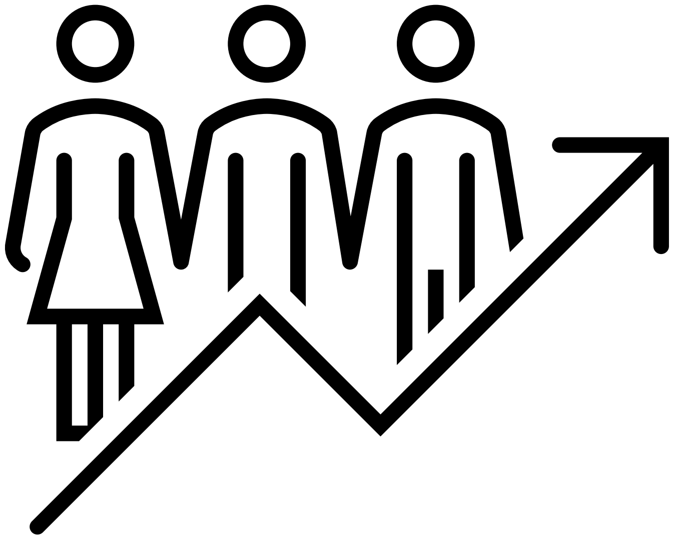

BAN402

Decision Modelling in Business

<u>Project 3</u>

Group: 22

Candidates: 35 & 64

Table of Contents

Part A: Golden Mayden

1: Same price

Using given demand functions for concert tickets we are tasked with formulating a decision model to maximise total revenue from ticket sales for a Golden Mayden concert. The venue has a capacity of 55 000 and the two segments "general" and "student" should be facing the same ticket price. Our model is formulated as follows:

Sets			
$i \in I$		Market segments	
Parameters			
Cap		Venue capacity	
Decision variables			
p_i		Ticket price for all segments	
Q_i		Demand for segment i	
S_i		Tickets sold to segment i	
Objective function			
$maximize$			
$\sum_{i \in I} S_i p_i$	$\forall\, i \in I$	Maximize revenue from ticket sales	
Constraints			
$p_g = p_s$		Same price for both segments	(1)
$Q_i = 120\,000 - 3\,000 p_i$	$i \in I: i = g$	Demand general segment	(2)
$Q_i = \ \ 20\,000 - 1\,250 p_i$	$i \in I: i = s$	Demand student segment	(3)
$S_i \leq Q_i$	$\forall\, i \in I$	Sales cannot exceed demand	(4)
$S_i \geq 0.2 Cap$	$\forall\, i \in I$	At least 20% capacity sold to each segment	(5)
$\sum_{i \in I} S_i \leq Cap$	$\forall\, i \in I$	Venue capacity must not be exceeded	(6)
$p_i\,,\ Q_i\,,\ S_i \geq 0$	$\forall\, i \in I$	Non-negativity	(7)

This is a non-linear model with 1 set, 1 parameter, 3 variables and 7 constraints. The demand functions in constraint (2) and (3) specifies how many tickets will be demanded in each segment for a given price p_i. This demand is used as an upper bound for the number of tickets sold to each segment in constraint (4). The venue capacity is used as both a lower and an upper bound in constraints (5) and (6), respectively. The objective function is maximising revenue from ticket sales.

Running the model in AMPL, the entire venue capacity is used in this scenario, with most tickets being sold to the "general" segment:

	Tickets sold	*Price in hundreds*
General segment	44 000	7.2
Student segment	11 000	7.2
Total revenue	396 000	

In this scenario, there is a massive gap between available "general" tickets and the demand for sold tickets, allowing the price of "general" tickets to be increased significantly.

2: Differing prices

To allow differing prices, we remove the constraint that requires equal prices from the model in A1:

$p_g = p_s$		Same price for both segments	(1)

Running the new model in AMPL, the entire venue capacity is still used with the majority of tickets still being sold to the general segment, but at a higher price:

	Tickets sold	*Price in hundreds*
General segment	44 000	25.3333
Student segment	11 000	7.2
Total revenue	1 193 866.667	

Since the "student" demand cannot be lower than 11 000, this price stays the same. The "general" price on the other hand, increases significantly. This in turn, results in a massive increase in total revenue.

