## Intro to Parametric Curves Terminology:

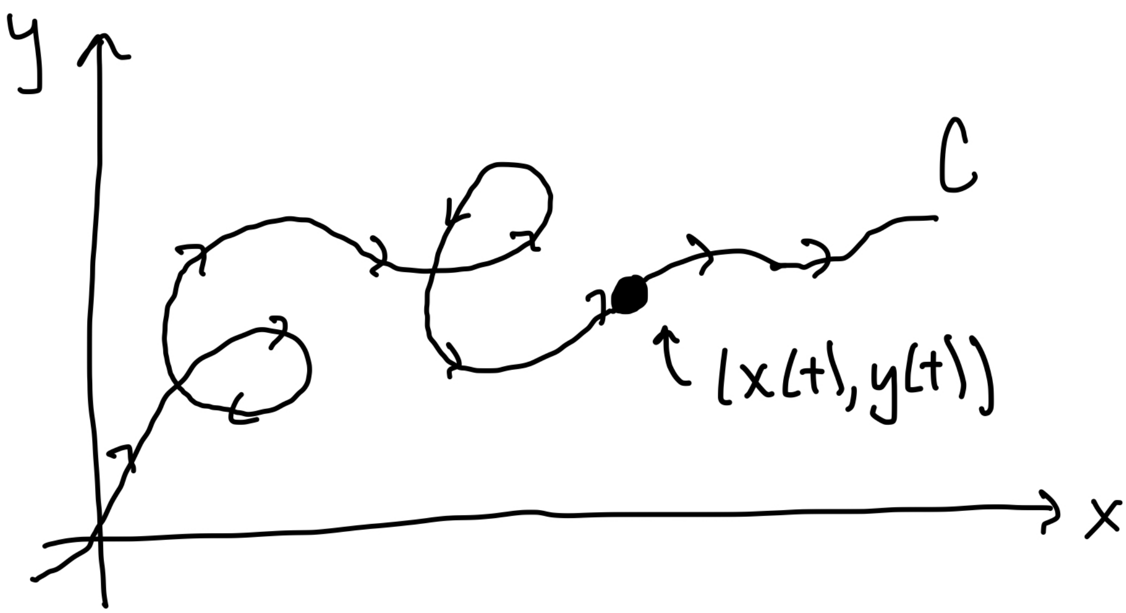


Figure 1: Figure illustrating a general parametric curve.

The **parametric equations** for the curve $C$:

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$

give the coordinates of a point on the curve in terms the independent variable $t$, which is referred to as the **parameter**.

The **parametric curve**, $C$, can be created by tracing out points $(x, y) = (f(t),\ g(t))$.

If we are given a restricted interval for $t$, we will only get part of the curve.

$$x = f(t), \quad y = g(t), \quad a \leq t \leq b$$

gives the part of the curve from the **initial point:** $(x, y) = (f(a), g(a))$
to the **terminal point:** $(x, y) = (f(b), g(b))$.

Note that parametric curves have a direction associated with them, which is indicated by drawing an arrow in the direction of increasing $t-$values.

# Example:

*Note that this example was covered in one of the pre-class videos for today's class. In the future, we will not repeat examples from the videos in class, but I will be happy to answer questions about them!*

Example 1: Sketch the curve defined by the parametric equations, then eliminate the parameter $t$ to obtain an equation in terms of only $x$ and $y$, which is referred to as a **Cartesian equation**.

$$x = t^2 - 2t, \quad y = t + 1, \quad 0 \le t \le 4$$

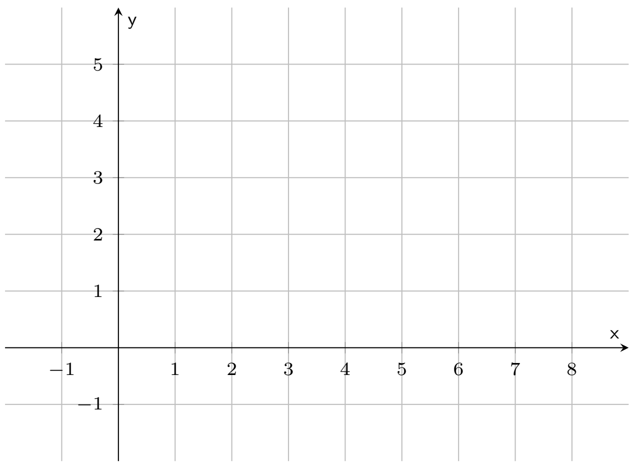


Figure 2: Blank set of coordinate axes for sketching the parametric curve from Example 1.

*Followup Discussion: What do you think are some advantages of parametric equations? disadvantages?*

# Group Work:

## Introductions:

Introduce yourself to your neighbors. Share one unusual thing that you did over the break.

## Work with your partners on the following problems:

Problem 1: Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as $t$ increases.

(a) $x = t, \qquad y = t^2, \qquad -2 \le t \le 4$

(b) $x = t - 1, \qquad y = t^3 + 1, \qquad -2 \le t \le 2$

(c) $x = \sin(t), \qquad y = \cos^2(t), \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

Problem 2: Describe the motion of a particle with position $(x, y)$ as $t$ varies in the given interval.

(a) $x = \cos(t), \qquad y = \sin(t), \qquad 0 \le t \le 2\pi$

(b) $x = \cos(2t), \qquad y = \sin(2t), \qquad 0 \le t \le 2\pi$

(c) $x = \sin\left(\frac{1}{2}t\right), \qquad y = \cos\left(\frac{1}{2}t\right), \qquad -\pi \le t \le \pi$

Problem 3: Eliminate the parameter to find a Cartesian equation of the curve.

(a) $x = t, \qquad y = t^2, \qquad -2 \le t \le 4$

(b) $x = t - 1, \qquad y = t^3 + 1, \qquad -2 \le t \le 2$

(c) $x = \sin(t), \qquad y = \cos^2(t), \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$