1

i $\vec{u} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$
Ser først på divergensen

$$\nabla \cdot \vec{u} = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}y + \frac{\partial}{\partial z}z$$
$$\nabla \cdot \vec{u} = 1 + 1 + 1$$
$$\nabla \cdot \vec{u} = 3$$

Ser så på virvlingen

$$\nabla \times \vec{u} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$
$$\nabla \times \vec{u} = \left(\frac{\partial}{\partial y}z - \frac{\partial}{\partial z}y\right)\hat{\mathbf{i}} + \left(\frac{\partial}{\partial z}x - \frac{\partial}{\partial x}z\right)\hat{\mathbf{j}} + \left(\frac{\partial}{\partial x}y - \frac{\partial}{\partial y}x\right)\hat{\mathbf{k}}$$
$$\nabla \times \vec{u} = \vec{0}$$

ii $\vec{u} = r\cos(\theta)\hat{\mathbf{i}}_r + r\sin(\theta)\hat{\mathbf{i}}_\theta + z\hat{\mathbf{k}}$ Ser først på divergensen

$$\nabla \cdot \vec{u} = \frac{1}{r}\left(\frac{\partial}{\partial r}\Big(r\left(r\cos(\theta)\right)\Big) + \frac{\partial}{\partial \theta}\left(r\sin(\theta)\right)\right) + \frac{\partial}{\partial z}z$$
$$\nabla \cdot \vec{u} = \frac{1}{r}\left(\cos(\theta)\frac{\partial}{\partial r}r^2 + r\frac{\partial}{\partial \theta}\sin(\theta)\right) + 1$$
$$\nabla \cdot \vec{u} = \frac{1}{r}\big(2r\cos(\theta) + r\cos(\theta)\big) + 1$$
$$\nabla \cdot \vec{u} = 2\cos(\theta) + \cos(\theta) + 1$$
$$\nabla \cdot \vec{u} = 3\cos(\theta) + 1$$

Ser så på virvlingen

$$\nabla \times \vec{u} = 0\hat{\mathbf{i}}_r + 0\hat{\mathbf{i}}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}\Big(r\left(r\sin(\theta)\right)\Big) - \frac{\partial}{\partial \theta}\left(r\cos(\theta)\right)\right)\hat{\mathbf{k}}$$
$$\nabla \times \vec{u} = \frac{1}{r}\left(\sin(\theta)\frac{\partial}{\partial r}\left(r^2\right) - r\frac{\partial}{\partial \theta}\left(\cos(\theta)\right)\right)\hat{\mathbf{k}}$$
$$\nabla \times \vec{u} = \frac{1}{r}\big(2r\sin(\theta) + r\sin(\theta)\big)\hat{\mathbf{k}}$$
$$\nabla \times \vec{u} = \big(2\sin(\theta) + \sin(\theta)\big)\hat{\mathbf{k}}$$
$$\nabla \times \vec{u} = 3\sin(\theta)\hat{\mathbf{k}}$$

iii $\vec{u} = \hat{\mathbf{i}}_r + \hat{\mathbf{i}}$
Konverterer først over til rene sylinder koordinater

$$\boxed{\hat{\mathbf{i}} = \cos(\theta)\hat{\mathbf{i}}_r - \sin(\theta)\hat{\mathbf{i}}_\theta}$$

$$\begin{aligned}
\vec{u} &= \hat{\mathbf{i}}_r + \cos(\theta)\hat{\mathbf{i}}_r - \sin(\theta)\hat{\mathbf{i}}_\theta \\
\vec{u} &= \big(1 + \cos(\theta)\big)\,\hat{\mathbf{i}}_r - \sin(\theta)\hat{\mathbf{i}}_\theta
\end{aligned}$$

Ser først på divergensen

$$\begin{aligned}
\nabla \cdot \vec{u} &= \frac{1}{r}\left(\frac{\partial}{\partial r}\Big(r\,\big(1 + \cos(\theta)\big)\Big)\,\frac{\partial}{\partial \theta}\,\big(-\sin(\theta)\big)\right) \\
\nabla \cdot \vec{u} &= \frac{1}{r}\,\big(1 + \cos(\theta) - \cos(\theta)\big) \\
\nabla \cdot \vec{u} &= \frac{1}{r}
\end{aligned}$$

