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Computational Physics

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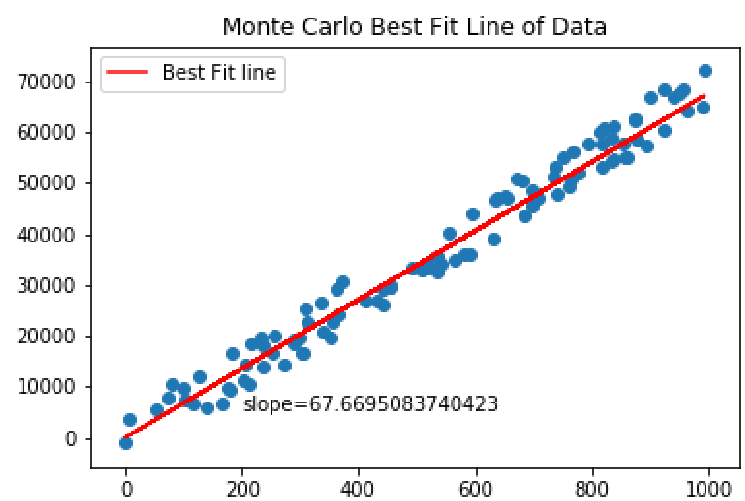
Final Exam

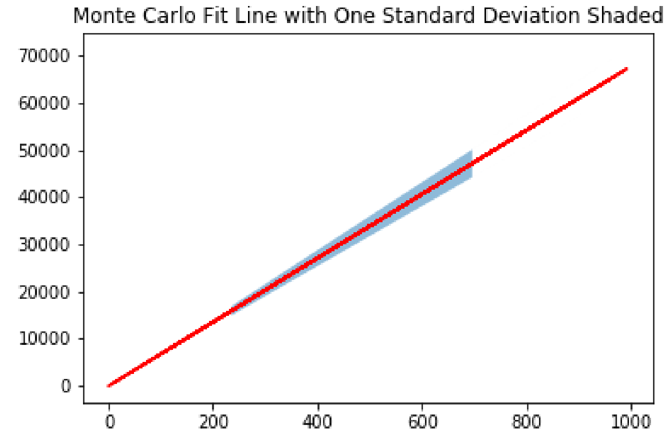
Part 1. Hubble's Law

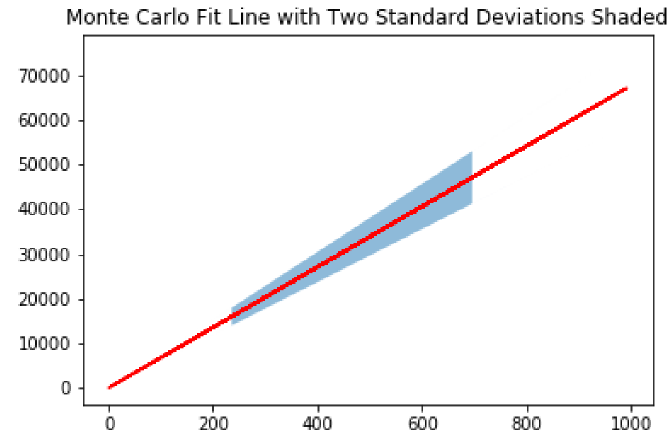
Hubble's Law Hubble's law is the relationship between the recessional velocity of galaxy calculated with Doppler Shift and its distance from us, determined using a variety of methods. It is given the form v = Hod where v is the velocity in km s−1 , d is the distance in Mpc and Ho is Hubble's constant. You can assume that at a distance of zero, the recessional velocity is also zero. With your exam you have been given a data file which contains distances in Mpc (millions of parsecs, where 1 parsec = 3.26 light years) and velocities in km s−1 for a set of galaxies.

(a) Using Monte Carlo techniques determine a the best fit value of Hubble's constant and the standard deviation. Plot the data, the best fit line for Hubble's Law, and a shaded region for Hubble's Law for 1 and 2 standard deviations above and below the best fit value.

Best fit value for Ho was 67.6695083740423, and standard deviation was 4.215235697619593







(b) Hubble's constant can be used to measure the age of the Universe, τ where $\tau = 1/Ho$. Using you best fit Ho and its standard deviations to calculation the age of the Universe with error bars. We know that the oldest stellar populations are around 13.5 Gyr old. Is that knowledge consistent with your value of Ho? Does it constrain your value further?

By using the best fit Ho of 67.6695083740423 and converting it seconds-1 The age of the universe was found to be 4.49e11 seconds old.

