CSC 488S/CSC 2107S Lecture Notes

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Shift Reduce Parsing Bottom Up - LR(k), SLR(k), LALR(k)

Parser Model



Parser Actions

Shift Next input symbol is pushed onto the stack

Reduce A sequence of symbols (the handle) starting

at the top of the stack is reduced using a production

rule to replace the symbols with one nonterminal symbol.

Accept Successful end of parse.

Error Call recovery routine to handle syntax error.

- Choice of actions is based on the *contents* of the stack (the *left context*) and the next *k* input tokens (*k-symbol lookahead*).
- See Fischer, Cytron, LeBlanc Figure 6.3 for a generic bottom up parsing engine.

Reading Assignment

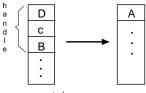
Fischer, Cytron, LeBlanc

Chapter 6

Section 5.9

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- A handle is the right hand side (RHS) of some rule in the grammar. Bottom up
 parsing allows more than one rule to have the same RHS iff the rules can be
 distinguished using the left context and k-symbol lookahead.
- Given a grammar rule: A → B c D
 a possible Reduce action would be



parse stack

- Issue: efficiently detecting when a handle is present on top of the parse stack.
- Issue: deciding which reduction to perform.
- See Fischer, Cytron, LeBlanc Figure 4.6 for an example of a complete bottom up parse.

LR(k) Parsing

- The contents of the parser stack (left context) represents a string from which the past input can be derived.
- Inputs are stacked until the top elements in the stack (the handle) are a complete alternative (RHS) for some rule.
- When a handle is recognized, a reduction is performed and the handle on the stack is replaced by the nonterminal symbol (LHS) of the applicable rule.
- Initial parser stack is

 ¬ and parsing continues until the stack contains
 S

 ¬ and the next input is \$
- At each stage the the top elements in the stack represent the initial portion of one or more alternatives. The next input symbol narrows the number of number of possible alternatives. If the number of choices in narrowed to zero, a syntax error has occurred.

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Rightmost Derivation Example^a

Parse Tree

For the grammar:

$$S \rightarrow AE$$
 $A \rightarrow aA$

$$egin{array}{cccc} {\sf B} &
ightarrow & {\sf B}\,{\sf b} \ & & & & {\sf b} \end{array}$$

Rightmost derivation of a a a b b

S

$$\rightarrow$$
 ABb

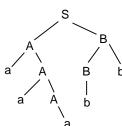
$$\rightarrow$$
 A b b

$$\rightarrow$$
 a A b b

$$\rightarrow$$
 aaAbb

$$\rightarrow$$
 aaabb

^aSee Slide 75



Rightmost derivation in reverse

a a a b b

ightarrow a a A b b

 \rightarrow a A b b

 \rightarrow A b b

.

ightarrow A B b ightarrow A B

→ S

- Finally, an input symbol is stacked that completes one or more alternatives.
 If there is more than one alternative, the language is not LR(0).
- At this point the next k input symbols must provide enough information to distinguish among the alternatives. If it doesn't, the language isn't LR(k).
- For LL(k) we had to know at the start of an alternative, given k input symbols which alternative to choose.

For LR(k) we do not need to know which alternative to choose until we reach the end of a rule. *Then* the next k input symbols must be sufficient to decide if a reduction can be performed.

Parsing decisions can be made *later* in an LR(k) parser than in an LL(k) parser. This is the reason that L(LL(k)) ⊂ L(LR(1))



LR(k) parsers effectively perform a rightmost derivation in reverse

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LR(k) Definition^a

- LR(k) parsers are the most powerful class of deterministic bottom up parsers using k-symbol lookahead.
 - If a grammar G can be parsed by any deterministic parser with k-symbol lookahead, then it can be parsed by an LR(k) parser.
- A grammar G is LR(k) if and only if the conditions

1.
$$S \Rightarrow_{rm}^* \alpha A \ w \Rightarrow \alpha \beta w$$
 Identical prefix $\alpha \beta$

2.
$$S \Rightarrow_{rm}^* \gamma B \ x \Rightarrow \alpha \beta y$$

3.
$$First_k(w) = First_k(y)$$
 Identical lookahead

imply that $\alpha A y = \gamma B x$

- ullet This means that a reduction $A \ o \ eta$ can be performed whenever
 - 1) $\alpha \beta$ is on top of the parse stack
- and 2) the k-symbol lookahead is $First_k(w)$.
- ⇒ the parser always has enough information to make a parsing decision.