Ser så på virvlingen

$$\begin{aligned}
\nabla \times \vec{u} &= 0\hat{\mathbf{i}}_r + 0\hat{\mathbf{i}}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}\Big(r\,\big(-\sin(\theta)\big)\Big) - \frac{\partial}{\partial \theta}\,\big(1 + \cos(\theta)\big)\right)\hat{\mathbf{k}} \\
\nabla \times \vec{u} &= \frac{1}{r}\,\big(-\sin(\theta) + \sin(\theta)\big)\,\hat{\mathbf{k}} \\
\nabla \times \vec{u} &= \vec{0}
\end{aligned}$$

2

a

For å finne enhetsvektorene så må man først finne skaleringsfaktorene. Starter med skaleringsfaktoren til u

$$h_u = \left|\frac{\partial \vec{r}}{\partial u}\right|$$

$$h_u = \left|a\cos(v)\frac{\partial}{\partial u}\cosh(u)\hat{\mathbf{i}} + a\sin(v)\frac{\partial}{\partial v}\sinh(u)\,\hat{\mathbf{j}}\right|$$

$$\boxed{a = 1}$$

$$h_u = \left|\cos(v)\sinh(u)\,\hat{\mathbf{i}} + \sin(v)\cosh(u)\right|$$

$$h_u = \sqrt{\big(\cos(v)\sinh(u)\big)^2 + \big(\sin(v)\cosh(u)\big)^2}$$

$$h_u = \sqrt{\cos^2(v)\sinh^2(u) + \sin^2(v)\cosh^2(u)}$$

$$\boxed{\cosh^2(u) = 1 + \sinh^2(u)}$$

$$h_u = \sqrt{\cos^2(v)\sinh^2(u) + \sin^2(v)\Big(1 + \sinh^2(u)\Big)}$$

$$h_u = \sqrt{\cos^2(v)\sinh^2(u) + \sinh^2(u)\sin^2(v) + \sin^2(v)}$$

$$h_u = \sqrt{\sinh^2(u)\Big(\cos^2(v) + \sin^2(v)\Big) + \sin^2(v)}$$

$$h_u = \sqrt{\sinh^2(u) + \sin^2(v)}$$

Finner så skaleringsfaktoren til v

$$h_v = \left|\frac{\partial \vec{r}}{\partial v}\right|$$

$$h_v = \left|\cosh(u)\frac{\partial}{\partial v}\cos(v)\hat{\mathbf{i}} + \sinh(u)\frac{\partial}{\partial v}\sin(v)\hat{\mathbf{j}}\right|$$

$$h_v = \left|-\cosh(u)\sin(v)\hat{\mathbf{i}} + \sinh(u)\cos(v)\hat{\mathbf{j}}\right|$$

$$h_v = \sqrt{\big(-\cosh(u)\sin(v)\big)^2 + \big(\sinh(u)\cos(v)\big)^2}$$

$$h_v = \sqrt{\cosh^2(u)\sin^2(v) + \sinh^2(u)\cos^2(v)}$$

$$h_v = \sqrt{\Big(1 + \sinh^2(u)\Big)\sin^2(v) + \sinh^2(u)\cos^2(v)}$$

$$h_v = \sqrt{\sin^2(v) + \sinh^2(u)\sin^2(v) + \sinh^2(u)\cosh^2(v)}$$

$$h_v = \sqrt{\sin^2(v) + \sinh^2(u)\Big(\sin^2(v) + \cos^2(v)\Big)}$$

$$h_v = \sqrt{\sinh^2(u) + \sin^2(v)}$$

Ser da at $h_u = h_v$ Enhetsvektorer er da gitt ved

$$\mathbf{e}_u = \frac{1}{h_u}\frac{\partial \vec{r}}{\partial u}$$
$$\mathbf{e}_u = \frac{1}{\sqrt{\sinh^2(u) + \sin^2(v)}}\left(\cos(v)\sinh(u)\,\hat{\mathbf{i}} + \sin(v)\cosh(u)\,\hat{\mathbf{j}}\right)$$

Og

$$\mathbf{e}_v = \frac{1}{h_v}\frac{\partial \vec{r}}{\partial v}$$
$$\mathbf{e}_v = \frac{1}{\sqrt{\sinh^2(u) + \sin^2(v)}}\left(-\cosh(u)\sin(v)\hat{\mathbf{i}} + \sinh(u)\cos(v)\hat{\mathbf{j}}\right)$$

