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# VORLESUNG 'HIGH PERFORMANCE COMPUTING'

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Summer Term 2013

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# A Reimplementation of the Finite Element Method Software Package using FLENS

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July 20, 2013

## Contents

<b>0</b>	<b>Introduction</b>	<b>3</b>
<b>1</b>	<b>Part I (Christopher Davis)</b>	<b>4</b>
1.1	Matrix and Vector Types . . . . .	4
1.1.1	Equivalent Types and Index Base . . . . .	4
1.1.2	TypeI and TypeII . . . . .	4
1.1.3	FLENSDataVector . . . . .	5
1.1.4	BLAS Overloading . . . . .	7
1.2	The CG Solver . . . . .	8
1.2.1	Implementation . . . . .	8
1.2.2	Testing . . . . .	9
<b>2</b>	<b>Part II (Mazen Ali)</b>	<b>11</b>
2.1	GS Solver . . . . .	11
2.2	Data Structures . . . . .	15
2.2.1	Mesh . . . . .	15
2.2.2	Coupling . . . . .	16
2.3	Debugging notes . . . . .	16
<b>3</b>	<b>Flens_supl (Mazen+Chris)</b>	<b>18</b>
<b>A</b>	<b>Appendix</b>	<b>19</b>
A.1	Desktop . . . . .	19
A.2	Pacioli . . . . .	20
A.3	GotoBLAS . . . . .	20

## 0 Introduction

The basis of this lecture course has been the parallelisation of various numerical methods, with a particular focus on the Finite Element Method. For this project we have been supplied with a software package (which we shall henceforth call *the FEM package*) that computes the solution to the Poisson partial differential equation using the Finite Element Method. This package is self contained - it includes its own custom matrix/vector types, and its own implementations of linear algebra operations on these types. Our goal is to make improvements to the package with the help of FLENS.

FLENS (*Flexible Library for Efficient Numerical Solutions*) is a C++ library written by Dr. Michael Lehn, which offers a comprehensive collection of matrix and vector types. Included is a C++ -based BLAS (*Basic Linear Algebra Subfunctions*) implementation, which provides linear algebra operations, such as matrix-vector multiplication, on these types.

The advantages of using FLENS in the FEM package are numerous. Firstly, use of an external library for matrix/vector types adds some standardisation to the package, aiding, for example, a user who is new to the FEM package, but has experience of FLENS from other projects. Secondly, the library can be linked, with almost trivial effort, to any optimised BLAS or LAPACK libraries available, such as *ATLAS* or *Goto BLAS*, for instant speed improvements of BLAS operations. Thirdly, the FLENS offers overloaded operators for the BLAS operations. We recognise that users from different backgrounds may have a preference regarding the notation used for linear algebra operations, be it the tradition BLAS notation:

```
1 || blas::mv(NoTrans, One, A, p, Zero, Ap);
```

or a notation more akin to that of MATLAB:

```
1 || Ap = A*p;
```

(for matrix A, vector p). With FLENS, we have the choice.

We therefore summarise the aims of this project as follows:

1. Replace all data storage objects with FLENS-based objects.
2. Where possible, replace linear algebra operations with their BLAS equivalents, for speed-up from optimised BLAS libraries.
3. Offer two versions of solvers, one with BLAS notation, one with overloaded operators.

Thus it is worth noting that we are primarily improving the existing implementation from a software design point of view, with possible performance improvements, rather than adding new methods.

## 1 Part I (Christopher Davis)

### 1.1 Matrix and Vector Types

A major part of converting the FEM package to the FLENS framework is the transition from the package's custom storage types to FLENS-based types. Of course, some types, such as the package's `Vector` class, have exact FLENS equivalents. Others, however, contain bespoke objects and methods for the MPI communications. Thus we must create our own custom storage types in these cases.

#### 1.1.1 Equivalent Types and Index Base

We adopt the following direct conversions from the FEM package to FLENS framework:

$$\begin{aligned} \text{Vector} &\rightarrow \text{DenseVector}\langle \text{Array}\langle \text{double} \rangle \rangle \\ \text{IndexVector} &\rightarrow \text{DenseVector}\langle \text{Array}\langle \text{int} \rangle \rangle \end{aligned}$$

However, we must note that the default index base in FLENS, which we are using here, is 1, as opposed to that of the package, which is 0. We make this change, despite the awkwardness it adds to the transition, because this is the natural index base regarding the mesh geometry. Numbering of the mesh nodes starts at 1, and assembly of the system of linear equations frequently accesses vector elements corresponding to node identities. For example in the FEM package:

```
1 || someVec_FEMpackage(nodeID - 1) = someValue;
```

as opposed to using FLENS:

```
1 || someVec_FLENS(nodeID) = someValue;
```

Whilst this is purely cosmetic, it may help to avoid bugs caused by forgetting to subtract 1 from node identities. For consistency, we implemented all FLENS matrices/vectors with index base 1.