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^aSee Fischer, Cytron, LeBlanc Page 190 . The symbol \Rightarrow_{rm}^* specifies a rightmost derivation

LR(1) Example^a

	Rule Grammar		ammar	Input Tokens								
		1 L	ightarrow L , E									
	:	2	\rightarrow E		a, b <mark>\$</mark>							
	;	3 E	\rightarrow a									
		4	\rightarrow b			Note left reci	ursion in gram	mar ⇒ no	ot LL(k)			
Stack		LF	R(1) Table			Stack S	napshots					
Confg	а	b	,	\$	St#	Parse	State	Input	Action			
∇	Shift a	Shift b				∇	∇	а	Next			
	Next	Next	Error	Error	0	a $ abla$	1 🗸	,				
a $ aggreyation$	Error	Error	Reduce 3	Reduce 3	1	E♡	3 🗸	,				
p 🛆	Error	Error	Reduce 4	Reduce 4	2	L♡	4 🗸	,	Next			
E▽	Error	Error	Reduce 2	Reduce 2	3	, L 🗸	5 4 🔽	b	Next			
L▽			Shift ,			b , L $ abla$	7 5 4 \bigtriangledown	\$				
	Error	Error	Next	Accept	4	E,L $ abla$	8 5 4 \bigtriangledown	\$				
, L ▽	Shift a	Shift b				L♡	4 🗸	\$	Accept			
	Next	Next	Error	Error	5							
a , L▽	Error	Error	Reduce 3	Reduce 3	6							
b ,L▽	Error	Error	Reduce 4	Reduce 4	7							
E ,L▽	Error	Error	Reduce 1	Reduce 1	8							

Define: Reduce j, use grammar rule j to replace handle on top of parse stack.

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• Redefine: Shift i

Stack input symbol Advance input Go to State i

- After a Reduce, need a list of states to restart in, the new state table.
 With this table we don't need to represent the stack configurations directly in the parser tables.
- Redefine Reduce i
 - Remove handle by doing pop^l where l is the length of the alternative i. Need a table giving the length of each alternative.

This popping uncovers some state. Note state 0 is never popped.

Push a new state where new state is a function of top item in the state stack and
the nonterminal symbol that is being reduced to (the LHS of alternative i)
State 0 on top signifies the empty stack. For example in the next slide:

Reducing	Top state	Push state	From state	Rule(s)
E	0	3	1,2	3,4
E	5	6	1,2	3,4
L	0	4	3	2
	6	4		4

LR(1) Parse Tables

- Some rows in table are the same as others, e.g. rows 1/6, 2/7 in the
 previous slide. To reduce table size these rows can be merged and assigned
 multiple indices.
- If the grammar is right recursive, the number of different parse stacks is
 infinite, but the number of different rows is finite (bounded by number of
 actions × number of columns) so we *must* merge the rows that are the
 same.
- Don't want the inefficiency of pattern matching the top elements in the stack against alternative stack configurations.
- We want parsing action to be determined by the single top element on the stack and a single input symbols (as in LL(1)).
- Assign a state number to each row in the table and stack the state number as a synonym for a complete stack configuration.

The state number is labeled St# in the previous Slide.

Top state on parse stack represents entire stack configuration

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LR(1) Example Revisited

Grammar						Condensed Parse Table						
						Stack	St#	a b ,				
1	L	\rightarrow	L,E			∇	0	Shift 1	Shift 2			
2		\rightarrow	E		a $ abla$	a , L ▽	1			Reduce 3	Reduce 3	
3	Е	\rightarrow	а		b ▽	b , L ▽	2			Reduce 4	Reduce 4	
4		\rightarrow	b			E▽	3			Reduce 2	Reduce 2	
						L♡	4			Shift 5	Accept	
						, ∟ ▽	5	Shift 1	Shift 2			
						E,L▽	6			Reduce 1	Reduce 1	

Example:	narse of	a . I	h \$

Old Stack	State Stack	Input	Action
∇	0	а	Shift 1
a $ abla$	1 0	,	Reduce 3
E♡	3 0	,	Reduce 2
L♡	4 0	,	Shift 5
, L ▽	5 4 0	b	Shift 2
b , L ▽	2540	\$	Reduce 4
E , L ▽	6540	\$	Reduce 1
L♡	4 0	\$	Accept

^aAlso see the larger example in *Fischer, Cytron, LeBlanc* Figures 6.4, 6.5, 6.6 and 6.7 .