Hvis de er ortogonale så må $\mathbf{e}_u \cdot \mathbf{e}_v = 0$

$$\mathbf{e}_u \cdot \mathbf{e}_v = \frac{1}{\sqrt{\sinh^2(u) + \sin^2(v)}}\begin{pmatrix}\cos(v)\sinh(u)\\ \sin(v)\cosh(u)\end{pmatrix} \cdot \frac{1}{\sqrt{\sinh^2(u) + \sin^2(v)}}\begin{pmatrix}-\cosh(u)\sin(v)\\ \sinh(u)\cos(v)\end{pmatrix}$$
$$\mathbf{e}_u \cdot \mathbf{e}_v = \frac{1}{\sinh^2(u)\sin^2(v)}\left(-\cos(v)\sinh(u)\cosh(u)\sin(v) + \sin(v)\cosh(u)\sinh(u)\cos(v)\right)$$
$$\mathbf{e}_u \cdot \mathbf{e}_v = \left(-\sin(v)\cos(v)\sinh(u)\cosh(u) + \sin(v)\cos(v)\sinh(u)\cosh(u)\right)$$
$$\mathbf{e}_u \cdot \mathbf{e}_v = 0$$

Med dette så observeres det at enhetsvektorene er ortogonale

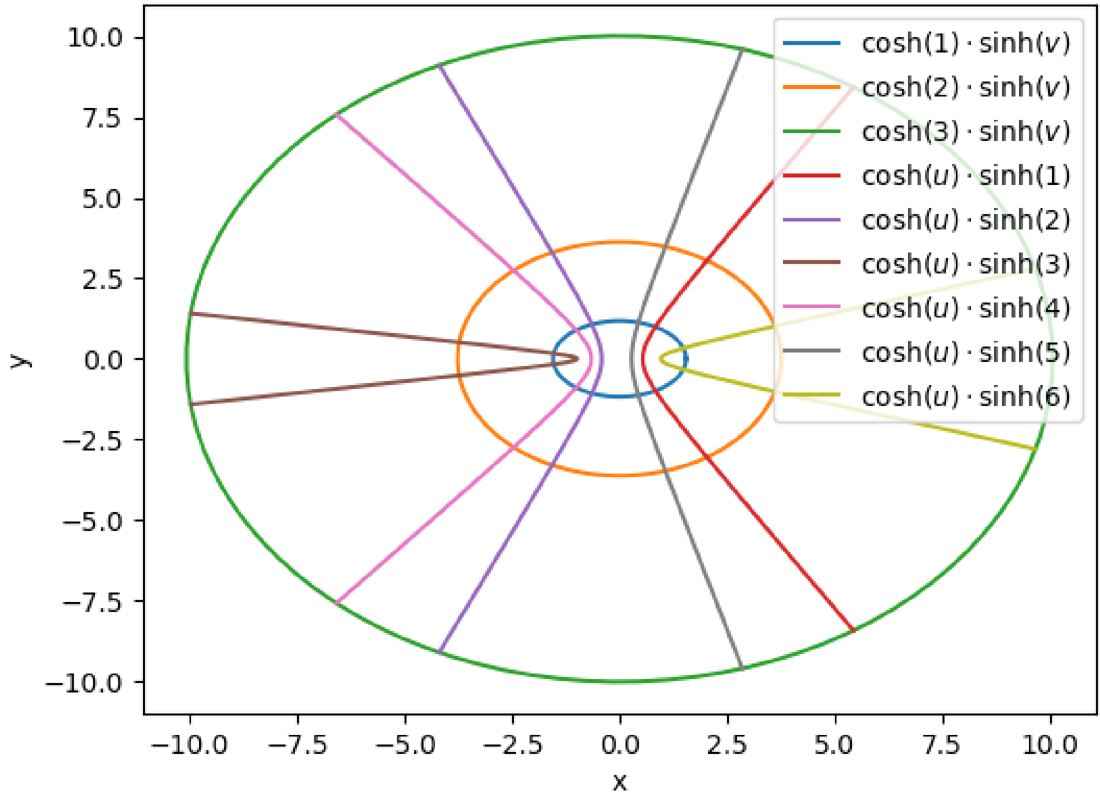
c

Python kode til skissen er

```python
import numpy as np
import matplotlib.pyplot as plt

N = 100
u = np.linspace(-3, 3, N)
v = np.linspace(0, 2*np.pi, N)

for i in range(1, 4):
    x = np.cosh(i)*np.cos(v)
    y = np.sinh(i)*np.sin(v)
    plt.plot(x, y, label=f"$\cosh({i}) \cdot \sinh(v)$")

for i in range(1, 7):
    x = np.cosh(u)*np.cos(i)
    y = np.sinh(u)*np.sin(i)
    plt.plot(x, y, label=f"$\cosh(u) \cdot \sinh({i})$")

plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.savefig("2c.png")
plt.show()
```

