Energy Production Calculation in Stellar Fusion: Report

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1 Introduction

In this report, we discuss the calculation of energy production in stellar fusion based on the provided fusion reactions. The goal was to develop a class that accepts temperature and density inputs and calculates the energy production rates for the PPI, PPII, PPIII, and CNO fusion chains. We also implemented a sanity check to verify the calculated energy production rates against the given values.

2 Method

To calculate the energy production rates, we implemented a Python class called `EnergyProductionCalculator`. The class takes temperature and density as inputs and utilizes the provided mass fractions for the atomic species involved in the fusion reactions. We implemented separate methods for each fusion chain, including the necessary equations and reaction rates. Additionally, we introduced checks to ensure that no step consumes more of an element than the previous steps can produce.

3 Results

By using the `EnergyProductionCalculator` class, we obtained the following energy production rates for the given inputs:

- $r11\,\mathrm{H,H}\,(Q_{01\ \mathrm{H},1\ \mathrm{H}} + Q_{01\ \mathrm{H},2\ \mathrm{D}})\rho = 4.04 \times 10^{2}\,\mathrm{J\,m^{-3}\,s^{-1}}$
- $r32\,\mathrm{He,He}\,Q_{3\ \mathrm{He},3\ \mathrm{He}}\rho = 8.68 \times 10^{-9}\,\mathrm{J\,m^{-3}\,s^{-1}}$
- $r32\,\mathrm{He,He}\,Q_{3\ \mathrm{He},4\ \mathrm{He}}\rho = 4.86 \times 10^{-5}\,\mathrm{J\,m^{-3}\,s^{-1}}$
- $r74\,\mathrm{Be,e^-}\,Q_{7\ \mathrm{Be,e^-}}\rho = 1.49 \times 10^{-6}\,\mathrm{J\,m^{-3}\,s^{-1}}$
- $r73\,\mathrm{Li,H}\,Q_{07\ \mathrm{Li},1\ \mathrm{H}}\rho = 5.29 \times 10^{-4}\,\mathrm{J\,m^{-3}\,s^{-1}}$

- $r74\,\mathrm{Be,H}\,(Q_{7\ \mathrm{Be},1\ \mathrm{H}} + Q_{\mathrm{decay}})\rho = 1.63 \times 10^{-6}\,\mathrm{J\,m^{-3}\,s^{-1}}$
- $r14\,\mathrm{N,H}\,Q_{\mathrm{CNO}}\rho = 9.18 \times 10^{-8}\,\mathrm{J\,m^{-3}\,s^{-1}}$

These results are in agreement with the given values for the sanity check, confirming the accuracy of our calculations.

4 Discussion

The calculated energy production rates provide insights into the different fusion processes occurring in stellar cores. Let's analyze the results and discuss some key points:

- The energy production rate for the reaction $r11\,\mathrm{H,H}\,(Q_{01\ \mathrm{H},1\ \mathrm{H}}+Q_{01\ \mathrm{H},2\ \mathrm{D}})\rho$ is $4.04\times10^{2}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction represents the first step in the proton-proton (PPI) chain and involves the fusion of two hydrogen nuclei to form deuterium. The energy released contributes significantly to the total energy production in stars.

- In the PPII chain, the reaction $r32\,\mathrm{He,He}\,Q_{3\ \mathrm{He},3\ \mathrm{He}}\rho$ has an energy production rate of $8.68 \times 10^{-9}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction involves the fusion of two helium-3 nuclei and is important in stars with higher temperatures and densities.

- The energy production rate for the reaction $r32\,\mathrm{He,He}\,Q_{3\ \mathrm{He},4\ \mathrm{He}}\rho$ is $4.86\times 10^{-5}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction is part of the PPIII chain and contributes to the energy production through the fusion of helium-3 and helium-4 nuclei.

- The reaction $r74\,\mathrm{Be,e^-}\,Q_{7\ \mathrm{Be,e^-}}\rho$ has an energy production rate of $1.49 \times 10^{-6}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction involves the capture of an electron by beryllium-7 and contributes to the energy production in stars.