Theoretical LR(1) Table Construction

- for all input symbols ($\Sigma \cup \{\$\}$) /* table columns */
 for all possible stack configurations /* table rows */
 /* Fill one parse table entry */
 Compute action(parseStackConfiguration , inputSymbol)
 /* Compute one next state table entry */
 Compute nextState(parseStackConfiguration , inputSymbol)
- There are a finite number of possible stack configurations for a finite grammar.
- Use parser state numbers to encode stack configurations so decisions can be made based on state number instead of a pattern match on the stack.
- LR(1) tables for real grammars are very large due to the large number of possible stack configurations
 e.g. > 1240 states and > 10,000 table entries for for Pascal.
- ullet For LR(k) iterate the columns over $\Sigma^k \cup \{~\$~\}^k$

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LR(0) Table Construction

- Each parser state in an LR parser is associated with a unique item set of LR(0) items (partially completed phrases).
- The LR(0) item represents what has been seen *prior* to entering the state.
- Define: LR(0) item

A LR(0) item is a marked production rule

 α seen so far $\beta \gamma$ to be seen

• An LR(0) item with the bookmark * at the right end. e.g.

$$G \rightarrow \alpha \beta \gamma$$

is a *REDUCE* production , $\alpha \beta \gamma$ is the handle , reduce to G

Practical LR(1) Parse Table Construction

- 1. First Compute LR(0) tables:
 - (a) LR(0) uses no lookahead
 - (b) Apply Closure and Completion to enumerate all possible stack configurations
 - (c) Note Conflicts in tables when lookahead is needed. Rules for which grammar is not LL(0)
- 2. Upgrade From LR(0) to LR(1)
 - Use exact lookahead to resolve LR(0) table conflicts
 - Split states as required to force unique left context and lookahead for every conflicting rule.
- SLR Simple LR, uses Follow sets instead of lookahead to resolve LR(0) conflicts.
- LALR LookAhead LR, use Item lookahead to make specific parsing decisions.

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- To generate an LR(0) table, start with an item set that contains all of the
 productions with a marker at the start of the right hand side of each rule.
 This is the start state of the parser (nothing has been see yet).
- Generate additional item sets (parser states) by applying closure and completion until all item sets have been generated.
 Derive parser state transitions from the item sets.
- Define: closure
 - if ullet is immediately to the left of a nonterminal symbol B Add to the item set all new LR(0) items such that B is the left hand side of a rule , i.e. $B \to {}^{ullet}$ ω
- Define: completion

```
Collect together in a new item set all LR(0) items that have the same symbol after the \ ^{\bullet} (e.g. \beta ) Complete by moving the \ ^{\bullet} past the symbol , e.g. G \rightarrow \alpha \ \beta \ ^{\bullet} \ \gamma if \beta \ \epsilon \ \Sigma then this corresponds to SHIFT \beta
```

• Save space by eliminating duplicate configuration sets as they are generated.

LR(0) Table Construction Example

Grammar:

Augment the set of productions with the rule

```
0 : Accept \rightarrow L §
```

7 : { Accept → L \$ • }

12: $\{P \rightarrow (E), E \rightarrow E + T\}$

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Another Example - not LR(0)

Grammar:

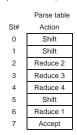
LR(0): Statemachine view

- s set of LR(0) items (table row)
- Building parse table P(s):
 - {If $B \to \rho$ $\in s$ and $B \to \rho$ is numbered i then Reduce i }
 - {If $A \to \alpha$ $a\beta \in s$ for terminal symbol a then Shift else \emptyset }
- If $\forall s \cdot |P(s)| = 1$, the grammar is LR(0).



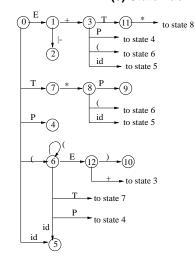
(6)

Ε



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LR(0) Statemachine for Example 2



- A table conflict occurs when the constructor algorithm tries to assign more than one type of action to some table entry.
- Types of LR(0) conflicts:
 - Shift/Reduce conflict, shift action and reduce action
 - Reduce/Reduce conflict, two different reduce actions
 - Example: Shift/Reduce conflicts in states 7 and 11 Reduce to E or shift *
- Solution: Have more states (i.e., split states 7 and 11!)