3: A third segment

Alternative a

To account for the new segment, we add this new segment "r" in set I and their demand in constraint (3). Constraint (1) ensures that students and seniors face the same price. In constraints (6) and (7) we ensure that the minimum quantities are available for each segment.

Constraints

$p_s = p_r$		Same price for students and seniors	(1)
$Q_i = 120\ 000 - 3\ 000 p_i$	$i \in I: i = g$	Demand general segment	(2)
$Q_i = \ \ 20\ 000 - 1\ 250 p_i$	$i \in I: i = s$	Demand student segment	(3)
$Q_i = \ \ 15\ 000 - 1\ 400 p_i$	$i \in I: i = r$	Demand senior segment	(4)
$S_i \leq Q_i$	$\forall\ i \in I$	Sales cannot exceed demand	(5)
$S_i \geq 0.2 Cap$	$i \in I: i = g$	At least 20% capacity sold to general segment	(6)
$S_s + S_r \geq 0.2 Cap$		At least 20% capacity sold to senior and student	(7)
$\sum_{i \in I} S_i \leq Cap$	$\forall\ i \in I$	Venue capacity must not be exceeded	(8)
$p_i\ ,\ Q_i\ ,\ S_i \geq 0$	$\forall\ i \in I$	Non-negativity	(9)

Running the new model in AMPL, the entire venue capacity is still used with most tickets still being sold to the general segment, but at a higher price:

	Tickets sold	*Price in hundreds*
General segment	44 000	25.3333
Student segment	8 679,25	9.0566
Senior segment	2 320,75	9.0566
Total revenue	1 214 289.308	

The relaxation of having an extra segment increases the total revenue further.

Alternative b

For this scenario we remove the first constraint of the model in A3a to allow differing prices for all segments. Constraints (5), (6) and (7) ensure each segment is allocated at least their minimum required venue capacity:

Constraints

$Q_i = 120\,000 - 3\,000p_i$	$i \in I: i = g$	Demand general segment	(1)
$Q_i = \ \ 20\,000 - 1\,250p_i$	$i \in I: i = s$	Demand student segment	(2)
$Q_i = \ \ 15\,000 - 1\,400p_i$	$i \in I: i = r$	Demand senior segment	(3)
$S_i \leq Q_i$	$\forall\, i \in I$	Sales cannot exceed demand	(4)
$S_i \geq 0.2Cap$	$i \in I: i = g$	At least 20% capacity sold to general segment	(5)
$S_i \geq 0.1Cap$	$i \in I: i = r$	At least 10% capacity sold to senior segment	(6)
$S_i \geq 0.1Cap$	$i \in I: i = s$	At least 10% capacity sold to senior segment	(7)
$\sum_{i \in I} S_i \leq Cap$	$\forall\, i \in I$	Venue capacity must not be exceeded	(8)
$p_i\,,\ Q_i\,,\ S_i \geq 0$	$\forall\, i \in I$	Non-negativity	(9)

Running the new model in AMPL, the entire venue capacity is still used. Most tickets are still being sold to the general segment at the highest price, while the seniors pay the lowest price:

	Tickets sold	*Price in hundreds*
General segment	44 000	25.3333
Student segment	5 500	11.6
Senior segment	5 500	6.78571
Total revenue	1 215 788.095	

The relaxation of allowing differing prices for all segments increases the total revenue further.

Alternative c

First, we add constraint (1) that requires all prices to be greater than 600kr. Constraint (6) ensures that each segment gets its minimum required venue capacity, while constraints (8) through (13) ensures that each segments price cannot be greater than double the price of any other segments.

