Foundational Equations of Modern Physics

Structural map — all foundations on one page

Classical Mechanics

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{p} = \vec{F}$$
Newton (1687)

$$S[q] = \int L(q, \dot{q}, t)\, dt, \quad \delta S = 0$$
Maupertuis–Euler-Lagrange (18th c.)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q}$$
Euler–Lagrange

$$H = \sum_i p_i \dot{q}_i - L$$
Hamilton (1833)

$$\dot{q}_i = \partial H/\partial p_i,\ \dot{p}_i = -\partial H/\partial q_i$$
Hamilton (canonical)

Waves, Fields, Continua

$$\Box\phi = 0$$
d'Alembert (1747)

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u$$
Fourier (1822)

$$\nabla^2\Phi = 4\pi G\rho$$
Poisson / Newton

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla p + \mu\nabla^2\vec{v}$$
Navier–Stokes (1822–45)

Special Relativity

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$
Minkowski (1908)

$$E = mc^2$$
Einstein (1905)

$$E^2 = p^2c^2 + m^2c^4$$
Relativistic dynamics

$$p^\mu = (E/c, \vec{p})$$
Poincaré–Minkowski

Electromagnetism

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu,\ \partial_{[\lambda} F_{\mu\nu]} = 0$$
Maxwell (1861–65)

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$
Lorentz (1892)

$$\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t},\ \vec{B} = \nabla\times\vec{A}$$
Potentials

$$\Box A^\mu = \mu_0 J^\mu$$
Lorenz gauge (1867)

Thermodynamics / Statistics

$$dU = TdS - PdV + \mu\, dN$$
Clausius–Kelvin–Gibbs

$$S = k_B \ln\Omega$$
Boltzmann (1877)

$$Z = \sum_i e^{-\beta E_i}$$
Gibbs (1902)

$$F = -k_B T \ln Z$$
Helmholtz free energy

Quantum Mechanics

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi$$
Schrödinger (1926)

$$[\hat{x}, \hat{p}] = i\hbar$$
Heisenberg (1925)

$$\Delta x\, \Delta p \geq \hbar/2$$
Uncertainty principle

$$\langle A\rangle = \langle\psi|\, \hat{A}\, |\psi\rangle$$
Born / Dirac

Quantum Field Theory

$$Z = \int \mathcal{D}\phi\, e^{\frac{i}{\hbar}S[\phi]}$$
Feynman (1948)

$$(\Box + m^2)\phi = 0$$
Klein–Gordon (1926)

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$
Dirac (1928)

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$
QED Lagrangian

Gauge Structure / Standard Model

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$
Standard Model

$$D_\mu = \partial_\mu - igA_\mu$$
Weyl gauge principle

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$
Yang–Mills (1954)

$$|D_\mu H|^2 - V(H)$$
Higgs mechanism (1964)

General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$
Einstein (1915)

$$S_{\mathrm{EH}} = \frac{c^3}{16\pi G}\int\sqrt{-g}(R - 2\Lambda)\, d^4x$$
Hilbert–Einstein

$$\Gamma^\rho_{\mu\nu} = \tfrac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$$
Levi-Civita

Symmetry & Information

$$\text{symmetry} \iff \text{conservation}$$
Noether (1918)

$$\rho \mapsto U\rho U^\dagger$$
von Neumann (1932)

$$H(p) = -\sum_i p_i \log p_i$$
Shannon (1948)

$$S(\rho) = -k_B\,\mathrm{Tr}(\rho\ln\rho)$$
von Neumann entropy

Illustrated claim: Most of modern physics arises from these equations via symmetry choice, quantization, and limiting procedures ($\hbar \to 0$, $c \to \infty$, coarse graining).