BAN402

Decision Modelling in Business

23. September 2022

Project 1

Linear Programming

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Group number: 7

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Group: 7, members: 35, 64

Table of Contents

Group: 7, members: 35, 64

Part A

A1: Implementing the model in AMPL

Formulating the problem mathematically:

Variables:

$F_1 = \textit{ton of fish processed at facility 1}$

$F_2 = \textit{ton of fish processed at facility 2}$

$F_3 = \textit{ton of fish processed at facility 3}$

Objective function:

MIN: $30F_1 + 20F_2 + 40F_3$

Constraints:

$0.10F_1 + 0.20F_2 + 0.40F_3 \geq 25$ (Pollutant 1)

$0.45F_1 + 0.25F_2 + 0.30F_3 \geq 35$ (Pollutant 2)

Running the model in AMPL results in the following:
Total cost = 2730.77
F_1 = 11.5385
F_2 = 119.231
F_3 = 0

The optimal solution is to process 11.54 tons of fish at facility 1 and 119.23 tons of fish at facility 2. Consequently, resulting in the optimal objective value of $ 2 730,77.

A2: Constraint satisfaction in relation to the optimal solution

Pollutant 1: $0.10 * 11.54 + 0.20 * 119.23 = 25$

Pollutant 2: $0.45 * 11.54 + 0.25 * 119.23 = 35$

Both constraints are satisfied and at the minimum level. This means that we have reduced pollution by the minimum required from the government, which is intuitive given that this is a minimization problem, resulting in a solution at the lower bounds.

A3: Optimal cost sensitivity to government's targets

Since this is a minimizing problem, the model would always want to minimize costs. Consequently, a decrease in the pollutant 2 reduction target will result in a smaller quantity of fish processed with the new technology, which in turn results in a smaller reduction of pollution. On the other hand, if the reduction target increases to 70, the total cost will increase as the facilities will have to process more fish using the new technology.

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A4: Effects of changes to cost coefficients on the optimal solution

Facility	Down	Current	Up
F1	10	30	36
F2	16.6667	20	25.3333
F3	27.6923	40	1e+20

Table 1: Sensitivity analysis

Table 1 summarizes the sensitivity analysis and displays the variables' allowable increase and decrease. For instance, processing at facility 1 is allowed to increase to 36 (everything else unchanged), without changing the optimal solution while increasing further would change the optimal solution. Since the optimal solution is to have no processing at facility 3, the cost at this facility can increase infinitely without changing the optimal solution.

If the cost of processing at facility 1 decreases from \$30 to \$20, the optimal solution will remain the same, since processing at facility 1 is allowed to decrease to 10 before the optimal solution changes.

Part B

B1: Implementing the model in AMPL

Our objective is to formulate a model to determine a weekly blending plan such that the refinery maximizes profit. The model will be implemented in the decision modelling tool AMPL. See folder "Task B" for all the files used to build the model in AMPL.

We refer to the assignment's description on how the problem is formulated. By interpreting the assignment's description, we can illustrate the process as follows:

We have four types of crude oils to purchase. These types of crude oils are mixed in a blender that produces five types of gasoline. Each kind of gasoline can then be sold in any of the three markets.

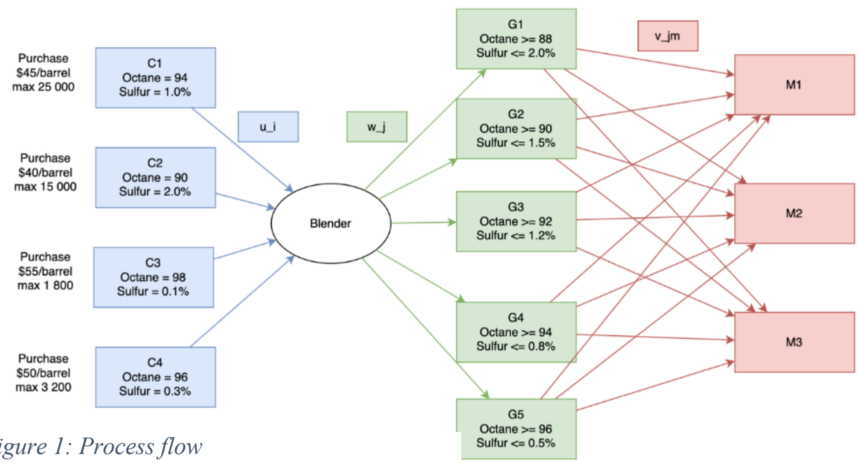

Figure 1: Process flow

We assume that there is no separation between part-time and full-time employees regarding their competency. Available work hours are therefore treated as total.

