

MATH 1336: Calculus III

Section 5.5: Alternating Series & Absolute Convergence

Sections 5.5 & 5.6 - More Series Tests!:

In these sections, we'll learn tests that can be applied to series that may have some negative terms.

Alternating Series Test (AST):

If an alternating series can be written in one of the following forms:

$$\sum_{n=1}^{\infty} (-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \dots = \sum_{n=1}^{\infty} (-1)^{n-1} b_n$$

-OR-

$$\sum_{n=1}^{\infty} (-1)^n b_n = -b_1 + b_2 - b_3 + b_4 - b_5 + b_6 - \dots$$

and satisfies both of the conditions listed below, then the series is convergent.

(i) $0 < b_{n+1} \leq b_n$

(ii) $\lim_{n \rightarrow \infty} b_n = 0$

Alternating Series Estimation/Remainder Theorem:

If S is the sum of a convergent alternating series

$$\sum_{n=1}^{\infty} (-1)^{n+1} b_n \quad \text{-OR-} \quad \sum_{n=1}^{\infty} (-1)^n b_n,$$

then

$$|R_N| = |S - S_N| \leq b_{N+1}$$

Absolute & Conditional Convergence:

A series $\sum a_n$ is called **absolutely convergent** if the series $\sum |a_n|$ is convergent.

A series $\sum a_n$ is called **conditionally convergent** if it is convergent, but *not* absolutely convergent.

Theorem 5.15:

If a series $\sum |a_n|$ converges, then $\sum a_n$ converges.

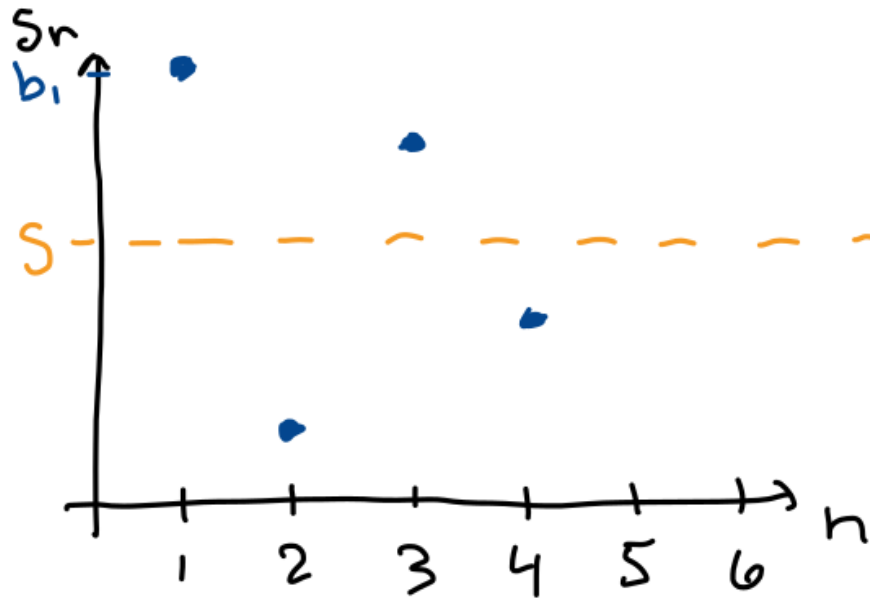


Figure 1: Illustration of a convergent alternating series to demonstrate the Alternating Series Estimation Theorem.

Examples:

Example 1: Use the AST to show that the Alternating Harmonic Series converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

Follow-up Question: How many terms would we need to use to obtain an estimate for the sum of the series that is within 0.01 of the exact value?

Example 2: Determine the convergence behavior of the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5n}{6n-2}$$

Example 3: Let's determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Example 4: Apply the Alternating Series Test to the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$$

Example 5: Find an upper bound on the error from using only the first 5 terms of the series in Example 4 to approximate the sum of the series.

Problems for Group Work:

Be sure to fully justify your reasoning as a part of your solutions.

The answers are upside-down on the bottom of this page.

For Problems 1-4, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{2/3}}$$

2.
$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^3}$$

3.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1 + \frac{1}{n}}$$

4.
$$\sum_{n=1}^{\infty} \frac{1}{5^n - 3^n}$$