FLENS includes a storage scheme for sparse matrices of CRS (Column-Row-Storage) type, offering an alternative to the FEM package's `CRSMatrix` type. However, these must be initialised from a FLENS sparse matrix with a coordinate storage scheme, effecting a change in the implementation of the type, but not requiring the creation of a custom type.

#### 1.1.2 TypeI and TypeII

The FEM package uses the nomenclature defined in the lecture course, of *TypeI* and *TypeII* to distinguish between vectors that contain values corresponding to the problem posed on a compute node's local mesh, or on the global mesh:

- **TypeI**: global values.
- **TypeII**: local values.

We adopt this definition in this paper and in our code, and refer to these types as *MPI vector types*.

### 1.1.3 FLENSDataVector

(Christopher Davis)

The FEM package’s `DataVector` class is the package’s primary custom vector type. Included members are:

- **Vector** object: stores the values of the `DataVector`.
- **Coupling** object: contains mesh geometry information required for MPI communications.
- A `vectorType` enumerated type: determines the MPI vector type of the vector (see Section 1.1.2).

We propose a FLENS-based replacement for this class called `FLENSDataVector`, which incorporates a few small yet profound modifications to the structure.

Firstly, we make the obvious choice of using a FLENS `DenseVector<Array<double> >` to store our vector values. However, we choose to *derive* our class from this FLENS type, rather than specifying a `DenseVector` as a member object. I.e. we use a ‘is-a’ `DenseVector` approach, rather than a ‘has-a’ approach. The advantage here is that our class inherits all methods and overloaded operator from the `DenseVector` class, and can be passed directly to the FLENS BLAS functions. This lends itself to a more parsimonious implementation - under the ‘has-a’ approach we would need to overload every BLAS function, such as:

```
1 | double
2 | dot(FLENSDataVector x, FLENSDataVector y) {
3 |     blas::dot(x.vec, y.vec);
4 | }
```

and clutter our `FLENSDataVector` class with trivial operators, such as:

```
1 | double &
2 | operator()(int index) {
3 |     return vec(index);
4 | }
```

neither of which are desirable.

The `Coupling` object is stored as a constant reference - the same way as in the FEM package.

Next we consider the enumerated type that specifies the MPI vector type. Here we wish to change the structure somewhat - whilst setting the MPI vector type in this fashion is easy and flexible, allowing the type to be changed as and when required, we find this to be *too* flexible, and not ideal from a software design perspective. To help explain the situation, we consider a blind man and his socks. The man in question loves to wear socks, and therefore does so at all times. He commands an extensive collection, consisting of many different colours. Each morning he changes his socks, taking care to pair the dirty socks together so that they remain paired after washing. His system appears to work well - he always wears matching socks, and as a result leads a successful life. But what about if one morning whilst in the middle of changing his socks he is distracted by the telephone ringing, and as such forgets to change one of his socks - now his socks don't match! Whilst we would like to think that some kind person may notify him of his mistake, the world can be a cruel place. Odd socks are rarely tolerated by modern societies, so we can imagine that his life's achievements would probably crumble around him. Returning to the world of the FEM package, we hope that the difficulties that could arise from the enumerated type are clear - there is no way of determining whether the values contained in a `TypeI DataVector` are actually global values. There is always the chance that some rogue function changed the type without modifying the values. Such a problem would not make for an enjoyable debugging task.

Thus our `FLENSDataVector` is implemented as a template class, requiring the MPI vector type to be defined (permanently) at instantiation. The following classes are permitted as specialisation types:

- `class FLvNonMPI`
- `class FLvTypeI`
- `class FLvTypeII`

Clever implementation of the `FLENSDataVector` constructors limits specialisation of the class to these types *only*, as well as ensuring the specification of a Coupling object for MPI types (and not for the non-MPI). We use specialisations of the constructor function for these types, for example:

```

1 | template <>
2 | FLENSDataVector<FLvNonMPI>::FLENSDataVector(int n)
3 |     : DenseVector<Array<double> >(n),
4 |       coupling(Coupling())
5 | {
6 |     //Permits instantiation of FLvNonMPI specialisation.
7 | }
```

and we add a line to the unspecialised constructor that will cause an error at *compile time* if scope ever reaches there (which would be due to a wrong type specialisation):

```

1 | template <typename VTYPE>
2 | FLENSDataVector<VTYPE>::FLENSDataVector(int n)
3 |     : coupling(Coupling())
4 | {
5 |     VTYPE::CHK; //<-- If scope ever reaches here,
```

```

6 | // compilation will fail.
7 | //e.g. if FLENSDataVector<double> instantiated.
8 | }

```

Thus the following instantiation would cause a compiler error:

```

1 | FLENSDataVector<double> oops(5);

```

As such, we have limited the potential for undefined behaviour.