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Our model is described with a general model notation as follows:

Indexes and sets

$i \in I$	set of crude oils
$j \in J$	set of gases
$b \in B$	set of components
$m \in M$	set of markets

Parameters

R_{ib}	component b obtained from one barrel of crude i
H_{jb}	maximum component b in one barrel gas j
L_{jb}	minimum component b in one barrel gas j
P_i	purchase price one barrel crude i
C_j	production cost one barrel gas j
S_{jm}	sale price one barrel gas j in market m
W_j	work hours one barrel gas j
T	available work hours
A_i	available barrels of crude i
D_{jm}	demand for gas j in market m

Decision variables

u_i	amount of crude oil i purchased
w_j	amount of gas j produced
r_{ij}	amount of crude oil i in gas j
v_{jm}	amount of gas j sent to market m

Objective function

$$Max\ profit = \sum_{j \in J} \sum_{m \in M} v_{jm} S_{jm} - \sum_{i \in I} u_i P_i - \sum_{(j \in J)} w_j C_j$$

Constraints

Material continuity: all barrels of crude oil purchased is used in gas production.

(1) $MatCont$ $\quad \sum_{j \in J} r_{ij} = u_i \quad i \in I \quad j \in J$

Production continuity: all gas produced originates from allocated crude oils.

(2) $ProdCont$ $\quad \sum_{i \in I} r_{ij} = w_j \quad i \in I \quad j \in J$

Sales continuity: all barrels of gas produced needs to be sold in a market as there is no storage facilities.

(3) $SellCont$ $\quad \sum_{m \in M} v_{jm} = w_j \quad j \in J \quad m \in M$

Raw material availability: purchased barrels of crude oil cannot exceed amount available.

(4) $RawAvail$ $\quad u_i \leq A_i \quad i \in I$

Work availability: hours used in production of gas cannot exceed number of hours available.

(5) $WorkAvail$ $\quad \sum_{i \in I} w_j Wj \leq T \quad j \in J$

Market demand: The amount of gas sold to each market needs to be greater or equal to the demand in said market.

(6) $MarkDem$ $\quad v_{jm} \geq D_{jm} \quad j \in J \quad m \in M$

Quality min: amount of each attribute in each gas needs to be greater or equal to the minimum levels allowed.

(7) $QualMin$ $\quad L_{jb} w_j \leq \sum_{i \in I} R_{ib} r_{ij} \quad i \in I \quad j \in J \quad b \in B$

Quality max: amount of each attribute in each gas needs to be less than or equal to the maximum levels allowed.

(8) $QualMax$ $\quad H_{jb} w_j \geq \sum_{i \in I} R_{ib} r_{ij} \quad i \in I \quad j \in J \quad b \in B$

The optimal solution to the problem is summarized in table 2. The table displays the weekly blending plan of each crude oil mixed to produce types of gas. The optimal solution results in a profit of 1 371 570,-.

Original	**C1**	**C2**	**C3**	**C4**	**Total Gas**
G1	1 775,0	8 725,0			10 500,0
G2	4 625,0	4 625,0			9 250,0
G3	6 600,0	1 650,0			8 250,0
G4	5 714,3			2 285,7	8 000,0
G5	1 542,9		1 542,9	914,3	4 000,0
Total Crude Oil	20 257,2	15 000,0	1 542,9	3 200,0	

Table 2: Blending plan, original data

Original	**G1**	**G2**	**G3**	**G4**	**G5**
M1	3 000,0	3 000,0	1 500,0	2 000,0	1 000,0
M2	2 500,0	2 000,0	1 000,0	2 000,0	1 000,0
M3	5 000,0	4 250,0	5 750,0	4 000,0	2 000,0

Table 3: Gas allocation in markets, original data

All gasoline type allocations satisfy minimum demand quantities, displayed in table 3. Gasoline type G3 sold in market M3 is the only occurrence where demand for that gasoline type at that market is higher than the minimum demand quantity.

