol{r}}~'(t)}{\|\vec{\boldsymbol{r}}~'(t)\|}$$

Definite Integrals:

$$\int_a^b \vec{r}(t)dt = \langle \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt \rangle$$

Arclength:

$$L = \int_{a}^{b} \sqrt{(\frac{dx}{dt})^{2} + (\frac{dy}{dt})^{2} + (\frac{dz}{dt})^{2}} dt = \int_{a}^{b} ||\vec{r}|'(t)||dt$$

# Warmup: Parallel, Perpendicular, or Neither?

For each of the following, determine whether the given items are parallel, perpendicular, or neither:

**Warmup 1:** The plane with the standard equation 2x - y - 5z = 0 and each of the planes listed below:

$$-6x + 3y + 15z = 3$$
$$x + 2y = -4$$
$$2x - 5z = -1$$

**Warmup 2:** The line with vector equation  $\vec{r}(t) = <5-3t, 2+4t, 4-2t>$  and each of the planes listed below:

$$-4.5x + 6y - 3z = -21$$
$$3x + 5y - 2z = -30$$
$$4x + 7y + 8z = 11$$

## Space Curve Examples:

### Example 1: We will revisit the example from the pre-class video in a Mathematica demonstration:

Answer the following question about the vector function:

$$\vec{r}(t) = \langle e^{2t}, t^2 - t, \ln t \rangle = e^{2t} \hat{\mathbf{i}} + (t^2 - t) \hat{\mathbf{j}} + \ln t \hat{\mathbf{k}}$$

- a) What is the domain of  $\vec{r}(t)$ ?
- b) Find a tangent vector at the point where t = 0.2.
- c) Find an equation for the tangent line to  $\vec{r}(t)$  at the point where t=0.2.
- d) What is the unit tangent vector  $\vec{T}(t)$  for  $\vec{r}(t)$ ? At t = 0.2?

**Example 2:** Find  $\vec{r}(t)$  if  $\vec{r}'(t) = \langle e^t, t^2, \cos 2t \rangle$  and  $\vec{r}(0) = \langle 2, 1, -1 \rangle$ .

## Space Curve Problems:

**Problem 1:** Calculate the length along the helix from t=0 to  $t=3\pi$ . Does your answer make sense? Why or why not?

$$\vec{r}(t) = \langle 2\cos t, 2\sin t, t \rangle$$

**Problem 2:** For the vector function shown below, show that  $\vec{T}'(t) \perp \vec{T}(t)$ .

$$\vec{r}(t) = \langle 2\cos t, 2\sin t, t \rangle$$

**Problem 3:** Prove that  $\vec{T}'(t) \perp \vec{T}(t)$  in general.

*Hint:* Start from the fact that  $\vec{T}(t)$  is a unit vector.

**Problem 4:** Find vector functions that represents the curves of intersection of the sphere  $x^2 + y^2 + z^2 = 4$  and the cylinder  $x^2 + y^2 = 1$ .