All communication-related member methods of the FEM package's `DataVector` are added to the `FLENSDataVector` with no significant changes. For conversion methods, we require the object to be of the destination type. For example:

```

1 | FLENSDataVector<FLvTypeI> myVec(5, Coupling());
2 |
3 | ///////////////////////////////////////////////////
4 | // *** fill with local values *** //
5 | ///////////////////////////////////////////////////
6 |
7 | myVec.typeII_2_I();           <-- perfect
8 | //myVec.typeI_2_II();        <-- would cause compiler error

```

#### 1.1.4 BLAS Overloading

Most BLAS functions can be used in the FEM package in their usual form. The copy and dot product functions, however, require attention.

Copying between two `FLENSDataVectors` of the same MPI vector type can use the BLAS copy function without further assistance. The types match, and we can't do anything about the constant reference to the `Coupling` object (this must be left to the user to ensure).

When copying vectors of differing MPI vector types, we overload the BLAS copy function. Within this overloaded function, we call the BLAS copy function to copy the vector values by *upcasting* the `FLENSDataVectors` to their parent class `DenseVector<Array<double>>`, then perform the appropriate conversion, for example:

```

1 | void
2 | copy(FLENSDataVector<FLvTypeII> &orig, FLENSDataVector<FLvTypeI> &dest)
3 | {
4 |
5 |     //Copy data as usual (masquerading as a DenseVector :) ):
6 |     blas::copy(*static_cast<DenseVector<Array<double>>>*>(&orig),
7 |               *static_cast<DenseVector<Array<double>>>*>(&dest));
8 |
9 |     //Perform vector type conversion:
10 |    dest.typeII_2_I();
11 | }

```

We use a similar technique for the dot product - the dot product of the two supplied vectors is calculated using BLAS via upcasting, and then the appropriate MPI communication is performed.

By ensuring that our overloaded functions still use the FLENS BLAS implementation, we maintain the possibility for objective (2) in the Introduction.

We point out that our use of ‘proper’ object type to define the MPI vector types means that the many type asserts present in the FEM package’s linear algebra subroutines, such as:

```

1 double dot(DataVector &u, DataVector &v)
2 {
3     if(u.type==nonMPI && v.type==nonMPI)
4         return u.values.dot(v.values);
5
6     // we only multiply typeI and typeII vectors
7     assert(u.type != v.type);           //<--assert

```

are not required. All type checking is moved to compile time, a significant advantage in terms of both runtime efficiency and ease of debugging.

## 1.2 The CG Solver

### 1.2.1 Implementation

In this section we look at the implementation of the conjugate gradient method for solving a system of linear equations. The solver was initially integrated into the FEM package via a wrapper, the details of which are described below in Section 2.1.

The CG solver uses many linear algebra operations, and is therefore a prime candidate for using BLAS functions. For example, we replace FEM package lines such as:

```

1 CRSmatVec(p,A,x);
2 add(r2,p,-1.);

```

with the more universally recognised:

```

1 blas::mv(NoTrans, One, A, x, Zero, p);
2 blas::axpy(-One, p, r2);

```

As claimed in objective (3) in the Introduction, we also offer a version of the CG solver where such BLAS functions are overloaded, providing a MATLAB-style notation:

```

1 p = A*x;
2 r2 = r2 - p;

```

The CG solver containing this notation is contained in `Flens_supl/overloaded`. Its functionality does not differ.



### 1.2.2 Testing

Here we undertake some benchmarking on the Pacioli compute cluster. We examine the performance of the new FEM implementation, with respect to:

1. Serial vs. parallel performance.
2. Performance boost from GotoBLAS (provided by the OpenBLAS library).

Testing was conducted using a mesh with four domains (hence requiring 4 computation processes). For full homogeneity, the program was executed such that each process was run on separate (identical) nodes, rather than making use of multiple processors/cores on single nodes. Thus all process communications were performed over the Infiniband network using the MPI.

Figure 1 shows the compute times required to assemble the FEM system. This part of computation requires no communication between processes, and incorporates no BLAS subroutines. The results follow our logical predictions, and can be summarised as follows:

- The parallel implementation is, on average, 4.2x faster than the serial implementation for a given number of mesh elements.
- GotoBLAS is of no benefit here.

Figure 2 shows the compute times required to solve the system of linear equations. This part of computation does require communication between processes, and does make extensive use of BLAS subroutines. The results again follow our predictions, and can be summarised as follows:

- The parallel implementation is, on average, only 1.8x faster than the serial implementation for a given number of mesh elements. Thus we see that the communication here acts as a bottleneck, with a detrimental effect on compute times.
- The near-trivial linking of the OpenBLAS library effects a 1.34x performance increase in the serial implementation, and a 1.43x performance increase in the parallel implementation, on average. This ‘free’ improvement was only possible due to the use of FLENS.