Som produserer skissen

Figur 1: Skissen til 2c



d

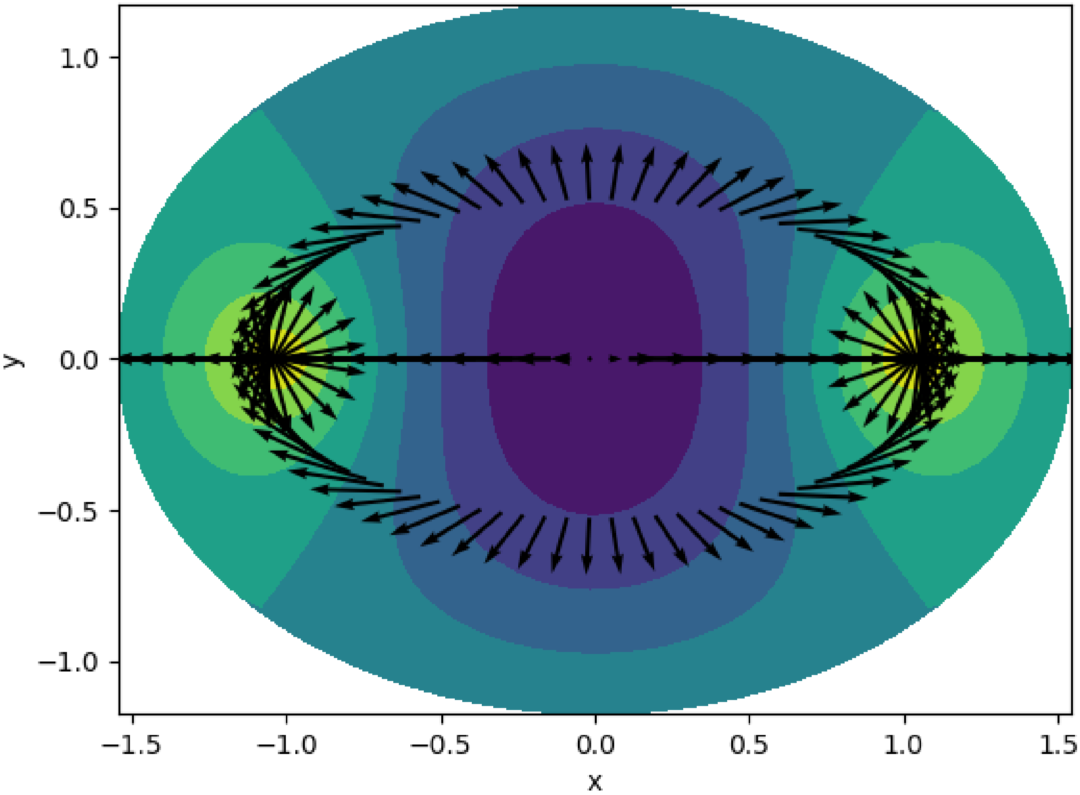
Python kode til dette er

```
import numpy as np
import sympy as sp
import matplotlib.pyplot as plt

u, v = psi = sp.symbols("u, v", real=True)
r = (sp.cosh(u)*sp.cos(v), sp.sinh(u)*sp.sin(v))

def basisvektor(psi, r):
    b = np.zeros((len(psi), len(r)), dtype=object)
    for i, ui, in enumerate(psi):
        for j, rj in enumerate(r):
            b[i, j] = sp.simplify(rj.diff(ui, 1))
    return b

def skaleringsfaktor(b):
    h = np.zeros(b.shape[0], dtype=object)
    for i, s in enumerate(np.sum(b**2, axis=1)):
        h[i] = sp.simplify(sp.sqrt(s))
    return h
```

```python

def enhetsvektor(psi, r):
    b = basisvektor(psi, r)
    hi = skaleringsfaktor(b)
    return b/ hi[None, :], hi

e, h = enhetsvektor(psi, r)

f = (1 - u**2)*sp.cos(2*v)

N = 100
ui = np.broadcast_to(np.linspace(0, 1, N)[:, None], (N, N))
vi = np.broadcast_to(np.linspace(0, 2*np.pi, N)[None, :], (N, N))
fj = sp.lambdify((u, v), f)(ui, vi)

mesh = []
for rj in r:
    mesh.append(sp.lambdify((u, v), rj)(ui, vi))
x, y = mesh

plt.contourf(x, y, fj)

df = np.array((1/h[0]*f.diff(u, 1), 1/h[1]*f.diff(v, 1)))

gradf = e[0]*df[0] + e[1]*df[1]

dfdxi = sp.lambdify((u, v), gradf[0])(ui, vi)
dfdyi = sp.lambdify((u, v), gradf[1])(ui, vi)
plt.contourf(x, y, fj)
plt.quiver(x[::50], y[::50], dfdxi[::50], dfdyi[::50], scale=15)

plt.xlabel("x")
plt.ylabel("y")

plt.savefig("2d.png")
plt.show()
```