Part 2. Real Projectile Motion

Many elementary mechanics problems deal with the physics of objects moving through the air, but they almost always ignore friction and air resistance. Consider a spherical cannon ball shot through air. The air resistance on a moving sphere is a force in the opposite direction with the magnitude:

$$\mathrm{F} = \frac{1}{2}\,\pi R^2 \rho C v^2$$

where R is the sphere's radius, ρ the density of air, v is the velocity, and C is the coefficient of drag (dependent of the shape of the object).

(a) Starting from Newton's second law, F = ma, show that the equations of motion for the position (x,y,) of the cannonball are:

$$F_{drag} = \overbrace{\xi}^{\text{constant}} |\vec{v}|^2 \overbrace{\left(-\frac{\vec{v}}{|\vec{v}|}\right)}^{\text{unit vector}}$$

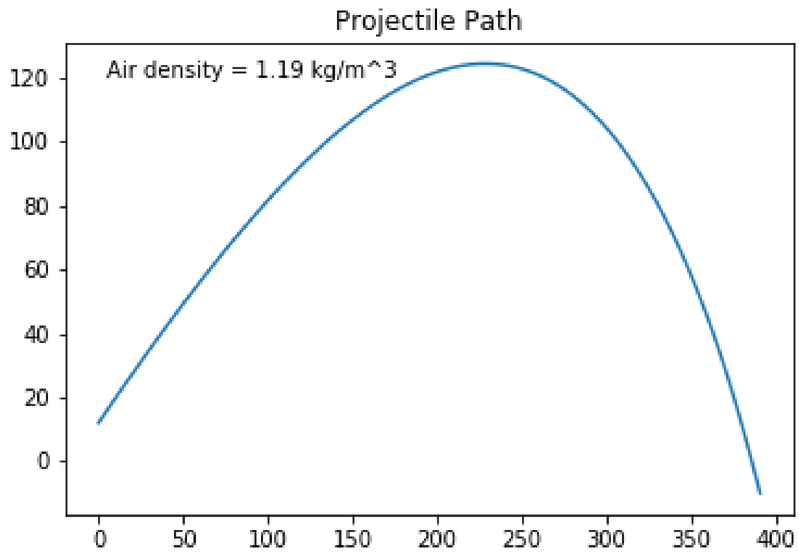
$$= -\xi |\vec{v}|\,\vec{v}$$

$$= -\xi \sqrt{V_x^2 + V_y^2}\left(V_x \hat{x} + V_y \hat{y}\right)$$

$$F_g = -mg\,\hat{y}$$

$$\begin{cases} \dfrac{d^2x}{dt^2} = -\dfrac{\xi}{m}\sqrt{V_x^2 + V_y^2}\,(V_x) = \dfrac{dV_x}{dt} \\[2ex] \dfrac{d^2y}{dt^2} = -g - \dfrac{\xi}{m}\sqrt{V_x^2 + V_y^2}\,(V_y) = \dfrac{dV_y}{dt} \\[2ex] \dfrac{dx}{dt} = V_x \\[2ex] \dfrac{dy}{dt} = V_y \end{cases}$$

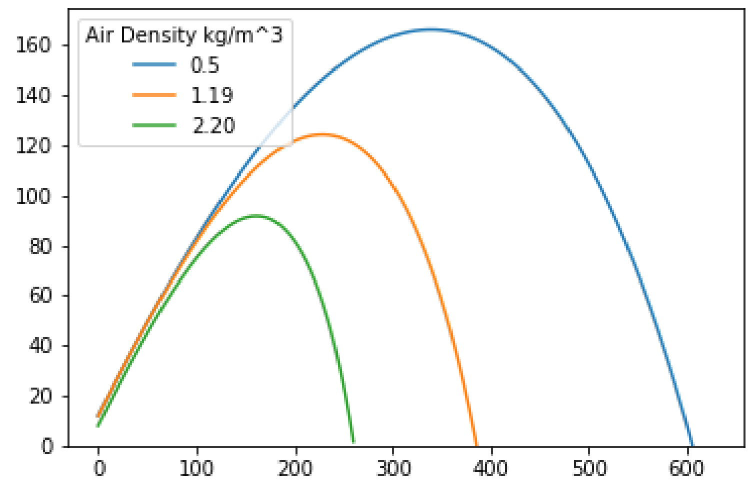
(b) Change these two second-order equations into four first-order equations using the methods you have learned, then write a program that solves the equations for a cannon ball of: density (Pb) = 11.3 g/cm3 , radius = 6 cm , firing angle = 38◦ to the horizontal, firing elevation = 12 m initial velocity = 110 m s−1 . density of air ρ = 1.19 kg m−3 coefficient of drag C = 0.47 (Assume all other ground is flat beside the firing platform). Measure the total distance travelled by the projectile and plot the projectiles paths.