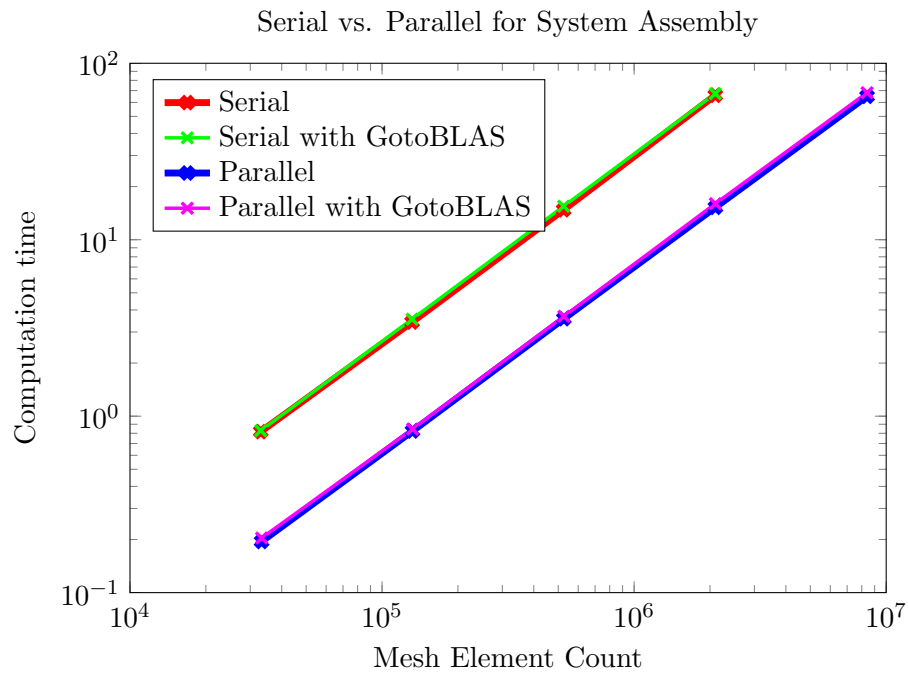


Figure 1: Compute times to perform system assembly.

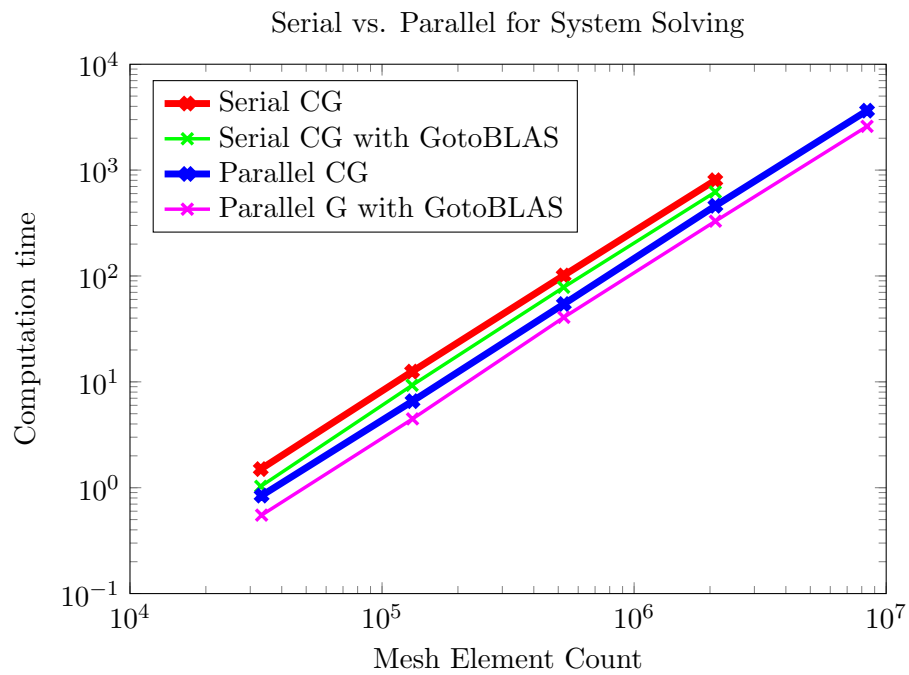


Figure 2: Compute times to perform system assembly.