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LR(0) Conflict Diagnosis

- The LR(0) parse table construction has a conflict when it trys to assign more than one value to the parse table entry for some state.
 - shift/reduce conflicts Two alternatives exist:
 - Shift the incoming terminal symbol onto the stack
 - Reduce the top of the stack using some rule
 - reduce/reduce conflicts. The right hand side of two or more rules match the handle on top of the stack.
- Conflicts may arise because the grammar is ambiguous or because the parse table construction method isn't powerful enough.
- ullet LR(0) conflicts are resolved by using some form of lookahead, i.e. using the next k input symbols to resolve the conflict. Usually k=1.
- Lookahead only matters in cases where the is at the right end of a production.

Use the lookahead sets to decide which of several productions to apply. Lookahead sets for each **state** and **production** must distinguish productions uniquely.

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SLR(1) Parsing

- Goal: get more information than LR(0) but do not create non-essential LR(1) states
- s set of LR(0) items (table row), a a terminal symbol (table column)
- Augment CFSM from LR(0) with different notion of lookahead:
- Building P(s,a):
 - {If $B \to \rho$ $^{\bullet} \in s, \, a \in Follow(B)$ and $B \to \rho$ is numbered i then Reduce i }
 - $\{\operatorname{lf} A \to \alpha \bullet a\beta \in s \text{ for terminal } a, \text{ then } Shift \\ \operatorname{else } \emptyset \ \}$
- If $\forall s \forall a \cdot | P(s, a) | = 1$, the grammar is SLR(1).

- There are several strategies, e.g. SLR(k), LALR(k) and LR(k) for using lookahead to resolve LR(0) conflicts. These strategies differ
 - in how lookahead information is used
 - in the size of the resultant parse tables
 - in the complexity of the table building algorithm
- SLR(k) (Simple LR) uses Followk to resolve conflicts.
- LALR(k) (LookAhead LR) Build LR(k) then merge all states that differ in their lookahead. Use the lookahead to make the parsing decision.
- LR(K) Construct LR(0) states, then augment with lookahead. May require splitting states to force unique actions for a given lookahead.
 May result in a very large number of states.

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SLR(1) Example^a

 $Follow(E) = \{ \$, +, \}$, so reduce if we see these, shift when we see *.

Action tableb

	State	Lookahead								
		+	*	id	()	\$			
Ī	0			S	S					
	1	S					Α			
	2									
	3			S	S					
	4	R5	R5			R5	R5			
	5	R6	R6			R6	R6			
	6			S	S					

State						
	+	*	id	()	\$
7	R3	S			R3	R3
8			S	S		
9	R4	R4			R4	R4
10	R7	R7			R7	R7
11	R2	S			R2	R2
12	S				S	

^aGrammar from example in Slides 138

^bRn is Reduce n , S is Shift

Lookahead and Follow Sets

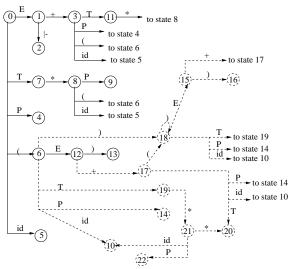
- ullet Begin with LR(0) but augment with lookahead $\ L \in \Sigma \cup \{\ \$\ \} \cup \{\lambda\}$
- Define: Lookahead Set The lookahead set for a production $A \to \alpha$ is the set of **terminal symbols** that can legally follow A or α during a **rightmost canonical parse**.
- ullet A terminal symbol b is in Follow(A) if b can legally follow A in a sentential form during **any** parse. Therefore

$$lookahead(A) \subset follow(A)$$

Therefore lookahead sets provide more decision making power than Follow sets.

