Here’s an expanded explanation of **integration** with detailed examples to fill a full page.

Integration: A Fundamental Concept

Integration is one of the two main operations in calculus (the other being differentiation). It is the process of finding a function when its rate of change is known or computing the total accumulation of quantities. Mathematically, it is the reverse operation of differentiation.

Integration is widely used in mathematics, physics, engineering, economics, and many other fields. It helps in determining areas, volumes, central points, work done by forces, probabilities, and much more. There are two main types of integration:

1. **Indefinite Integration**: Represents the collection of all antiderivatives of a function and includes a constant of integration (CC). ∫f(x) dx=F(x)+C\int f(x) \, dx = F(x) + C
2. **Definite Integration**: Computes the accumulated value of a function over a specific interval [a,b][a, b]. ∫abf(x) dx=F(b)−F(a)\int_{a}^{b} f(x) \, dx = F(b) - F(a)

Example 1: Calculating the Area Under a Curve

Find the area under f(x)=x2f(x) = x^2 from x=1x = 1 to x=3x = 3.

Solution:
The definite integral gives the area under the curve:

∫13x2 dx=[x33]13=333−133=273−13=263.\int_{1}^{3} x^2 \, dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}.

Thus, the area is 263\frac{26}{3}.

Example 2: Physics Application – From Velocity to Distance

A particle’s velocity is given by v(t)=4t−2v(t) = 4t - 2 (in meters per second). Find the total distance traveled from t=0t = 0 to t=3t = 3.

Solution:
Distance is the integral of velocity over time:

$$\int_{0}^{3} (4t - 2) \, dt = \left[2t^2 - 2t\right]_0^3 = \left(2(3)^2 - 2(3)\right) - \left(2(0)^2 - 2(0)\right).$$ $$= (18 - 6) - 0 = 12 \, \text{meters}.$$

The particle travels 12 meters.

Example 3: Indefinite Integration

Find the general antiderivative for $f(x) = 3x^2 + 4x - 5$.

Solution:
Apply the power rule of integration:

$$\int (3x^2 + 4x - 5) \, dx = \int 3x^2 \, dx + \int 4x \, dx - \int 5 \, dx.$$ $$= x^3 + 2x^2 - 5x + C.$$

Thus, the antiderivative is $x^3 + 2x^2 - 5x + C$, where C is the constant of integration.

Example 4: Volume of a Solid of Revolution

Using integration, calculate the volume of a solid obtained by rotating $y = x^2$ about the x-axis from $x = 0$ to $x = 2$.

Solution:
The volume is given by the formula:

$$V = \pi \int_{a}^{b} [f(x)]^2 \, dx.$$

Here, $f(x) = x^2$, $a = 0$, and $b = 2$:

$$V = \pi \int_{0}^{2} (x^2)^2 \, dx = \pi \int_{0}^{2} x^4 \, dx = \pi \left[\frac{x^5}{5}\right]_0^2 = \pi \left(\frac{2^5}{5} - 0\right) = \frac{32\pi}{5}.$$

The volume is $\frac{32\pi}{5}$.

Applications of Integration

1. **Physics**: Calculating work, energy, and motion problems.

2. **Engineering**: Determining the center of gravity or moments of inertia.
3. **Economics**: Finding consumer and producer surplus.
4. **Statistics**: Calculating probabilities in continuous distributions.

Conclusion

Integration is a powerful mathematical tool that plays a critical role in solving real-world problems involving accumulation, area, and volume. Its versatility makes it indispensable in various scientific and engineering disciplines.