MATH 1336: Calculus III

Section 5.6: Ratio Test & Root Test

Sections 5.5 & 5.6 - More Series Tests!:

Both of the tests listed below can be used on series that have some negative terms.
Both tests are built on the idea of comparing how fast the terms are going to zero with geometric series.
Note: Both tests will be inconclusive when applied to p-series or series with terms that are rational functions!

Ratio Test:

Consider the series $\sum a_n$.

(i) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L < 1$:
then $\sum a_n$ is absolutely convergent.

(ii) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L > 1$ -OR- $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$:
then $\sum a_n$ is divergent.

(iii) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$:
then the test is is inconclusive.
(Try something else!)

<u>Useful for Series:</u>
that contain a factorial

Root Test:

Consider the series $\sum a_n$.

(i) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L < 1$:
then $\sum a_n$ is absolutely convergent.

(ii) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = L > 1$ -OR- $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \infty$:
then $\sum a_n$ is divergent.

(iii) $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$:
then the test is is inconclusive.
(Try something else!)

Useful for Series:
where the whole expression is raised to the n^{th} power

Examples we will work through together:

Determine the Convergence of the following Series:

Example 1: $\sum_{n=1}^{\infty} \frac{10^n}{n!}$

Example 2: $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

Use the Ratio Test on the following Series:

Example 3: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

Example 4: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

Example 5: $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Use the Root Test on the following Series:

Example 6: $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

Example 7: $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$

Problems for Group Work:

For each of the following series, apply either the Ratio Test or the Root Test, or state that one of those tests is inconclusive.

1. $$\sum_{n=0}^{\infty}(-1)^n\frac{n!}{1000^n}$$

2. $$\sum_{n=1}^{\infty}\frac{2^n}{n^{2n}}$$

3. $$\sum_{n=1}^{\infty}\frac{n}{n^2+1}$$

4. $$\sum_{n=0}^{\infty}\left(\frac{5n}{3n+1}\right)^n$$

ers: **Problem 1:** Diverges, **Problem 2:** Converges Absolutely, **Problem 3:** Ratio Test is Inconclusive, **Problem 4:** Diverges