Constraints

$p_i \geq 6$	$\forall\, i \in I$	Minimum price of 600kr	(1)
$Q_i = 120\,000 - 3\,000p_i$	$i \in I: i = g$	Demand general segment	(2)
$Q_i = \ \ 20\,000 - 1\,250p_i$	$i \in I: i = s$	Demand student segment	(3)
$Q_i = \ \ 15\,000 - 1\,400p_i$	$i \in I: i = r$	Demand senior segment	(4)
$S_i \leq Q_i$	$\forall\, i \in I$	Sales cannot exceed demand	(5)
$S_i \geq 0.05Cap$	$\forall\, i \in I$	At least 5% capacity sold to each segment	(6)
$\sum_{i \in I} S_i \leq Cap$	$\forall\, i \in I$	Venue capacity must not be exceeded	(7)
$p_g \leq 2p_s$			(8)

$p_g \leq 2p_r$			(9)
$p_s \leq 2p_g$			(10)
$p_s \leq 2p_r$			(11)
$p_r \leq 2p_g$			(12)
$p_r \leq 2p_s$			(13)
$p_i\ ,\ Q_i\ ,\ S_i \geq 0$	$\forall\ i \in I$	Non-negativity	(14)

Running the new model in AMPL, the entire venue capacity is still used. Most tickets are still being sold to the general segment at twice the price of seniors. Both students and seniors are allocated the minimum required quantity:

	Tickets sold	*Price in hundreds*
General segment	49 500	17.5
Student segment	2 750	13.8
Senior segment	2 750	8.75
Total revenue	928 262.5	

The "double price" constraint drives down the price for the "general" segment and subsequently reduces the total revenue.

Part B: Romeo and Juliet

1: New medical requirement

Romeo and Juliet are required to consume at least 250 units of k_1 in every set of three consecutive days during the time horizon. We will have to implement this requirement in the $\alpha_{p,k}^{min}$ parameter, so that $\alpha_{1,1}^{min} = 250$. Further, we'll use the $\omega_{k,i}$ parameter and the $y_{p,i,j,t}$ variable to construct the following constraint:

$$\sum_{j \in M} \sum_{i \in I} \omega_{k,i} y_{p,i,j,t} + \sum_{j \in M} \sum_{i \in I} \omega_{k,i} y_{p,i,j,t-1} + \sum_{j \in M} \sum_{i \in I} \omega_{k,i} y_{p,i,j,t-2} \geq \alpha_{p,k}^{min}$$

$p \in P, k \in K, t \in T: p = 1\ and\ k = 1\ and\ t \geq 3$

This constraint strictly ensures that for every day in $t \geq 3$, the sum of nutrition k_1 consumed in the past 3 consecutive days is greater or equal to 250.

2: Happy Saturday

In this task Romeo and Juliet asks us to make sure that there is at least two Happy Saturdays in the planning horizon. A Happy Saturday is when they are served "pasta Bolognese" together with either "vanilla ice cream" or "strawberry ice cream". This task can be solved by implementing the following decision variable, set and constraints:

New decision variable:

g_t binary variable indicating whether Pasta Bolognese is served with either vanilla or strawberry ice cream ($g_t = 1$) or not ($g_t = 0$) on day t.

New subset of T:

$T': subset\ of\ days\ T\ that\ is\ Saturdays$

New constraints:

1:

$$x_{"pasta\ bolognese", \text{"dinner"}, \text{t}} + x_{\text{"vanilla icecream", "dinner", t}} + x_{\text{"strawberry icecream", "dinner", t}} \leq 1 + g_t$$

2:

$$2g_t \leq x_{"pasta\ bolognese", \text{"dinner"}, \text{t}} + x_{\text{"vanilla icecream", "dinner", t}} + x_{\text{"strawberry icecream", "dinner", t}}$$

3:

$$\sum_{t \in T'} g_t \geq 2$$

Constraints 1 and 2 ensures that the binary variable g_t takes the value 1 if there is a "happy serving", that is pasta bolognese is served with vanilla or strawberry ice cream on day t. Constraint 3 ensures that there is a "happy serving" on at least two Saturdays, making at least two happy Saturdays in the planning horizon.

Part C: Nord Pool

1: Exploring equilibrium

Based on our candidate numbers we have plotted supply and demand curves for period 2. Figure 1 displays supply and demand curves, both step function curves and linearized curves.