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Example LR(1) Statemachine



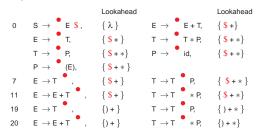
Computing LR(1) Lookahead Sets

• For each marked production the lookahead sets are calculated using closure. If $A \to \alpha$ • $B \beta$ { LA } is a configuration item and we are adding

$$B o {}^{ullet} \gamma \quad \{ \textit{newLA} \} \quad \text{to the configuration set then}$$
 $\textit{newLA} = \textit{first}(\beta) \qquad \beta \text{ is not nullable}$
 $\textit{newLA} = \textit{first}(\beta) \cup \textit{LA} \qquad \beta \text{ is nullable}.$

• For a marked production obtained by moving a marker past a terminal symbol, the lookahead set is unchanged.

Example



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Building LR(1) Tables

- Building parse table P(s, a):
 - $\begin{array}{l} \textbf{-} \; \{ \text{If } B \to \rho \quad \bullet \quad \text{, } \{ \text{ } a \text{ } \} \in s \text{ and } B \to \rho \text{ is numbered } i \text{ then Reduce } i \} \\ \textbf{-} \; \{ \text{If } A \to \alpha \quad \bullet \quad a\beta \quad \text{, } \{ a \} \in s \text{ for terminal } a \text{, then Shift else } \emptyset \} \\ \end{array}$
- If $\forall s, \forall a \cdot | P(s, a) |= 1$, the grammar is LR(1).

For our example: 23 states instead of 13

State	е	Lookahead				State			Lookahead					
	+	*	id	()	\$		State						
0			S	S					+	*	id	()	\$
			3	3				12	S				S	
1	S					Α		13	R7	R7				R7
2														
3			S	S				14	R5	R5			R5	
4	R5	R5				R5		15	R7	R7			R7	
								16	S				S	
5	R6	R6				R6		17			S	S		
6			S	S							S			
7	R3	S				R3		18			5	S		
8			S	S				19	R3	S			R3	
			3	3		_		20	R2	S			R2	
9	R4	R4				R4		21			S	S		
10	R6	R6			R6				Б.4	Б.4	0	0		
11	R2	S				R2		22	R4	R4			R4	

LALR(1) Parsing

- LALR(1) differs from LR(1) in that states with identical reductions but different lookahead sets are merged in LALR(1) and kept distinct in full LR(1).
 Goal: merge non-essential LR(1) states
- Construct LR(1) table, then merge states (table rows) with the same reductions.
- Define Cognate(s'):
 Those states with the same table row as s', union lookaheads.

Example (from LR(1) table in Slide 147):

- Building parse table P(s,a):
 - {If $B \to \rho$, $\{a\} \in \textit{Cognate}(s) \text{ and } B \to \rho \text{ is numbered } i \text{ then } \text{Reduce } i\}$
 - {If A → α aβ ∈ s for terminal a, then Shift else \emptyset }
- If $\forall s, a \cdot | P(s, a) |= 1$, the grammar is LALR(1).

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Parser Error Recovery

- LL(k) and LR(k) parsers have the valid prefix property:
 - if no error has been detected then

the input thus far is a valid prefix of one or more programs.

This is a consequence of processing the input token stream strictly left-to right

- Both kinds of parsers can give error messages in terms of input symbols without mentioning the grammar (i.e. names of nonterminal symbols)
- Just look along current row of parse table for non-error entries and list the column headings.
- Changing the parser stack can cause problems in later compiler phases, e.g.
 compiler internal data structures may be left in an inconsistent state.
 If and only if the parser has the valid prefix property, we can avoid changing
 the parser stack.

LALR(1) Example (Cont'd)

Thus, in both cases, reduce on $\{$), \$, + $\}$, shift on *.

Action table (same as SLR(1))

State			Looka	ahead					Lookahead					
	+	*	id	,	١.	\$	State							
	т	7		- (Ψ		+	*	id	()	\$	
0			S	S			7	R3	S			R3	R3	
1	S					Α	8			S	S			
2							9	R4	R4	•	Ŭ	R4	R4	
3			S	S										
4	R5	R5			R5	R5	10	R7	R7			R7	R7	
							11	R2	S			R2	R2	
5	R6	R6			R6	R6	12	S				S		
6			S	S			12	3				3		

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Syntax Error Repair

- Goal: transform syntactically incorrect program into the *closest* correct program, mainly to be able to continue parsing.
 Try to maximize the number of *useful* error messages per compilation.
- Measure of closeness: smallest number of changes. For example: insert 1 token or delete 1 token or change 1 token.
- Problems
 - closest correct program may not be unique e.g. delete **begin** or insert **end**?
 - Algorithms to find a closest correct program are non-linear.
 - Any measure of closeness may not give the intuitively closest program in all cases.
 Examples

