# MATH 1336: Calculus III

## Section 5.5: Alternating Series & Absolute Convergence

## Sections 5.5 & 5.6 - More Series Tests!:

In these sections, we'll learn tests that can be applied to series that may have some negative terms.

### Alternating Series Test (AST):

If an alternating series can be written in one of the following forms:

$$\sum_{n=1}^{\infty}(-1)^{n+1}\ b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \ldots = \sum_{n=1}^{\infty}(-1)^{n-1}\ b_n$$

-OR-

$$\sum_{n=1}^{\infty}(-1)^{n}\ b_n = -b_1 + b_2 - b_3 + b_4 - b_5 + b_6 - \ldots$$

and satisfies both of the conditions listed below, then the series is convergent.

(i) $0 < b_{n+1} \leq b_n$

(ii) $\lim\limits_{n\to\infty} b_n = 0$

#### Alternating Series Estimation/Remainder Theorem:

If $S$ is the sum of a convergent alternating series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\ b_n \qquad \text{-OR-} \qquad \sum_{n=1}^{\infty}(-1)^{n}\ b_n,$$

then

$$|R_N| = |S - S_N| \leq b_{N+1}$$

### Absolute & Conditional Convergence:

A series $\sum a_n$ is called **absolutely convergent** if the series $\sum |a_n|$ is convergent.

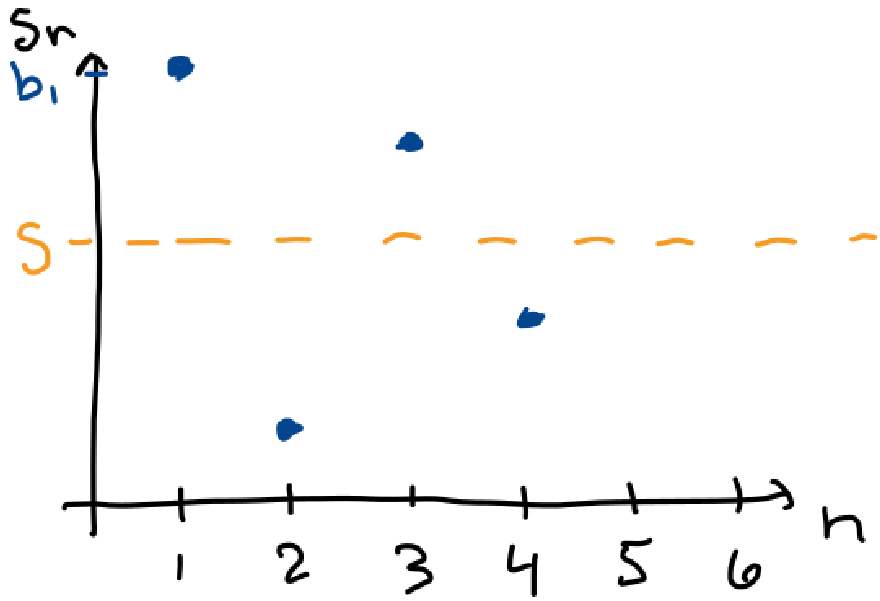


Figure 1: Illustration of a convergent alternating series to demonstrate the Alternating Series Estimation Theorem.

A series $\sum a_n$ is called **conditionally convergent** if it is convergent, but *not* absolutely convergent.

**Theorem 5.15:**
If a series $\sum |a_n|$ converges, then $\sum a_n$ converges.

# Examples:

Example 1: Use the AST to show that the Alternating Harmonic Series converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

*Follow-up Question:* How many terms would we need to use to obtain an estimate for the sum of the series that is within $0.01$ of the exact value?

Example 2: Determine the convergence behavior of the following series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5n}{6n-2}$$

Example 3: Let's determine if the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$$

Example 4: Apply the Alternating Series Test to the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$$

Example 5: Find an upper bound on the error from using only the first 5 terms of the series in Example likesection4 to approximate the sum of the series.

# Problems for Group Work:

**Be sure to fully justify your reasoning as a part of your solutions.**
The answers are upside-down on the bottom of this page.

For Problems likesection1-likesection4, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{2/3}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^3}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+\frac{1}{n}}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n - 3^n}$

**Problem likesection2:** Absolutely Convergent,**Problem likesection3:** Divergent, **Problem likesection4:** Absolutely Convergent