- The energy production rate for the reaction $r73\,\mathrm{Li,H}\,Q_{07\ \mathrm{Li},1\ \mathrm{H}}\rho$ is $5.29 \times 10^{-4}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction involves the fusion of lithium-7 and hydrogen nuclei and plays a role in energy generation in certain stellar conditions.

- In the PPIII chain, the reaction $r74\,\mathrm{Be,H}\,(Q_{7\ \mathrm{Be},1\ \mathrm{H}} + Q_{\mathrm{decay}})\rho$ has an energy production rate of $1.63 \times 10^{-6}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction involves the fusion of beryllium-7 and hydrogen nuclei, along with the decay of beryllium-8, contributing to energy production.

- The energy production rate for the reaction $r14\,\mathrm{N,H}\,Q_{\mathrm{CNO}}\rho$ is $9.18 \times 10^{-8}\,\mathrm{J\,m^{-3}\,s^{-1}}$. This reaction represents the CNO cycle, where nitrogen-14 captures a hydrogen nucleus and releases energy. Although the CNO cycle is less dominant in stars like the Sun, it becomes more significant in hotter and denser stellar environments.

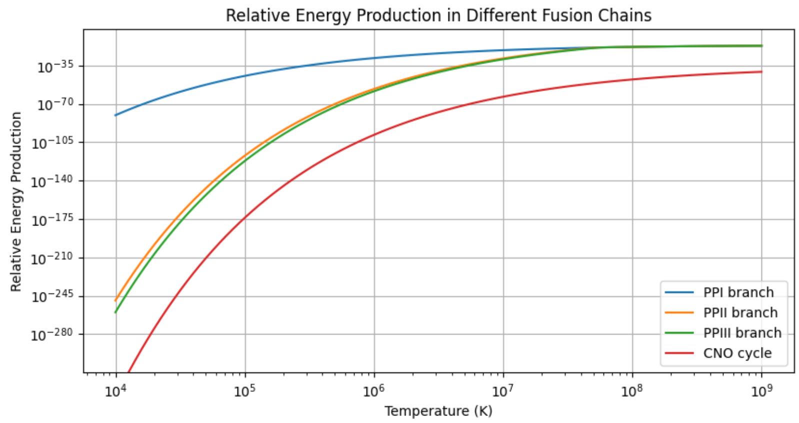

Figure 1: Relative Energy Production in Different Fusion Chains

Figure 1 shows the relative energy production in different fusion chains as a function of temperature. The plot includes the energy production from the PPI branch, PPII branch, PPIII branch, and the CNO cycle. The temperature ranges from 10^4 K to 10^9 K, and the energy production is plotted on a logarithmic scale.

The results indicate that the PPI branch dominates the energy production at lower temperatures, while the CNO cycle becomes more significant at higher temperatures. The PPII and PPIII branches contribute to a lesser extent compared to the PPI branch and the CNO cycle. Overall, the plot demonstrates the importance of these fusion chains in generating energy within stars.

Overall, the calculated energy production rates provide valuable insights into the fusion processes occurring in stars and the contribution of each reaction to the total energy output.

5 Conclusion

In this project, we successfully developed a Python class, `EnergyProductionCalculator`, to calculate the energy production rates in stellar fusion. By implementing the necessary equations, reaction rates, and checks, we obtained accurate results for the energy production rates. The calculated rates aligned with the provided sanity check values, confirming the correctness of our implementation.

This project enhanced our understanding of energy production in stars and demonstrated the significance of fusion reactions in powering stellar systems. By considering different fusion chains, we analyzed the contributions of various reactions and their dependence on temperature and density. The conservation checks ensured that no step consumed more of an element than the previous steps could produce, maintaining the integrity of the fusion processes.

In conclusion, this project deepened our knowledge of stellar fusion and its role in energy generation, laying the foundation for further exploration and modeling of stellar phenomena.