PL/I DOWHILE(I = 1);
$$\Rightarrow$$
 DOWHILE(I) = 1;
Turing gut S \Rightarrow get S \Rightarrow put S

Syntax Error Repair Strategy

- General strategy: isolate the error in a replaceable phrase. Replaceable implies that the parse can continue.
- Making the closest correction may require changing the parser stack.
 Example:

```
A = B then I = 1 else I = 0
```

The closest correction would be to insert an **if** and compile A = B as a boolean expression. But it may have already been compiled as an assignment statement before we know about the error.

 Three (of many) possible strategies are the recovery token strategy, the panic strategy and spelling correction.

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Summary LL(k) and LR(k) Parsing

- A parser which can determine the correct derivation (reduction) at each stage as it scans the input stream in one direction is called *deterministic*.
- A grammar is LL(k) if a parser can parse strings scanning from left to right using left derivations with k-symbol lookahead deterministically.
- A grammar is LR(k) if a parser can parse strings scanning from left to right using right reductions with k-symbol lookahead deterministically.
- LR(k) is the most general deterministic left-to-right language.
 Language properties:

$$\mathcal{L}(LL(k)) \subset \mathcal{L}(LR(1))$$

$$\mathcal{L}(LR(1)) \subset \mathcal{L}(LR(k))$$

 $\mathcal{L}(L(k)) \subset \mathcal{L}(L(k))$

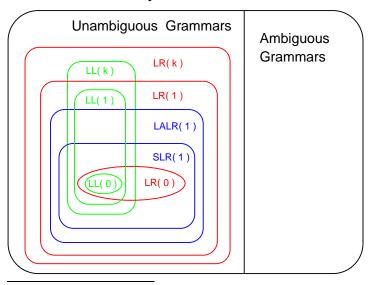
There are LR(0) grammars which are not LL(k) for any k.

Syntax Error Repair Strategy

- Recovery token repair strategy
 - For each row in the table, one input token is designated as the recovery token.
 - Reserved words, identifiers, numbers and strings are long tokens.
 All other tokens are short tokens
 - if the incorrect token is long and the recovery token is short insert the recovery token in front of the input.
 - Otherwise, replace the current input symbol with the recovery token.
- Panic repair strategy
 - Some tokens are designated as hard tokens. For example ; , end
 All other tokens are soft
 - Discard input up to and including the first hard token.
 - Pop the parser stack down to a corresponding token or state.
- · Spelling correction strategy:
 - Replace identifier that is close to a reserved word with the reserved word.

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Hierarchy of Grammar Classes^a



^a Adapted from, Andrew Appel, *Modern Compiler Implementation in Java*, 2002

Parsers for Compilers - Recursive Descent, LL(1) and LR(1)

- Almost all compilers use Recursive Descent, LL(1) or LALR(1)
- LALR(1) is more powerful than LL(1), but LL(1) may be strong enough.
- If you have a parser generator, use it.
- If you have both LL(1) and LALR(1) parser generators, use LALR(1)
- If you have no parser generator, try to make the grammar LL(1) and then build an LL(1) parser or use recursive descent.
- if you don't have a parser generator do not try and build an LALR(1) parser from scratch.
- There are well designed and thoroughly tested open source parser generators for both LL(1) and LALR(1).
- If you're building a scanner/parser for an ugly or complicated language (i.e. C++, Java, Fortran), consider buying an off the shelf compiler front end from a specialist compiler company^a

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YACC, Bison, JCup, et. al.

- YACC is a widely available LALR(1) parser generator developed at Bell Labs.
 Bison is a freeware clone from GNU. Many other variants have been developed.
- YACC assumes input from Lex (or flex)lexical analyzer
- YACC allows the compiler writer to provide an arbitrary piece of C code for each rule in the grammar. This code gets executed just before the parser performs a reduction involving the rule. There is a convention for accessing information stored parallel to the parse stack. YACC does automatic rule splitting to handle code that is not at the end of a rule.
- YACC processes a grammar and produces tables that are used with a fixed parsing algorithm. The pieces of C code become the body of a giant switch statement.
- jflex, jcup versions of flex and bison for Java.

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^aFor example Edison Design Group www.edg.com