B2: Modifying the model

Scenario 1: Increased G1 sales price and decreased G1 demand in M1

In scenario 1 we have increased the sales price of gasoline G1 in market M1 from $75 to $78 and decreased the minimum demand quantity of gasoline G1 in market M1 from 3000 to 2700 barrels of gasoline. Table 4 displays the optimal solution to the problem in scenario 1, which tells us that compared to the solution with original data, the total amounts of different types of crude oils are unchanged. However, total gasoline type G1 has decreased by 300 and total gasoline type G3 has increased by 300. In other words, we have moved amounts of crude oils from production of gasoline type G1 to production of gasoline type G3.
The optimal solution to the problem in scenario 1 results in a profit of 1 384 470,- which is an increase of 12 900,- compared to the original data.

Scenario 1	**C1**	**C2**	**C3**	**C4**	**Total Gas**
G1	1 535,0	8 665,0			10 200,0
G2	4 625,0	4 625,0			9 250,0
G3	6 840,0	1 710,0			8 550,0
G4	5 714,3			2 285,7	8 000,0
G5	1 542,9		1 542,9	914,3	4 000,0
Total Crude Oil	20 257,2	15 000,0	1 542,9	3 200,0	

Table 4: Blending plan, scenario 1

The increased types of gasoline G3 is sold in market M3, which is intuitive since the sales price for this type of gasoline is highest in market M3. Table 5 displays the gasoline allocation in the different markets for scenario 1 and describes that moved resources has consequently resulted in an increase of gasoline type G3 in market M3

Scenario 1	**G1**	**G2**	**G3**	**G4**	**G5**
M1	2 700,0	3 000,0	1 500,0	2 000,0	1 000,0
M2	2 500,0	2 000,0	1 000,0	2 000,0	1 000,0
M3	5 000,0	4 250,0	6 050,0	4 000,0	2 000,0

Table 5: Gas allocation in markets, scenario 1

Scenario 2: Increased demand for G3 in M3, G1 in M1 and decreased demand for G1 in M2

In scenario 2 we have increased demand for gasoline G3 in market M3 and gasoline G1 in market M1 from respectively 3000 to 4000 and 3000 to 3100. Furter is demand for gasoline G1 in market M2 decreased from 2500 to 2400. Table 6 summarizes the results to the optimal solution of the problem.

Scenario 2	**C1**	**C2**	**C3**	**C4**	**Total Gas**
G1	1 775,0	8 725,0			10 500,0
G2	4 625,0	4 625,0			9 250,0
G3	6 600,0	1 650,0			8 250,0
G4	5 714,3			2 285,7	8 000,0
G5	1 542,9		1 542,9	914,3	4 000,0
Total Crude Oil	20 257,2	15 000,0	1 542,9	3 200,0	

Table 6: Blending plan, scenario 2

The optimal optimal solution in scenario 2 is identical to the optimal solution with the original data.

Scenario 2	**G1**	**G2**	**G3**	**G4**	**G5**
M1	3 100,0	3 000,0	1 500,0	2 000,0	1 000,0
M2	2 400,0	2 000,0	1 000,0	2 000,0	1 000,0
M3	5 000,0	4 250,0	5 750,0	4 000,0	2 000,0

Table 7: Gas allocation in markets, scenario 2

Gasoline type G1 is allocated on the minimum demand requirement. Consequently, means that we have sold 100 barrels more of gasoline type G1 in market M1 and 100 barrels less of gasoline type G1 in market M2. Since the sales price for gasoline type G1 is the same in both markets M1 and M2, the profit remains the same as originally at 1 371 570,-. Additionally, increased demand of gasoline type G3 in market M3 does not affect the solution because the production of G3 in M3 is nonetheless larger than the increased minimum demand. These effects are displayed in table 7.

Scenario 3: Decreased maximum available crude oil barrels for C3 and C4

In scenario 3 the maximum number of barrels available decreases from 1800 to 1350 for crude C3 and from 3200 to 2050 for crude C4. The model results in a profit of 1 368 690,- but becomes infeasible.

Scenario 3	C1	C2	C3	C4	Total Gas
G1	1 775,0	8 725,0			10 500,0
G2	4 625,0	4 625,0			9 250,0
G3	6 600,0	1 650,0			8 250,0
G4	6 401,6		2 405,6	- 807,1	8 000,0
G5	1 142,9			2 857,1	4 000,0
Total Crude Oil	20 544,5	15 000,0	2 405,6	2 050,0	

Table 8: Blending plan, scenario 3

Scenario 3 is an infeasible problem because we do not have enough barrels available for crude oil C3 and C4 in order to meet the demand and comply with all the constraints.