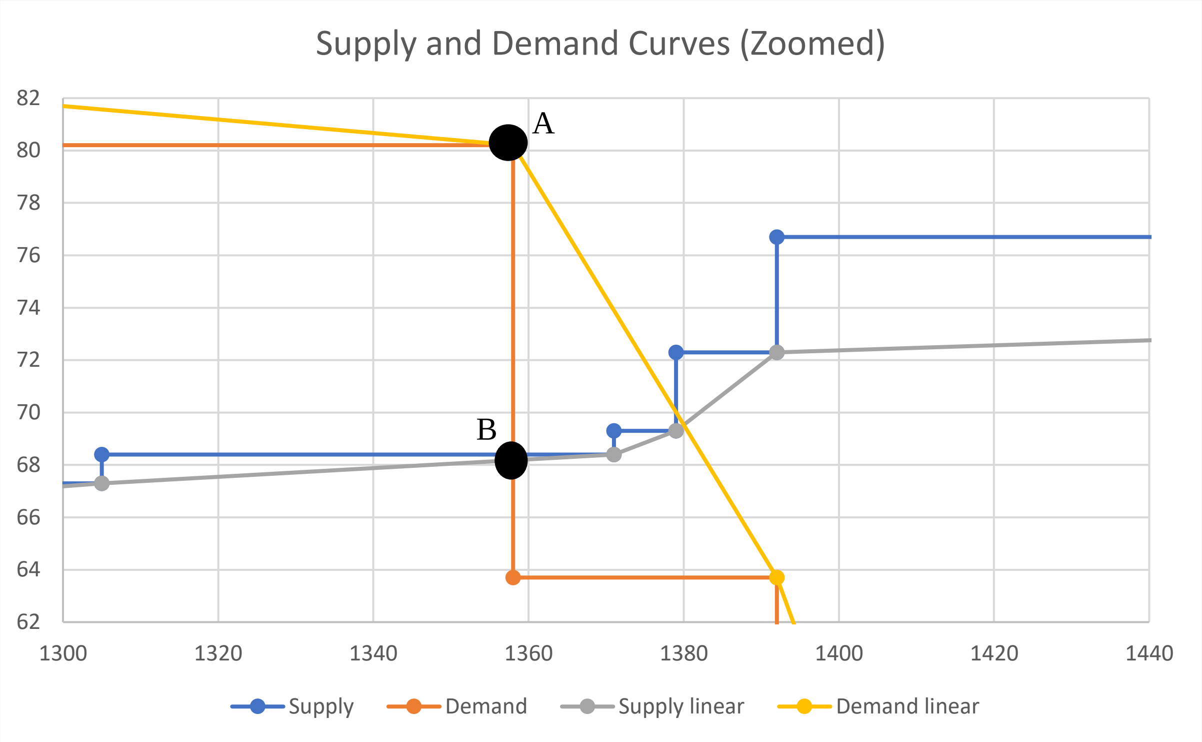

Figure 1: Supply and demand curves period 2

The intersection of the supply and demand linear curves is approximately where y = 69.5 and x = 1380, which is there we find the system price. The system price is therefore 69.5.

The method of finding the system price by observing the intersection of supply and demand curve in our case has its advantages and disadvantages. Firstly, the non-linearized curves have "steps" which makes it difficult or impossible to find an exact equilibrium. In our case, by interpreting the step function curves, equilibrium would be somewhere on the line between point A and B. Based on this, we cannot clearly decide what equilibrium price is. Equilibrium price would be between approximately 68 and 81, which is a wide span and a definite uncertainty when it comes to figuring out the correct price.

On the other hand, linearized curves are easier to interpret and finding equilibrium. The equilibrium is where these lines, the yellow and grey line, intercept. The disadvantage by using linearized curves is the overestimation of the price on the demand side and underestimation of the price on the supply side. However, the bidders are aware of this and will construct their bids accordingly.

It is the equilibrium price from linearized curves that will be the determining factor. Therefore, for suppliers all bids lower than the system price would be accepted and for buyers all bids higher would be accepted. When it comes to block orders, a sales block with an order price lower than the average price during the specified block interval would be accepted and a purchase block with an order price higher than the average price during the period would be accepted. See file BAN402_Proj3_PartC for all calculations in tasks related to part C.