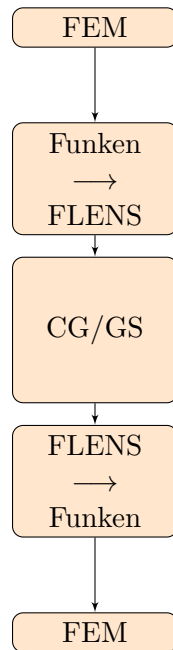
## 2 Part II (Mazen Ali)

This section continues describing the performed changes to the FEM package in order to replace the provided LinearAlgebra library with FLENS. Note that the Makefiles have to be slightly adjusted as well, please refer to the provided package. Further adjustments, not described here, were performed for the main functions (main-serial and main-parallel).

### 2.1 GS Solver

One of the main components of the FEM package is the solver that provides the end-result. Implemented in the FEM package were the CG (*Conjugate Gradient*) and the GS (*Gauss-Seidel*) solvers, parallel and serial versions. The adapting of the CG-Solver to FLENS was described in the previous section.

Analogue to the CG solver, the GS solver was initially integrated into FEM via a wrapper. Schematically the wrapper does the following:



Function wrapper

The wrappers transform the data from the original format to FLENS and from FLENS back to the original format upon completion of the solving procedure. This functionality is provided in `Flens_supl` by the headers `funk2flens.h` and `flens2funk.h`. For the solvers the transformation is required for a

matrix and a vector type. Although for the Poisson problem and the chosen basis functions it suffices to consider transformations for sparse matrices, Flens\_supl includes both transformations for sparse and dense matrices, as well as a GS-solver for dense matrix classes (possible future utilization as preconditioner for e.g. multigrid in problems with resulting non-sparse systems).

The implemented transformation in Flens\_supl copies the data manually from the original storing format into FLENS and back. However, FLENS also provides other alternatives for transforming data to FLENS using *Matrix/Vector-Views*. The concept is illustrated below, where data from the original storing format is transformed to FLENS:

```

1  int
2  main()
3  {
4      // Typedefs
5      typedef flens::DenseVector<flens::Array<double> >
6                                          Fl_Vec;
7      typedef flens::DenseVector<flens::Array<double>>::View>
8                                          VecView;
9      typedef flens::Array<double>::View
10                                         View;
11
12     // Initilaize with 0s
13     Vector v_funk(5); // Funken
14     VecView v_flens_view = View(5, v_funk.data()); // Flens View
15     Fl_Vec v_flens = v_flens_view; // Flens
16
17     // Change values
18     v_funk(0) = 1;
19     v_flens_view(5) = 1;
20
21     // Print
22     cout << "funk=" << endl;
23     print(v_funk, 0, 4);
24     cout << endl;
25     cout << "flens_view=" << endl;
26     print(v_flens_view, 1, 5);
27     cout << endl;
28     cout << "flens=" << endl;
29     print(v_flens, 1, 5);
30     return 0;
31 };
```

which would produce the output:

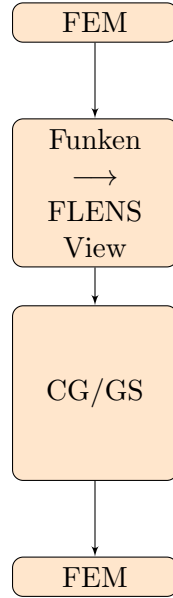
```

$ funk=
1 0 0 0 1
flens_view=
1 0 0 0 1
flens=
```

0 0 0 0 0 FLENS views can be thus used in two ways: reference data from any other linear algebra package, as long as it provides access to the underlying C-Array, maintaining all the functionality of FLENS; or initialize FLENS non-view data structures via the internal FLENS copy constructor, thus

avoiding copying the values manually.

Note that this offers another form of "wrapping" for the solvers which does not involve data copying. The original data can be referenced via FLENS Matrix/Vector-Views using non-const view for the solutions vector and const views for the rest. The idea is schematically illustrated below:



Function wrapper w. Views

After completing the modification process for all data structures in the FEM package the wrappers for the solvers are turned off, since not required anymore.

The implementation of the solvers themselves remained mostly unchanged. The required changes included:

- Index base: by default the index base in FLENS is 1 (FORTRAN style), as opposed to the original data structures where the index base is 0 (C style). FLENS provides the possibility to set the index base (and range), as well as operators to access first and last indexes of its data structures. Thus, it is possible to utilize these operators in order to provide maximum flexibility w.r.t. choice of index base. However, we decided not to follow this approach, since this would make the code longer and less readable and the mentioned flexibility does not provide any foreseeable advantages. Thus, the index base was assumed to be 1 in all FEM data structures, as by default in FLENS, and the code was modified accordingly.
- CRS matrices access: FLENS CRS matrices can be accessed much in the same way as the original CRS matrices:

```
1 || const auto &_rows = A.engine().rows();
```

```

2 | const auto &_cols = A.engine().cols();
3 | const auto &_vals = A.engine().values();

```

The keyword *auto* is recognized by the C++11 standard, i.e. this necessitates the use of a C++11 conform compiler, such as g++ version 4.7 or higher<sup>1</sup>. Compile via:

\$ g++ hello.cpp -std=c++11 <options> The keyword *auto* allows the compiler to deduce the type of the declared variable, applying rules similar to template argument deduction during function calls. Hence, this saves the programmer a lot of typing and makes the code more readable for the user.

