Our Universe Homework 1

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Exercise 1

(a) The number of metres in a light year is given by

$$
\begin{aligned}
&3 \times 10^8\,\mathrm{m\,s^{-1}} \times 1\ \text{year} \\
&= 3 \times 10^8\,\mathrm{m\,s^{-1}} \times (3600 \times 24 \times 365)\ \mathrm{s} \\
&= 9.46 \times 10^{15}\,\mathrm{m}.
\end{aligned}
$$

(**b**) The light travel time t is given by:

$$\begin{aligned} t &= \frac{\text{distance}}{c} \\ &= \frac{40 \times 1.5 \times 10^{11}\,\mathrm{m}}{3 \times 10^{8}\,\mathrm{m\,s^{-1}}} \\ &= 2 \times 10^{4}\,\mathrm{s} \\ &\approx 5 \text{ hours and } 33 \text{ minutes.} \end{aligned}$$

(**c**) The time t since its launch is given by

$$\begin{aligned} t &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{125 \times 1.5 \times 10^{11}\,\mathrm{m}}{17 \times 10^{3}\,\mathrm{m\,s^{-1}}} \\ &= 1.1 \times 10^{9}\,\mathrm{s} \\ &= 34.97 \text{ years} \\ &\approx 35 \text{ years.} \end{aligned}$$

So the launch date should be in $2020 - 35 = 1985$. The actual lunch date is 1977.

(i)

$$\begin{aligned} t_{\mathrm{i}} &= \frac{4.3 \times 9.46 \times 10^{15}\,\mathrm{m}}{17 \times 10^{3}\,\mathrm{m\,s^{-1}}} \\ &= 2.39 \times 10^{12}\,\mathrm{s} \\ &= 7.5876 \times 10^{4} \text{ years} \\ &\approx 7588 \text{ years.} \end{aligned}$$

(ii)

$$\begin{aligned} t_{\mathrm{ii}} &= t_{\mathrm{i}} \times \frac{10^{5}}{4.3} \\ &\approx 1.76 \times 10^{9} \text{ years.} \end{aligned}$$

(iii)

$$\begin{aligned} t_{\mathrm{iii}} &= t_{\mathrm{i}} \times \frac{2 \times 10^{6}}{4.3} \\ &\approx 3.53 \times 10^{10} \text{ years.} \end{aligned}$$

(d) Let the time that passes for the astronaut be t', and the time that passes for their twin sister be t. Then, due to time dilation from special relativity, we have

$$
\begin{aligned}
t' &= t\sqrt{1-\frac{v^2}{c^2}} \\
&= t\sqrt{1-\frac{0.995^2}{1^2}} \\
&= t\sqrt{9.975 \times 10^{-3}} \\
&= t \times 9.987 \times 10^{-2}.
\end{aligned}
$$

The time t as measured by the twin sister on Earth is given by

$$
\begin{aligned}
t &= \frac{2 \times 4.3 \text{ lightyears}}{0.995 \times \text{speed of light}} \\
&= \frac{2 \times 4.3 \text{ years}}{0.995} \\
&= 8.6432 \text{ years} \\
&\approx 8.6 \text{ years.}
\end{aligned}
$$

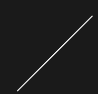
So, the twin sister will be 38 or 39 years old, depending on how far away her birthday was when the astronaut left Earth. The time t' that passed for the astronaut is

$$
\begin{aligned}
&8.6432 \text{ years} \times 9.987 \times 10^{-2} \\
&= 0.8632 \text{ years} \\
&\approx \text{about 10 months,}
\end{aligned}
$$

so the astronaut will be 30 or 31 years old, depending on how far away his birthday was when he left Earth.

Exercise 2

(e)



The earth is spinning in the eastward direction. Thus, the celestial sphere appears to us to spin in the westward direction, and so the stars in the sky appear to move to the west. Thus, the stars that pass through our zenith will all have different right ascensions. However, since the earth spins on a plane, there is no spin in the North or South direction, and so the celestial sphere too does not appear to spin in the North or South direction. Therefore, the declination value of all the stars that pass through the zenith will be the same.

(f) A lunar eclipse occurs when the Sun, Earth, and Moon line up, and the Earth blocks the Sun's light, preventing it from reaching the Moon, leaving the Moon in the Earth's shadow. So, it would be natural to expect a lunar eclipse once a month when the Moon lines up behind the Earth. However, the Moon's orbital plane is tilted slightly relative to the line joining the centres of the Earth and Sun, and so often, when we would expect a lunar eclipse, the Moon is actually below or above the Earth, so there is no lunar eclipse.

(g) A solar eclipse occurs when the Sun, Moon, and Earth line up, and the Moon blocks out the sunlight so that the Sun is hidden behind the Moon. The Moon's apparent size varies with time since the Earth-Moon orbit is elliptical, not circular. Sometimes, the Moon's apparent size is not large enough to completely cover the Sun, and so a narrow ring of sun is seen around the Moon's shadow. This is known as an **annular eclipse**.

(h) A **sidereal month** is the time it takes for the Moon to complete one orbit around the Earth. It is approximately 27.3 days.

A **synodic month** is the time between two successive New Moons. It is approximately 29.5 days. A synodic month is about 8% longer than a sidereal month because in the time the month has orbited the Earth one time, the Earth has moved about $30°$ through its orbit around the sun, and so the Moon must move an extra $30°$ around the Earth before we get a New Moon. Therefore, in a sidereal month, the Moon moves $360°$, whereas in a synodic month it moves $390°$, and so the ratio of the synodic month to the sidereal month is given by

$$
\begin{aligned}
\frac{\text{synodic month}}{\text{sidereal month}} &= \frac{390^\circ}{360^\circ} \\
&= \frac{13}{12} \\
&= 1.08333\ldots \\
&= \text{About } 8\% \text{ longer.}
\end{aligned}
$$

Exercise 3