

Christine Pho

Computational Physics

Mia Bovill

May 8, 2019

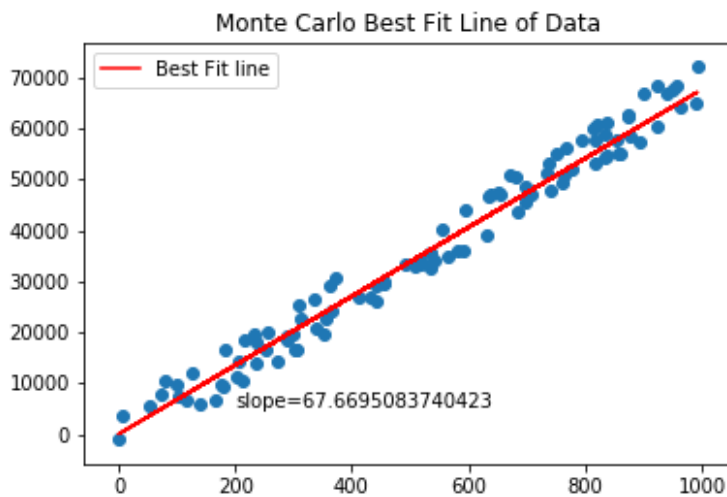
## Final Exam

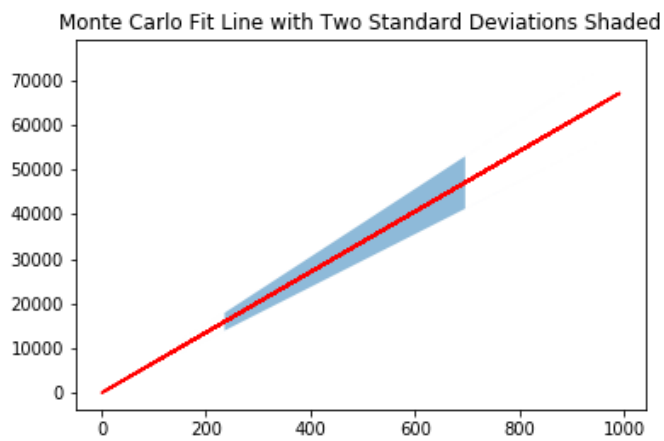
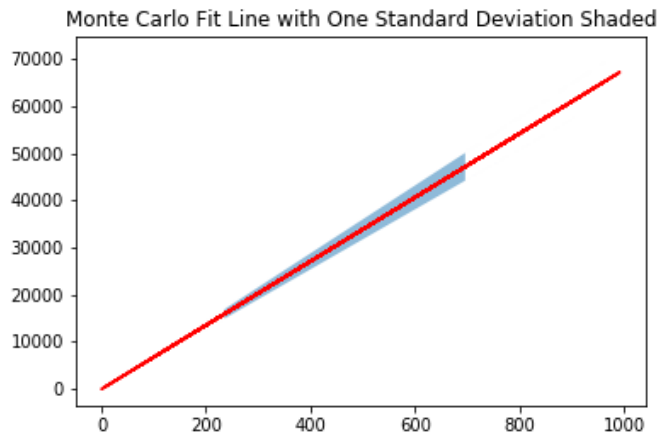
### Part 1. Hubble's Law

Hubble's Law Hubble's law is the relationship between the recessional velocity of galaxy calculated with Doppler Shift and its distance from us, determined using a variety of methods. It is given the form  $v = H_0 d$  where  $v$  is the velocity in  $\text{km s}^{-1}$ ,  $d$  is the distance in Mpc and  $H_0$  is Hubble's constant. You can assume that at a distance of zero, the recessional velocity is also zero. With your exam you have been given a data file which contains distances in Mpc (millions of parsecs, where 1 parsec = 3.26 light years) and velocities in  $\text{km s}^{-1}$  for a set of galaxies.

- (a) Using Monte Carlo techniques determine a the best fit value of Hubble's constant and the standard deviation. Plot the data, the best fit line for Hubble's Law, and a shaded region for Hubble's Law for 1 and 2 standard deviations above and below the best fit value.

Best fit value for  $H_0$  was 67.6695083740423, and standard deviation was 4.215235697619593





- (b) Hubble's constant can be used to measure the age of the Universe,  $\tau$  where  $\tau = 1/H_0$ . Using your best fit  $H_0$  and its standard deviations to calculate the age of the Universe with error bars. We know that the oldest stellar populations are around 13.5 Gyr old. Is that knowledge consistent with your value of  $H_0$ ? Does it constrain your value further?

By using the best fit  $H_0$  of 67.6695083740423 and converting it seconds<sup>-1</sup> The age of the universe was found to be 4.49e11 seconds old.