*engine()* is the storage mechanism for FLENS and for CRS matrices it contains the getter methods *rows()*, *cols()* and *values()*, that return the corresponding vectors of type *DenseVector*. In order to preserve the functionality of FLENS data structures, it is recommended to use references instead of pointers to the data (which would also be possible). Usage of the attained vectors is analogue to the standard usage of CRS data structures, e.g. as implemented in the original FEM package.

Note, however, that FLENS sparse matrices do not allow to access elements directly (which can otherwise result in unintentional densifying), i.e.:

```

1 | A(2,2) += 3; // works fine for CoordStorage
2 | double _tmp = A(2,2); // compile error

```

The second access would produce a compile error for all sparse FLENS classes. Thus, for retrieving values of the matrix a search loop is required.

- Underscore operator `_`: FLENS provides the utility of an underscore operator that can be used to set or copy portions of matrices and vectors, similar as in e.g. MATLAB:

```

1 | const Underscore<IndexType> _
2 | // Set MyVector to 3rd column of MyMatrix (must be same length)
3 | MyVector = MyMatrix(_,3);
4 | // Set Myvector's elements 1 to 3 to first elements of first row
5 | MyVector(_(1,3)) = MyMatrix(1,_(1,3));

```

The underscore operator substituted the original *set()* and *memcpy()* calls of the LinearAlgebra data structures.

- Type I,II vectors: the original *DataVectors* were replaced by templated *FLENSDataVectors* that were described in Part I. As already mentioned, this preserves type safety. Usage is in many ways identical to the original *DataVectors*, main difference being that type conversions are performed during copy operations or during explicit method calls, such as *typeII\_2.I* or vice versa.
- Templates: Both solvers were templated, as well as most of *Flens\_supl* headers. The only exceptions are some of the headers that were used as temporary "adapters" to introduce FLENS into the original FEM package. Apart from some minor flexibility, this would allow to utilize *Flens\_supl* via headers only. Note that the FLENS library is headers only as well.

---

<sup>1</sup>Note that this is required for the entire FLENS library.

The dense GS solver required the same modifications, CRS matrix access excluded. The GS solver was tested and debugged.

## 2.2 Data Structures

In this section the adjustments for the Mesh and Coupling data structures are briefly described.

### 2.2.1 Mesh

Apart from the self explanatory data type changes, the required adjustments included:

- Index Base: as described in previous section.
- Read and write methods: Flens\_supl includes the templated read and write methods, implemented similar to the original read and write methods of the FEM package.
- Const-Correctness: note that another advantage of the FLENS library is that it ensures const-correctness. Therefore the constructor calls had to be modified accordingly. E.g. in the original Mesh constructor the input vector *\_dirichlet* is declared as *const* although write access is required, which would cause a compile error in FLENS.
- Underscore operator \_: as described in the previous section.
- CRS\_sort: for the method provideGeometricData() the original FEM package utilized the constructor for the CRSMatrix class to sort the values of a vector. The constructor accepted 3 vectors of same size, the first contained the row index, the second the column index and the third the corresponding value in the matrix. This functionality is provided by the templated header function CRS\_sort.h, implemented analogue to the constructor call of the original FEM package. Alternatively, this can be done entirely without intermediary headers, utilizing the FLENS CoordStorage format and converting to GeCRSMatrix:

```

1 // Initialize (_numRows, _numCols assumed to be already computed in a preceding
  for-loop)
2 flens::GeCRSMatrix<flens::CRS<ElementType> >
3                               vals_crs;
4 flens::GeCoordMatrix<flens::CoordStorage<ElementType> >
5                               vals_coord(_numRows, _numCols);
6 // Fill
7 for (IndexType i=1; i<=rowi.length(); ++i)
8 {
9     if (vals(i) != 0)
10     {
11         vals_coord(rowi(i), coli(i)) += vals(i);
12     }
13 }
14 // Convert

```

```

15 | vals_crs = vals_coord;
16 | // Access
17 | const auto &_vals = vals_crs.engine().values();

```

The latter way requires less writing but internally does more than actually necessary. However, even for large vectors this does not have a substantial influence on run-time, so that the end-choice is a matter of taste.

After adjusting the Mesh class, the dependencies in Fem.cpp have to be adjusted accordingly. The modified data structures were tested and debugged.