Scenario 4: New supplying source analysis (sensitivity)

Sensitivity analysis:

	RawAvail	RawAvail.down	RawAvail.current	RawAvail.up
C1	-	20 257,1	25 000,0	1E+20
C2	5,0	10 271,1	15 000,0	16 775,0
C3	-	1 542,9	1 800,0	1E+20
C4	-	3 085,7	3 200,0	6 285,7

Table 9: Raw material availability shadow price and allowable increase/decrease

Raw material availability of crude oil type C2's shadow price is $5, which means that one additional available C2 results in an extra profit of $5 which will take effect until crude oil C2 available increases to 16 775. Thus, potential profit increase with the same purchase price is; 1 775 * 5 = 8 875,-.

If we buy the allowable increase of 1 775 barrels, from the alternative source, profit will increase by: 1 775 * (5-1) = 7 100,-. Since the allowable increase limits us to increase available crude oil type C2 up to 1 775, and cost increase of $1 is less than shadow price $5 we can comfortably buy 1 775 barrels from the alternative source.

Part C

C1: Implementing the model in AMPL

Our objective is to formulate a model to determine a transport plan in order to minimize transportation costs. The model will be implemented in the decision modelling tool AMPL. See folder "Task C" for all the files to build the model in AMPL.

We refer to the assignment's description on how the problem is formulated. By interpreting the assignment's description, we can illustrate the process as follows:

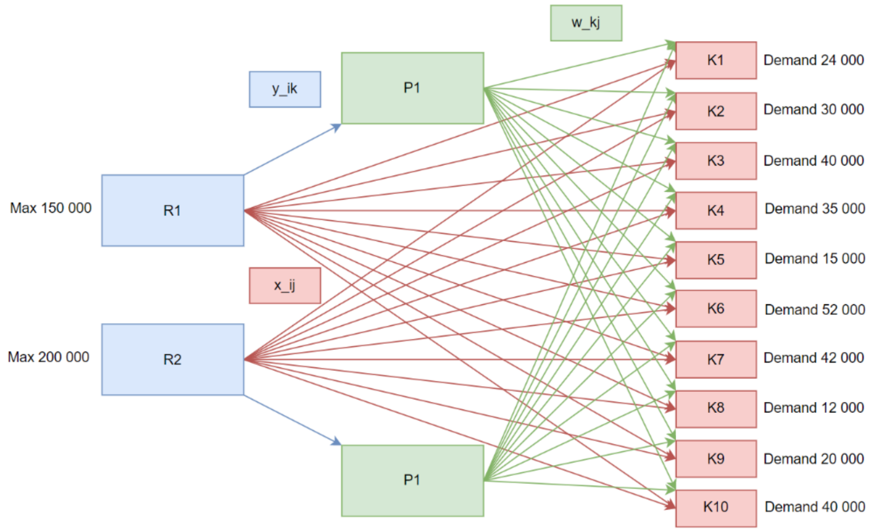

Figure 2: Transportation flow

There are two regions where the FreshFruits Company can obtain a given number of apples. These apples can either be transported directly to ten different markets, or via two different ports as figure 2 illustrates. The transport costs differ between the regions, ports and markets, which provides an opportunity for optimization. Each market has a demand that needs to be satisfied. In our model we assume that there are no possibilities to store apples in ports, which means that all apples sent to a port will have to be sent to a market immediately.

Given this scenario we have the following general model notation:

Indexes and sets

$i \in I$ set of regions
$j \in J$ set of markets
$k \in K$ set of ports

Parameters

s_i supply in region i
d_i demand in market i
c_{ij} transportation from region i to market j
e_{ik} transportation cost from region i to port k
g_{kj} transportation cost from port k to market j

Decision variables

x_{ij} quantity from region i to market j
y_{ik} quantity from region i to port k
w_{kj} quantity from port k to market j

Objective function

$$Min\ costs = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{i \in I} \sum_{k \in K} e_{ik} y_{ik} + \sum_{k \in K} \sum_{j \in J} g_{kj} w_{kj}$$

Constraints

Supply cap: total sum of apples sent from each region cannot be greater than the available supply in said region.

$$(1) \quad SupplyCap \qquad \sum_{j \in J} x_{ij} + \sum_{k \in K} y_{ik} \le s_i \qquad i \in I \quad k \in K$$

Transshipment: all apples sent to each port has to be distributed to markets, as there is no storage facilities.