2: Bidding

Alternative a

For this task we are placing exactly one bid per period, with the volume the same every period and a price that may vary from one period to another but always is an integer number greater than 1 euro. Figure 2 illustrates the optimal bidding process.

Placing one bid pr period, volume the same pr period, price may vary						
the bid:	QS	PS	Status bid			
Period 1	1200	32	Bid accepted			
Period 2	1200	47	Bid accepted			
Period 3	1200	54	Bid accepted			
Period 4	1200	39	Bid accepted			
Period 5	1200	53	Bid accepted			
Without block bids:						
Linearized social surplus = 310610.92						
Step function social surplus = 283695.30						
hour	Price	Volume	PS	PD	s	d
1	32,17	1 279,00	32,00	34,10	1 089,00	2,00
2	47,56	1 707,00	47,00	49,30	1 117,00	11,00
3	54,55	1 471,00	54,30	58,90	9,00	130,00
4	39,61	2 172,00	39,00	42,70	1 200,00	103,00
5	53,15	1 798,00	53,00	53,30	1 195,00	32,00
Cost	11					
Period	System price	Quantity	Profit			
1	32,17	1089	23 054,13			
2	47,56	1117	40 837,52			
3	54,55	1200	52 260,00			
4	39,61	1200	34 332,00			
5	53,15	1195	50 369,25			
TOTAL PROFIT			200 852,90			

Figure 2: Bidding task. 2A

In order for us to maximize profit, our goal is to bid as high of a price as possible without it being higher than the system price consequently dismissing our bids. Additionally, we would want to sell all of our available quantity. Our strategy in this case was to firstly bid a very low price that must be accepted, for example 11 that is larger than our fixed cost. Afterwards we would increase the price towards system price and stop right before it is too high to be accepted. Our bids are displayed on the top of figure 2 and results in a total profit of 200 852,90 EUR.

Alternative b

For this task we are placing at most one bid per period, with a volume and price that may vary between periods. Volume needs to be a multiple of 10 and price needs to be an integer greater than 1 euro. Figure 3 displays the results from this task.

Placing at most one bid per period, volume may vary (10,20,30 etc), price may vary						
the bid:	QS	PS	Status bid			
Period 1	1090	33	Bid accepted			
Period 2	1120	48	Bid accepted			
Period 3	1200	54	Bid accepted			
Period 4	1200	39	Bid accepted			
Period 5	1200	53	Bid accepted			
Without block bids:						
Linearized social surplus = 309134.70						
Step function social surplus = 281489.30						
hour	Price	Volume	PS	PD	s	d
1	33,78	1279,00	33,00	34,10	1089,00	2,00
2	48,30	1707,00	48,00	49,30	1117,00	11,00
3	54,55	1471,00	54,30	58,90	9,00	130,00
4	39,61	2172,00	39,00	42,70	1200,00	103,00
5	53,15	1798,00	53,00	53,30	1195,00	32,00
Cost	11					
Period	System price	Quantity	Profit			
1	33,78	1089	24 807,42			
2	48,30	1117	41 664,10			
3	54,55	1200	52 260,00			
4	39,61	1200	34 332,00			
5	53,15	1195	50 369,25			
TOTAL PROFIT			203 432,77			

Figure 3: Bidding task 2B

The strategy for the bidding process in this task was to use the solution from the previous task and edit it in order to increase profit. Firstly, we would change the bidding quantities closer to the actual quantity sold from task 2A. Afterwards we would increase the prices for each period towards system price. The reduction in bid quantities from task 2A enables the process to produce a larger system price, that is for period 1 and 2 since that is where we have changed bid quantity. The total profit is therefore 203 432,77 EUR.

3: Block bids

In this task we are placing one block bid, covering four consecutive periods (1 to 4, or 2 to 5), the price of our block bid cannot be the same as another block bid in the data and is greater than 1.0 EUR. The volume must be a multiple of 10.

After playing with the model, we have found some characteristics to what is happening after you place a bid.