### 2.2.2 Coupling

Other than the self explanatory data type changes, no further modifications for the Coupling class were required. The dependencies have to be adjusted accordingly in Mesh, Fem, FLENSDataVector and the GS solver. The modifications were tested and debugged.

## 2.3 Debugging notes

The modifications described above required extensive debugging for both parts, as well as the merging process. Since this amounted to the biggest portion of the entire work time, this section will illustrate on a simple example of a common segmentation fault how to debug using core dumps. This will hopefully save future users a lot of time when modifying the FEM package.

The FLENS library utilizes *assert()* to ensure the index is not out of bounds. However, since these assertions are "hidden" deep in the templates and there is no exception handling implemented, the resulting error message will not provide any useful information about the origin of the faulty access. The system will attempt to produce a core dump, which is however turned off by default on most modern OSs. For example, the following code will produce a segmentation fault:

```

1 | int
2 | main()
3 | {
4 |     flens::DenseVector <flens::Array<double> > V(5);
5 |     V(6) = 3;
6 |     return 0;
7 | };

```

Compile and run:

```

$ g++ seg_fault.cpp -o seg_fault -g
# ./seg_fault
seg_fault: /usr/local/FLENS/flens/storage/array/array.tcc:108:
flens::Array<T, I, A>::ElementType& flens::Array<T, I, A>::operator()
(flens::Array<T, I, A>::IndexType) [with T = double; I = flens::IndexOptions<>];

```



```
A = std::allocator<double>; flens::Array<T, I, A>::ElementType = double;
flens::Array<T, I, A>::IndexType = int]: Assertion 'index<=lastIndex()' failed.
Aborted (core dumped)
```

As can be seen at the end of the error message, it claims that a core was dumped, which does not have to be true. To indeed enable core dumping type in the command line:

\$ `ulimit -c unlimited` Instead of *unlimited* you can specify any other data size (in number of 512-byte blocks). After running the executable again, the crash now generates a core dump, by default in the same directory. Note that for debugging it is advisable to compile with the `-g` option, since the resulting machine code then contains more information. To analyze the produced core one can e.g. use the standard `gdb` debugger and proceed by typing:

\$ `gdb -c core seg_fault` where the executable has to be specified as well, otherwise the machine code messages will not be "linked" with the source code. The last statements will usually be some calls to the kernel, e.g. a request to terminate the program. Working up the core file, one then can get to the observed error message:

```
$ Program terminated with signal 6, Aborted.
#0 0x00007f3fa01f1425 in raise () from /lib/x86_64-linux-gnu/libc.so.6
(gdb) up
#1 0x00007f3fa01f4b8b in abort () from /lib/x86_64-linux-gnu/libc.so.6
(gdb)
#2 0x00007f3fa01ea0ee in ?? () from /lib/x86_64-linux-gnu/libc.so.6
(gdb)
#3 0x00007f3fa01ea192 in __assert_fail () from /lib/x86_64-linux-gnu/libc.so.6
(gdb)
#4 0x0000000000400835 in operator() (index=6, this=0x7fff85fc2108)
at /usr/local/FLENS/flens/storage/array/array.tcc:108
108 ASSERT(index<=lastIndex());
```

Working up further will lead to preceding calls and then finally to the origin of the faulty access:

```
$ 4 0x0000000000400835 in operator() (index=6, this=0x7fff85fc2108)
at /usr/local/FLENS/flens/storage/array/array.tcc:108
108 ASSERT(index<=lastIndex());
(gdb)
#5 operator() (index=6, this=0x7fff85fc2100)
at /usr/local/FLENS/flens/vectortypes/impl/densevector.tcc:201
201 return _array(index);
(gdb)
#6 main () at seg_fault.cpp:22
```

22 `v(6) = 3;` Obviously the benefits of analyzing core dumps extend to any sort of program crashes and it is generally a good way to make debugging much more efficient.

Unfortunately, the approach does not apply in general to MPI. Though a core dump can still be generated, the content of the core is usually useless for debugging purposes. This, together with the fact that standard debuggers are not suited for parallel programs, often forces the programmer to debug "by hand", i.e. via `cout` and `Barrier()`.

### 3 Flens\_supl (Mazen+Chris)

This section summarizes the headers in Flens\_supl.