## Part 2. Real Projectile Motion

Many elementary mechanics problems deal with the physics of objects moving through the air, but they almost always ignore friction and air resistance. Consider a spherical cannon ball shot through air. The air resistance on a moving sphere is a force in the opposite direction with the magnitude:

$$F = \frac{1}{2} \pi R^2 \rho C v^2$$

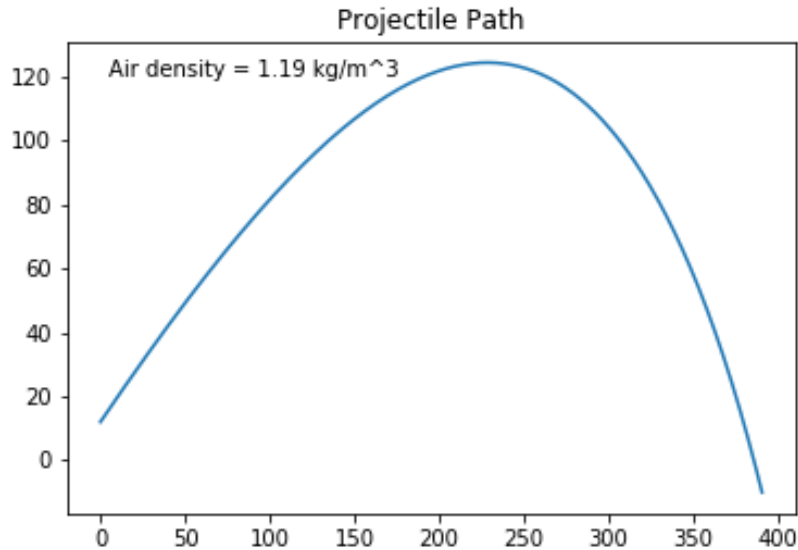
where  $R$  is the sphere's radius,  $\rho$  the density of air,  $v$  is the velocity, and  $C$  is the coefficient of drag (dependent of the shape of the object).

- (a) Starting from Newton's second law,  $F = ma$ , show that the equations of motion for the position  $(x, y)$  of the cannonball are:

The image shows a handwritten derivation of the equations of motion for a projectile with air resistance. The derivation starts with the drag force  $F_{\text{drag}}$  and the gravitational force  $F_g$ , then uses Newton's second law to derive the differential equations for position and velocity.

$$\begin{aligned} F_{\text{drag}} &= \underbrace{\frac{1}{2} \pi R^2 \rho C}_{\text{constant}} \underbrace{v^2}_{\text{unit vector}} \left( -\frac{\vec{v}}{|\vec{v}|} \right) \\ &= - \frac{1}{2} \pi R^2 \rho C |\vec{v}| \vec{v} \\ &= - \frac{1}{2} \pi R^2 \rho C \sqrt{v_x^2 + v_y^2} (v_x \hat{x} + v_y \hat{y}) \\ F_g &= -mg \hat{y} \end{aligned}$$
$$\left\{ \begin{aligned} \frac{d^2 x}{dt^2} &= - \frac{1}{m} \sqrt{v_x^2 + v_y^2} (v_x) = \frac{dv_x}{dt} \\ \frac{d^2 y}{dt^2} &= -g - \frac{1}{m} \sqrt{v_x^2 + v_y^2} (v_y) = \frac{dv_y}{dt} \\ \frac{dx}{dt} &= v_x \\ \frac{dy}{dt} &= v_y \end{aligned} \right.$$

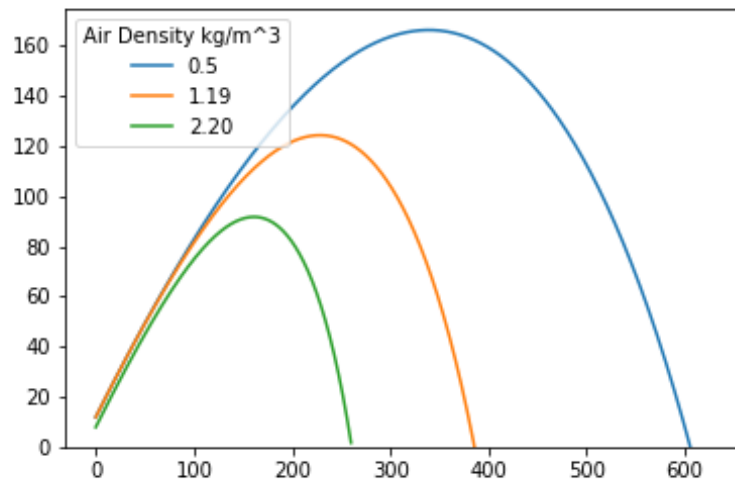
- (b) Change these two second-order equations into four first-order equations using the methods you have learned, then write a program that solves the equations for a cannon ball of: density ( $P_b$ ) =  $11.3 \text{ g/cm}^3$  , radius =  $6 \text{ cm}$  , firing angle =  $38^\circ$  to the horizontal, firing elevation =  $12 \text{ m}$  initial velocity =  $110 \text{ m s}^{-1}$  . density of air  $\rho = 1.19 \text{ kg m}^{-3}$  coefficient of drag  $C = 0.47$  (Assume all other ground is flat beside the firing platform). Measure the total distance travelled by the projectile and plot the projectiles paths.



The distance traveled by this projectile was 385 meters

- (c) Now experiment with different projectile materials to determine which would fire the farthest.

The projectile material density was not included as a parameter in the drag force equation. So, instead I varied the air density.



The distance traveled by each of the projectiles:

Air density kg/m <sup>3</sup>	Distance Traveled
0.5	602
1.19	385
2.2	260