Firstly, for our bid to be accepted, it needs to be lower 50 and 52.88 for respectively bids in period 1 to 4 and 2 to 5. These prices are also the average of the periods system prices after running the model with the original 10 block bids. The quantity we place in our bid does not affect the upper bound of bid price to be accepted.

Furthermore, in general when being a supplier you would want to bid the highest price you can while making sure that it is not too high for it to be unaccepted. Additionally, we can see that the system prices are the same when changing between very low prices and changing between prices close to the maximum accepted. For example, placing a block bid of 20 EUR with 1200 volume for period 2 to 5 constructs the same system prices as a 10 EUR more expensive bid with the same volume and periods.

However, when we stepwise increase the price, we arrive at a price that makes the system prices decrease. After testing this we have found that a bid price of 44.5 is where the system prices decrease, and it is the same for period 1 to 4 and 2 to 5. Therefore, prices need to be lower than or equal to these prices in order to keep the high system prices.

Based on all these findings we can say that for our purpose of finding the optimal maximized profit, we would choose the following: the block bid's period should be from 2 to 5, since that is the periods, we can charge the most for our product. Afterwards we would sell all volume and we would choose a price that is less than or equal to 44.5.

After placing a block bid with a price of 44.5, a volume of 1200 and periods from 2 to 5, we get a profit of 141 744,00 EUR.

Part D: Bike Sharing

A mathematical formulation

Sets

$i \in I$		set of locations
$n \in N$		set of inhabitants in the city
Parameters		
f_i		fixed cost of installing bike rack at location i
v_i		variable cost of installing bike rack at location i
c		cost of acquiring conventional bike
e		cost of acquiring electric bike
d_{in}		distance between location i and inhabitant n
t		max distance for a inhabitant to be a potential user
p		minimum bike coverage of inhabitants at a bike rack
Decision variables		
b_i		number of conventional bikes at rack i
x_i	Binary	1 if location i is used 0 if otherwise
a_{in}	Binary	1 if location i is a candidate location for inhabitant n 0 if otherwise
g_{in}	Binary	1 if location i is linked to inhabitant n 0 if otherwise

Objective function

$$minimize$$
$$\sum_{i \in I} x_i f_i + \sum_{i \in I} x_i v_i + \sum_{i \in I} x_i 2e + \sum_{i \in I} b_i c$$

Constraints

Only used locations can be considered candidate locations

$$a_{in} \leq x_i \qquad \forall\ i \in I, n \in N \tag{1}$$

The distance between inhabitant n and a candidate location i must be less than or equal to t

$$a_{in} d_{in} \leq t \qquad \forall\ i \in I, n \in N \tag{2}$$

The distance between inhabitant n and a linked location i must be less or equal to all other candidate locations j.

$$g_{in} d_{in} \leq a_{jn} d_{jn} \qquad \forall\ i \in I, j \in I, n \in N : i <> j \qquad i <> j \text{ means i is not equal to j} \tag{3}$$

At least 50% of inhabitants must be linked to a used location

$$\sum_{i \in I} \sum_{n \in N} g_{in} \geq 0.5 * |N| \qquad |N|\ is\ the\ cardinality\ of\ set\ N \tag{4}$$

Conventional bikes allocated to location i must be at least 5% of linked inhabitants n minus two e-bikes

$$\sum_{n \in N} g_{in} \cdot p - 2 \leq b_i \qquad \forall\ i \in I \qquad p = 0.05 \tag{5}$$

Explanation of the model

The objective function aims to minimize the total cost as a sum of fixed installation costs, variable installation costs, e-bike purchasing costs and conventional bike purchasing costs. The objective function explicitly demands 2 e-bikes for each bike rack. Constraints (1) and (2) makes sure that a location is only considered a candidate location for an inhabitant if a bike rack is installed, and it is within the maximum range of the inhabitant's house. Constraint (3) ensures that only the closest location is linked to an inhabitant while constraint (4) implies that at least 50% of the city's inhabitants are linked to an installed bike rack. The minimum bike quantity is enforced through constraint (5), which calculates 5% of the linked inhabitants and subtracts 2 e-bikes. This means that our model considers e-bikes and conventional bikes equally adequate to meet the bike demand of 5%.