Foundational Equations of Modern Physics

Structural map — all foundations on one page

Classical Mechanics

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{p} = \vec{F}$$

Newton (1687)

$$S[q] = \int L(q, \dot{q}, t)\, dt, \quad \delta S = 0$$

Maupertuis–Euler-Lagrange (18th c.)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$$

Euler–Lagrange

$$H = \sum_i p_i \dot{q}_i - L$$

Hamilton (1833)

$$\dot{q}_i = \partial H/\partial p_i,\ \dot{p}_i = -\partial H/\partial q_i$$

Hamilton (canonical)

Waves, Fields, Continua

$$\Box \phi = 0$$

d'Alembert (1747)

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u$$

Fourier (1822)

$$\nabla^2 \Phi = 4\pi G \rho$$

Poisson / Newton

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}\right) = -\nabla p + \mu \nabla^2 \vec{v}$$

Navier–Stokes (1822–45)

Special Relativity

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

Minkowski (1908)

$$E = mc^2$$

Einstein (1905)

$$E^2 = p^2 c^2 + m^2 c^4$$

Relativistic dynamics

$$p^\mu = (E/c, \vec{p})$$

Poincaré–Minkowski

Electromagnetism

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu,\ \partial_{[\lambda} F_{\mu\nu]} = 0$$

Maxwell (1861–65)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Lorentz (1892)

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t},\ \vec{B} = \nabla \times \vec{A}$$

Potentials

$$\Box A^\mu = \mu_0 J^\mu$$

Lorenz gauge (1867)

Thermodynamics / Statistics

$$dU = T dS - P dV + \mu\, dN$$

Clausius–Kelvin–Gibbs

$$S = k_B \ln \Omega$$

Boltzmann (1877)

$$Z = \sum_i e^{-\beta E_i}$$

Gibbs (1902)

$$F = -k_B T \ln Z$$

Helmholtz free energy

Quantum Mechanics

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi$$

Schrödinger (1926)

$$[\hat{x}, \hat{p}] = i\hbar$$

Heisenberg (1925)

$$\Delta x\, \Delta p \geq \hbar/2$$

Uncertainty principle

$$\langle A \rangle = \langle \psi |\, \hat{A}\, | \psi \rangle$$

Born / Dirac

Quantum Field Theory

$$Z = \int \mathcal{D}\phi\, e^{\frac{i}{\hbar} S[\phi]}$$

Feynman (1948)

$$(\Box + m^2)\phi = 0$$

Klein–Gordon (1926)

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

Dirac (1928)

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

QED Lagrangian

Gauge Structure / Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Standard Model

$$D_\mu = \partial_\mu - igA_\mu$$

Weyl gauge principle

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

Yang–Mills (1954)

$$|D_\mu H|^2 - V(H)$$

Higgs mechanism (1964)

General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein (1915)

$$S_{\mathrm{EH}} = \frac{c^3}{16\pi G} \int \sqrt{-g}(R - 2\Lambda)\, d^4 x$$

Hilbert–Einstein

$$\Gamma^\rho_{\mu\nu} = \tfrac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$$

Levi-Civita

Symmetry & Information

$$\text{symmetry} \iff \text{conservation}$$

Noether (1918)

$$\rho \mapsto U \rho U^\dagger$$

von Neumann (1932)

$$H(p) = -\sum_i p_i \log p_i$$

Shannon (1948)

$$S(\rho) = -k_B \operatorname{Tr}(\rho \ln \rho)$$

von Neumann entropy

Note: This document is educational and illustrative only. It does not claim to be complete or authoritative. Most of modern physics arises from these equations via symmetry choice, quantization, and limiting procedures ($\hbar \to 0$, $c \to \infty$, coarse graining).