Observerer da at vektorene peker ut i fra origo og brennpunktene $(-1, 0)$ og $(1, 0)$

Figur 2: Graf til oppgave 2d



3

a

Strømfunksjon

$$v_x = \frac{\partial \psi}{\partial y} \wedge v_y = \frac{\partial \psi}{\partial x}$$

$$\begin{aligned} \frac{\partial \psi}{\partial y} &= -v_x & \wedge && \frac{\partial \psi}{\partial x} &= v_y \\ \psi(x,y) &= -\int \cos(x)\sin(y)\mathrm{e}^{-2vt}\,\mathrm{d}y & \wedge && \psi(x,y) &= \int -\sin(x)\cos(y)\mathrm{e}^{-2vt}\,\mathrm{d}x \\ \psi(x,y) &= -\cos(x)\cos(y)\mathrm{e}^{-2vt} + C & \wedge && \psi(x,y) &= -\cos(x)\cos(y)\mathrm{e}^{-2vt} + C \end{aligned}$$

Ser så om den har ett skalarpotensial. Hvis det har det så må

$$
\begin{aligned}
&\frac{\partial \phi}{\partial x} = v_x && \wedge && \frac{\partial \phi}{\partial y} = v_y \\
&\phi(x,y) = \int \cos(x)\sin(y)\mathrm{e}^{-2vt}\,\mathrm{d}x && \wedge && \phi(x,y) = \int \left(-\sin(x)\cos(y)\mathrm{e}^{-2vt}\right)\mathrm{d}y \\
&\phi = \sin(x)\sin(y)\mathrm{e}^{-2vt} + f(y) && \wedge && \phi = -\sin(x)\sin(y)\mathrm{e}^{-2vt} + f(x)
\end{aligned}
$$

Disse kan ikke være like så det finnes ikke noe skalarpotensial

c

Kan bruke Gauss' divergensteorem siden det er en generalisering av Green Sett inn mer her siden

$$\int\limits_C \vec{u}\cdot\vec{n}\,\mathrm{d}s = \iint\limits_R \nabla\cdot\vec{u}\,\mathrm{d}x\,\mathrm{d}y$$

Regner ut $\nabla\cdot\vec{u}$

$$
\begin{aligned}
\nabla\cdot\vec{u} &= \frac{\partial}{\partial x}\big(\cos(x)\sin(y)\big) + \frac{\partial}{\partial y}\big(-\sin(x)\cos(y)\big) \\
\nabla\cdot\vec{u} &= -\sin(x)\sin(y) + \sin(x)\sin(y) \\
\nabla\cdot\vec{u} &= 0
\end{aligned}
$$