The distance traveled by this projectile was 385 meters

(c) Now experiment with different projectile materials to determine which would fire the farthest.

The projectile material density was not included as a parameter in the drag force equation. So, instead I varied the air density.



The distance traveled by each of the projectiles:

Air density kg/m^3	Distance Traveled
0.5	602
1.19	385
2.2	260

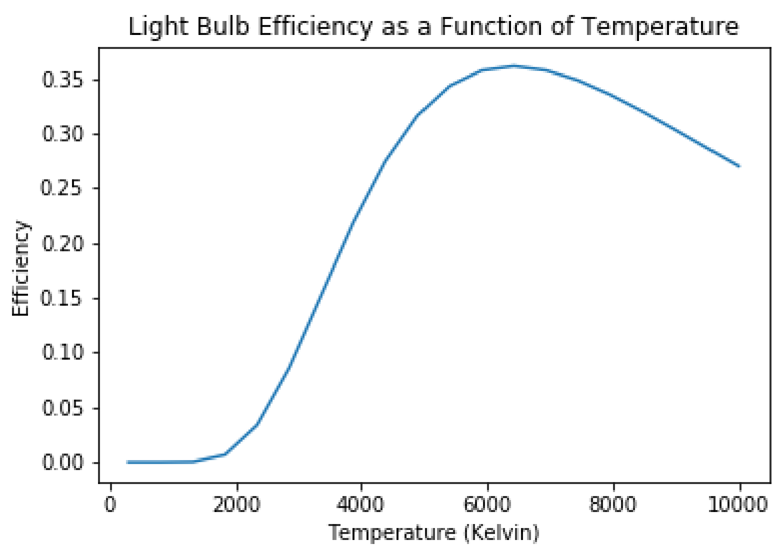
Part 3. The Temperature of a light bulb:

The temperature of a light bulb: An incandescent light bulb is a simple device—it contains a resistive filament, usually made of tungsten, heated by the flow of electricity until it becomes hot enough to radiate thermally. Essentially all of the power consumed by such a bulb is radiated as electromagnetic energy, but some of the radiation is not in the desired wavelengths, which means it is useless for desired lighting purposes. Let us define the efficiency of a light bulb to be the fraction of the radiated energy that falls in a specific band. It's a good approximation to assume that the radiation obeys the Planck radiation law, meaning that the power radiated per unit wavelength λ obeys

$$\mathrm{I}(\lambda) = 2\pi \mathrm{Ahc}^2 2 \frac{\lambda^{-5}}{e^{\mathrm{hc}/\lambda \mathrm{kB\,T}} - 1}$$

where A is the surface area of the filament, T is the temperature, h is Planck's constant, c is the speed of light, and kB is Boltzmann's constant. For a specific use, the desired wavelengths run from λ1 = 450 nm to λ2 = 750 nm, so the total energy radiated in the visible window is R λ2 λ1 I(λ)dλ and the total energy at all wavelengths is R ∞ o I(λ)dλ . Dividing one expression by the other and substituting for I(λ) from above, we get an expression for the efficiency η of the light bulb thus: η = R λ2 λ1 λ −5/(e hc/λk BT −1)dλ R ∞ o λ−5/(e hc/λk BT −1)dλ (4) where the leading constants and the area A have canceled out. Making the substitution x = hc/λkBT, this can also be written as Computational Physics - Final Exam 2 η = R hc/λ1kBT hc/λ2kBT x 3/(e x −1)dx R ∞ o x 3/(e x −1)dx = 15 π 4 Z hc/λ1kBT hc/λ2kBT x 3 e x −1 dx (5) where we have made use of the known exact value of the integral in the denominator.

(a) Write a Python function that takes a temperature T as its argument and calculates the value of η for that temperature from the formula above. The integral in the formula cannot be done analytically, but you can do it numerically using Simpson's Rule. Ensure that you are choosing a high enough number of steps that your answer is convergent. Use your function to make a graph of η as a function of temperature between 300 K and 10,000 K. You should see that there is an intermediate temperature where the efficiency is a maximum. What is that maximum temperature? (b) Is it practical to run a tungsten-filament light bulb at the temperature you found? If not, why not?



The temperature at maximum efficiency is 6426.3 Kelvin.

(b) Is it practical to run a tungsten-filament light bulb at the temperature you found? If not, why not?

No. The melting point of Tungsten is 3695 Kelvin. Running the filament at maximum efficiency would cause it to melt.