- CRS\_sort.h: sorting function required in Mesh; accepts a vector of values with the corresponding row and column coordinates, returns a sorted vector values; sorted according to the order in a CRS matrix; templated;
- FLENSDataVector.h: Type I and II (2 separate data types) data vectors derived from FLENS DenseVector; required for parallel computations; methods implemented as in the original FEM package; templated;
- FLENS\_read\_write.h: read method for FLENS and write methods for FLENS dense vectors and matrices; implemented as in the original FEM package; templated;
- cg\_mpi\_blas.h: CG sparse solver for MPI; implemented as in the original FEM package; templated;
- cg\_nompi\_blas.h: CG sparse solver; standard implementation; templated;
- gs\_mpi\_blas.h: GS sparse and dense solvers for MPI; implemented as in the original FEM package; templated;
- gs\_nompi\_blas.h: GS sparse and dense solvers; standard implementation; templated;
- flens2c.h: converts FLENS DenseVector to C array; initially required for wrapping the original package with FLENS data structures; not required in the final version; non-templated;
- flens2funk.h: converts data from FLENS data structures to the original FEM package; initially required for wrapping; not required in the final version; templated;
- funk2flens.h: converts data from the original FEM package to FLENS; initially required for wrapping; not required in the final version; templated;
- wrappers.h: wraps the CG and GS solvers for use with the data structures from the original FEM package; not required in the final version; non-templated;

## A Appendix

In this appendix we offer instructions on how to set up and run our software package on both a desktop computer and on the Pacioli compute cluster. We assume the target device is of the UNIX family, and already has the following:

- gcc of at least version 4.7
- git
- MPI

### A.1 Desktop

1. Make a new directory where you would like the magic to happen.

```
$ mkdir FEMtastic
```

2. Add our package (the directory called HPC\_FLENS) to this new directory.

3. Get and build FLENS:

```
$ git clone https://github.com/michael-lehn/FLENS.git
$ cd ./FLENS
$ make
$ cd ..
```

4. Our makefiles assume FLENS is located under /usr/local/FLENS, so copy it there. If this isn't possible, the makefiles will require minor modification.

```
$ cp -rvf ./FLENS /usr/local/FLENS
```

5. Build our package:

```
$ cd ./HPC_FLENS
$ make
```

6. Run our package.

Replace `n` with the number of mesh refinements you desire (less than 8 is sensible), and `cg` can be replaced by `gs` if you have a lot of time on your hands:

- (a) Serial implementation:

```
$ ./main-serial ./input/Square_one_domain n cg 1
```

- (b) Parallel implementation:

```
$ mpirun -np 4 main-parallel ./input/Square_four_domains n cg 1
```

7. Sit back and watch it work its magic.

## A.2 Pacioli

1. Log in to Pacioli and make a new directory where you would like the magic to happen.

```
$ mkdir FEMtastic
```

2. Use scp to add our package (the directory called HPC\_FLENS) to this new directory.

3. Get and build FLENS:

```
$ git clone https://github.com/michael-lehn/FLENS.git
```

```
$ cd ./FLENS
```

```
$ make
```

```
$ cd ..
```

(no copying of FLENS to /usr/local is required here, our makefiles know we're now on Pacioli ☺).

4. Load modules:

```
$ module load gcc/4.7.2
```

```
$ module load sge/6.2u5
```

```
$ module load openmpi/gcc/64/1.4.2
```

5. Build our package:

```
$ cd ./HPC_FLENS
```

```
$ make
```

6. Submit the appropriate jobscript to the queue (modify the commands in it first, if you so wish):

- (a) Serial:

```
$ qsub batchscript_hpc_flens_serial.qs
```

- (b) Parallel:

```
$ qsub batchscript_hpc_flens_parallel.qs
```

7. Wait for the job to be processed. Since the parallel version demands exclusive use of 4 nodes, this may take some time if the queue is busy. Once complete, there will be an output file created in the HPC\_FLENS directory.

## A.3 GotoBLAS

Notes:

- GotoBLAS offers highly machine-specific optimisations. We performed testing on the single processor (AMD Opteron 252) nodes of Pacioli (nodes 1-16). The frontend is identical to these nodes, and therefore can be used to build GotoBLAS. Creativity would be required to build the library for the other nodes.

- We found some issues with certain nodes (prime suspect: Node 16), whereby computation times were increased significantly (10x) when using GotoBLAS. We suspect that this is due to a node that isn't actually identical to the others, and as such the 'optimised' code is in fact the opposite.

To link FLENS with GotoBLAS (on Parcioli), perform steps 1-4 of A.2. Then:

1. Get GotoBLAS in the form of OpenBLAS:

```
$ git clone git://github.com/xianyi/OpenBLAS
```

2. Make OpenBLAS:

```
$ cd ./OpenBLAS
$ make FC=gfortran
$ cd ..
```

3. Install OpenBLAS:

```
$ mkdir gotoins
$ cd ./OpenBLAS
$ make PREFIX=../gotoins FC=gfortran install
$ cd ..
```

4. Add OpenBLAS to the linker directory path:

```
$ export LD_LIBRARY_PATH=/home/your_username/gotoins/lib:$LD_LIBRARY_PATH
```

5. Build our package with GotoBLAS options:

```
$ cd ./HPC_FLENS
$ make goto
```

6. Submit the appropriate jobscript with 'goto' in its name, in the same way as in A.2.