Følgelig er fluksen da 0
Sirkulasjonen blir da

$$\oint \vec{u}\cdot\mathrm{d}\vec{r} = \int\limits_0^{\frac{\pi}{2}} \vec{u}\cdot\mathrm{d}\vec{r}_{x,0} + \int\limits_0^{\frac{\pi}{2}} \vec{u}\cdot\mathrm{d}\vec{r}_{y,0} + \int\limits_{\frac{\pi}{2}}^{0} \vec{u}\cdot\mathrm{d}\vec{r}_{x,1} + \int\limits_{\frac{\pi}{2}}^{0} \vec{u}\cdot\mathrm{d}\vec{r}_{y,1}$$

$$
\boxed{
\begin{aligned}
&\vec{r}_{x,0}(t') = t'\hat{\mathbf{i}} + 0\hat{\mathbf{j}} \quad \wedge \quad \vec{r}_{y,0}(t') = \frac{\pi}{2}\hat{\mathbf{i}} + t'\hat{\mathbf{j}} \wedge \quad \vec{r}_{x,1}(t') = t'\hat{\mathbf{i}} + \frac{\pi}{2}\hat{\mathbf{j}} \quad \wedge \quad \vec{r}_{y,1}(t') = 0\hat{\mathbf{i}} + t'\hat{\mathbf{j}} \\
&\overrightarrow{r}'_{x,0}(t') = \hat{\mathbf{i}} \quad \wedge \quad \overrightarrow{r}'_{y,0}(t') = \hat{\mathbf{j}} \wedge \quad \overrightarrow{r}'_{x,1}(t') = \hat{\mathbf{i}} \quad \wedge \quad \overrightarrow{r}'_{y,1}(t') = \hat{\mathbf{j}}
\end{aligned}
}
$$

$$
\oint \vec{u}\cdot\mathrm{d}\vec{r} = \begin{aligned}
&\int\limits_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{x,0}(t')\right)\cdot\overrightarrow{r}'_{x,0}(t')\,\mathrm{d}t' + \int\limits_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right)\cdot\overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' \\
&+ \int\limits_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right)\cdot\overrightarrow{r}'_{x,1}\,\mathrm{d}t' + \int\limits_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{y,1}(t')\right)\cdot\overrightarrow{r}'_{y,1}\,\mathrm{d}t'
\end{aligned}
$$

For å få det litt mer oversiktelig så regner jeg ett integral av gangen. Starter med

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{x,0}(t')\right) \cdot \overrightarrow{r}'_{x,0}(t')\,\mathrm{d}t' = \int_0^{\frac{\pi}{2}} \left(\cos\left(t'\right)\sin(0)\mathrm{e}^{-2vt}\right)\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\,\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{x,0}(t')\right) \cdot \overrightarrow{r}'_{x,0}(t')\,\mathrm{d}t' = \int_0^{\frac{\pi}{2}} \left(\cos\left(t'\right)\sin(0)\mathrm{e}^{-2vt}\right)\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{x,0}(t')\right) \cdot \overrightarrow{r}'_{x,0}(t')\,\mathrm{d}t' = \int_0^{\frac{\pi}{2}} 0\,\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{x,0}(t')\right) \cdot \overrightarrow{r}'_{x,0}(t')\,\mathrm{d}t' = 0$$

Ser så på

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = \int_0^{\frac{\pi}{2}} \left(-\sin\left(\frac{\pi}{2}\right)\cos\left(t'\right)\mathrm{e}^{-2vt}\right)\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\,\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = \int_0^{\frac{\pi}{2}} \left(-\sin\left(\frac{\pi}{2}\right)\cos\left(t'\right)\mathrm{e}^{-2vt}\right)\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = -\mathrm{e}^{-2vt}\int_0^{\frac{\pi}{2}} \cos\left(t'\right)\mathrm{d}t'$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = -\mathrm{e}^{-2vt}\left[\sin\left(t'\right)\right]_0^{\frac{\pi}{2}}$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = -\mathrm{e}^{-2vt}\,(1-0)$$

$$\int_0^{\frac{\pi}{2}} \vec{u}\left(\vec{r}_{y,0}(t')\right) \cdot \overrightarrow{r}'_{y,0}(t')\,\mathrm{d}t' = -\mathrm{e}^{-2vt}$$

Ser så på

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right) \cdot \overrightarrow{r}'_{x,1}\,\mathrm{d}t' = \int_{\frac{\pi}{2}}^{0} \left(\cos\big(t'\big)\sin\left(\frac{\pi}{2}\right)\right)\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\,\mathrm{d}t'$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right) \cdot \overrightarrow{r}'_{x,1}\,\mathrm{d}t' = \mathrm{e}^{-2vt}\int_{\frac{\pi}{2}}^{0} \cos\big(t'\big)\,\mathrm{d}t'$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right) \cdot \overrightarrow{r}'_{x,1}\,\mathrm{d}t' = \mathrm{e}^{-2vt}\Big[\sin\big(t'\big)\Big]_{\frac{\pi}{2}}^{0}$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right) \cdot \overrightarrow{r}'_{x,1}\,\mathrm{d}t' = \mathrm{e}^{-2vt}\left(0-1\right)$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{x,1}(t')\right) \cdot \overrightarrow{r}'_{x,1}\,\mathrm{d}t' = -\mathrm{e}^{-2vt}$$