$$(2) \quad TransShip \qquad \sum_{i \in I} y_{ik} - \sum_{k \in K} w_{kj} = 0 \qquad i \in I \quad k \in K$$

Market demand: the number of apples sent to each market has to be equal to said market's demand.

$$(3) \quad MarkDem \qquad \sum_{i \in I} x_{ij} + \sum_{k \in N} x_{kj} = d_j \qquad j \in J$$

Non negativity: all variables need to be positive numbers.

$$(4) \quad NonNegat \qquad x_{ij}, y_{ik}, w_{kj} \ge 0 \qquad i \in I \quad j \in J \quad k \in K$$

The optimal solution to the problem in tons is summarized in table 10. Total cost is minimized to 10 760,- while the demand is satisfied in all markets.

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1	24	30				52			20	
P2			40		15		42	12		
R1										40
R2				35						
Total	24	30	40	35	15	52	42	12	20	40

	P1	P2
R1	110	
R2	16	109
Total	126	109

Table 10: Optimal transportation plan in tons, original

As we can see in the table above, it seems that it is often better to go via ports than to send directly from the regions to markets. Each region only sends one shipment each directly to markets. On the right of table 10 we also see that region 1 sends its majority to port 1, while region 2 sends its majority to port 2 while also sending smaller quantity to port 1. Region 1 ships all its supply, while there is some slack in the supply from region 2 as this region only ships $109 + 16 + 35 = 160$ out of its total 200 tons of apples.

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C2: Port 1 unavailable

In order to run this problem, we simply removed P1 from the dataset and ran the model again. The optimal objective value is now 18 438, which is a 7 678 increase from the original.

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1										
P2		30	40		15	52	42	12		
R1	24								20	40
R2				35						
Total	24	30	40	35	15	52	42	12	20	40

	P1	P2
R1		26
R2		165
Total	0	191

Table 11: Optimal transportation plan in tons, port 1 unavailable

The total cost increases when P1 becomes unavailable as more expensive transportation options must be used. Region 1 now starts sending apples to port 2 which it didn't do originally. The load of transporting the apples formerly distributed through port 1 is spread over direct shipments from region 1 and shipments through port 2.

C3: Decreased supply

Decreased supply

In order to run this problem, the supply was reduced in the dataset and the model was run again. This resulted in an error message, saying we have an infeasible problem. This is because the reduced supply is less than the demanded quantity in the markets.

$$total\ demand: \sum_{j \in J} d_j = 310\ 000$$

With the reduced supply, the total amount of apples that can be provided is $125\ 000 + 175\ 000 = 300\ 000$. This is 10 000 apples less than total demand, which is why the problem is infeasible. In order to solve the problem with this supply, the demand in the markets needs to be reduced.

Decreased supply and maximum level of unsatisfied demand

With the ability to deliver less apples than the original demand, the problem with decreased supply becomes feasible. However, two new constraints and one additional parameter needs to be implemented:

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New parameter

mud_j $\quad$ *maximum unsatisfied demand in market j*

New constraints

Min demand: the number of apples sent to each market has to be greater or equal to said market's min demand.

$$(5)\ MinDem, \quad \sum_{i \in I} x_{ij} + \sum_{k \in N} x_{kj} \geq d_j(1 - mud_j) \quad j \in J$$

Max transport: the entire supply of apples needs to be distributed

$$(6)\ MaxTrans, \quad \sum_{j \in J} x_{ij} + \sum_{k \in K} y_{ik} = s_i$$

In addition, the market demand constraint needs to be altered to allow lower deliveries:

Updated constraint

Max demand: the number of apples to each market has to be less than or equal to said market's max demand.

$$(3)\ MarkDem, \quad \sum_{i \in I} x_{ij} + \sum_{k \in N} x_{kj} = d_j \rightarrow (3)\ MaxDem, \quad \sum_{i \in I} x_{ij} + \sum_{k \in N} x_{kj} <= d_j \quad j \in J$$

These changes to the model allow transportation to the markets to be less than their original demand, while requiring it to be higher than their minimum demand. In order to ensure that the number of apples distributed will not be reduced to each market's minimum demand, constraint (6) requires the entire supply to be distributed.

The optimal objective value becomes 10 460.4, which is 299.6 lower than originally.