Ser så på

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{y,1}(t')\right) \cdot \overrightarrow{r}'_{y,1}\,\mathrm{d}t' = \int_{\frac{\pi}{2}}^{0} \Big(-\sin(0)\cos\big(t'\big)\mathrm{e}^{-2vt}\Big)\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\,\mathrm{d}t'$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{y,1}(t')\right) \cdot \overrightarrow{r}'_{y,1}\,\mathrm{d}t' = \int_{\frac{\pi}{2}}^{0} \Big(-\sin(0)\cos\big(t'\big)\mathrm{e}^{-2vt}\Big)\,\mathrm{d}t'$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{y,1}(t')\right) \cdot \overrightarrow{r}'_{y,1}\,\mathrm{d}t' = \int_{\frac{\pi}{2}}^{0} 0\,\mathrm{d}t'$$

$$\int_{\frac{\pi}{2}}^{0} \vec{u}\left(\vec{r}_{y,1}(t')\right) \cdot \overrightarrow{r}'_{y,1}\,\mathrm{d}t' = 0$$

Kan så gå tilbake til sirkulasjonen

$$\oint \vec{u}\cdot\mathrm{d}\vec{r} = 0 - \mathrm{e}^{-2vt} - \mathrm{e}^{-2vt} + 0 = -2\mathrm{e}^{-2vt}$$

Kan også bruke Stokes sats som sier

$$\oint_C \vec{u}\cdot\mathrm{d}\vec{r} = \int_{\sigma} \left(\nabla\times\vec{u}\right)\cdot\vec{n}\,\mathrm{d}\sigma$$

Der

$$
\begin{aligned}
\nabla\times\vec{u} &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(x)\sin(y)\mathrm{e}^{-2vt} & -\sin(x)\cos(y)\mathrm{e}^{-2vt} & 0 \end{vmatrix} \\
\nabla\times\vec{u} &= \mathrm{e}^{-2vt}\left(\frac{\partial}{\partial x}\big(-\sin(x)\cos(y)\big) - \frac{\partial}{\partial y}\big(\cos(x)\sin(y)\big)\right)\hat{\mathbf{k}} \\
\nabla\times\vec{u} &= \mathrm{e}^{-2vt}\big(-\cos(x)\cos(y) - \cos(x)\cos(y)\big)\hat{\mathbf{k}} \\
\nabla\times\vec{u} &= -2\mathrm{e}^{-2vt}\cos(x)\cos(y)\hat{\mathbf{k}}
\end{aligned}
$$

Og $\vec{n} = \hat{\mathbf{k}}$ siden den har positiv orientering om z. Da blir Stokes

$$
\begin{aligned}
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= \int\limits_{\sigma}\Big(-2\mathrm{e}^{-2vt}\cos(x)\cos(y)\Big)\,\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\,\mathrm{d}\sigma \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\int\limits_{\sigma}\big(\cos(x)\cos(y)\big)\,\mathrm{d}\sigma \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\int\limits_{0}^{\frac{\pi}{2}}\int\limits_{0}^{\frac{\pi}{2}}\cos(x)\cos(y)\,\mathrm{d}x\,\mathrm{d}y \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\int\limits_{0}^{\frac{\pi}{2}}\cos(y)\,\big[\sin(x)\big]_{0}^{\frac{\pi}{2}}\,\mathrm{d}y \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\int\limits_{0}^{\frac{\pi}{2}}\cos(y)\,(1-0)\,\mathrm{d}y \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\int\limits_{0}^{\frac{\pi}{2}}\cos(y)\,\mathrm{d}y \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\,\big[\sin(y)\big]_{0}^{\frac{\pi}{2}} \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}\,(1-0) \\
\int\limits_{\sigma}(\nabla\times\vec{u})\cdot\vec{n}\,\mathrm{d}\sigma &= -2\mathrm{e}^{-2vt}
\end{aligned}
$$