	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
P1	24	30				49,4			20	
P2			40		12,1		39,9	9,6		
R1										40
R2				35						
Total	24	30	40	35	12,1	49,4	39,9	9,6	20	40

	P1	P2
R1	87	
R2	44	99,5
Total	131	99,5

Table 12: Optimal transportation plan, C3

Demand	24	30	40	35	15	52	42	12	20	40
Delivered	1,00	1,00	1,00	1,00	0,81	0,95	0,95	0,80	1,00	1,00
Minimum	0,9	0,9	0,95	0,9	0,8	0,95	0,95	0,8	0,8	0,95
Difference	0,10	0,10	0,05	0,10	0,01	0,00	0,00	0,00	0,20	0,05

Table 13: Demand analysis

In Table 12 the new optimal transportation plan is presented. Table 13 displays a quick analysis of the transportation plan. The first row shows the original demand in each market. The second row describes the percentage of original demand delivered, with 100% colored green. Row three contains the minimum demand allowed in each market while the fourth shows the difference between demand delivered and minimum demand allowed. Marked in red is the three markets K6, K7, K8 that receives their minimum demand. Market K5 colored blue receives 1% more than their minimum demand.

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Part D

D1: Interpreting the model

D1A: Could the model be infeasible?

Yes, the model could be infeasible. Constraint (2) implies that the flow x must be less than or equal to supply s. This means that inventory $y_{ik,t}$ will never be less than $y_{ik,t-1}$ since inventory is defined as

$$y_{ik,t-1} + s_{ikt} - \sum_{j \in J} x_{ijkt} = y_{ikt}$$

This can be demonstrated as follows:

$$if\ x_t \leq s_t \ \rightarrow \ s_t - x_t \geq 0 \ \rightarrow \ y_{t-1} + s_t - x_t = y_t \geq y_{t-1}$$

Consequently, the inventory can never be reduced. Because the model does not allow a reduction of inventory, the demand d can never be greater than supply s. If this constraint was formulated differently, such that the flow x must be less or equal to supply s + inventory y, parts of the inventory could be used to satisfy a demand greater than s. This problem can be demonstrated as follows:

$$if\ d > s\ \ and\ x = d \rightarrow x > s \rightarrow infeasible\ due\ to\ constraint\ (2)$$

$$if\ d > s\ and\ x \leq s \rightarrow x > d \rightarrow infeasable\ due\ to\ constraint\ (3)$$

D1B: Could the model be unbounded?

Constraint (2) is the supply constraint that works as an upper bound for the problem. The flow exiting from each supply point cannot exceed the available supply for all assortments and time periods. Likewise, does constraint (3) work as a lower bound. All group assortments and time periods are equal to the amount of demand for each demand point. The model can therefore not be unbounded.

D2: Given objective value

Objective value = 234 560

D2A: Total demand quantity:

$$\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} 1 * x_{ijkt} = 234\ 560 \rightarrow x_{ijkt} = 234\ 560 = d_{jlt}$$

Because:

$$\sum_{i \in I} \sum_{k \in K} x_{ijkt} = d_{jlt}$$

D2B Total demand specifically in period 6:

We cannot determine total demand specifically for period 6 since we do not know how the flow x nor demand d is allocated over the different time periods t.

D2C: Total supply availability

Constraint (2) instructs aggregated flow of x to be less than or equal to the aggregated supply. We can therefore not conclude of a specific number, but can identify supply as greater than or equal to x.

$s_{ikt} \geq 234\ 560$

D2D: Total supply specifically in period 6:

Like task D2B, we cannot determine total supply for period 6 since we do not know the allocation of flow over the different time periods t.

D3: New condition

We would need to modify the model to include the following two constraints in order to capture the new condition:

The total flow exiting supply point i cannot be more than 10% lower than previous time period

$x_{ijkt} \geq x_{ijkt-1} * 0.9$

The total flow exiting supply point i cannot be more than 10% higher than previous time period

$x_{ijkt} \leq x_{ijkt-1} * 1.1$

These constraints are limiting the possible variation in demand for each time period. Limiting the demand to not fluctuate more than $\pm$ 10% between each time period since the total flow must be equal to the total demand.

$$x_{ijkt} \geq x_{ijkt-1} * 0.9 \;\; \& \;\; x_{ijkt} \leq x_{ijkt-1} * 1.1 \;\; \& \;\; \sum_{i \in I} \sum_{k \in K} x_{ijkt} = d_{jlt}$$

$$\downarrow$$

$$d_{jlt} \geq d_{jlt-1} * 0.9 \; \& \;\; d_{jlt} \leq d_{jlt-1} * 1.1$$