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Write my thesis

Example Template

John Doe

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Erklärung

Ich versichere hiermit, dass ich die vorliegende Arbeit selbstständig und unter Beachtung der Satzung des Karlsruher Instituts für Technologie (KIT) zur Sicherung guter wissenschaftlicher Praxis in der aktuellen Fassung angefertigt habe. Ich habe keine anderen als die angegebenen Quellen und Hilfsmittel benutzt und wörtlich oder inhaltlich übernommene Stellen als solche kenntlich gemacht.

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Abstract

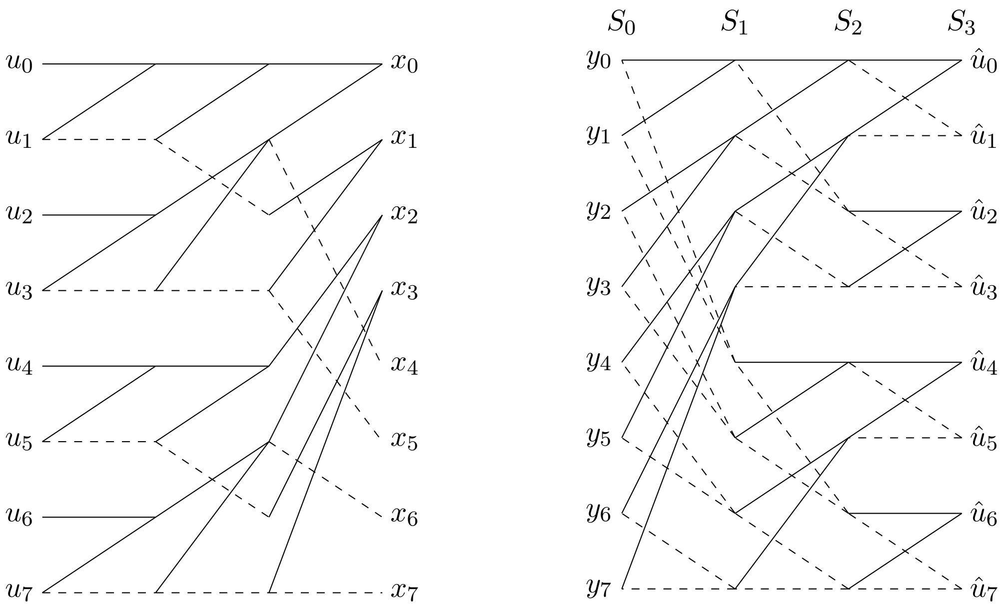
This is an abstract. CEL thesis rules require it to be about 3-5 pages. It is a summary of what you do in your thesis. Do not confuse this with the introduction! Use around 5 pictures and outline whatever you did. And now a few lines of information.

This is a todo example

Polar codes are the first codes to asymptotically achieve channel capacity with low complexity encoders and decoders. They were first introduced by Erdal Arikan in 2009 [**?**].

Channel coding has always been a challenging task because it draws a lot of resources, especially in software implementations. Software Radio is getting more prominent because it offers several advantages among which are higher flexibility and better maintainability. Future radio systems are aimed at being run on virtualized servers instead of dedicated hardware in base stations [**?**]. Polar codes may be a promising candidate for future radio systems if they can be implemented efficiently in software.

In this thesis the theory behind polar codes and a polar code implementation in GNU Radio is presented. This implementation is then evaluated regarding parameterization options and their impact on error correction performance. The evaluation includes a comparison to state-of-the-art **LDPC!** (**LDPC!**) codes.



(a) 8 bit polar encoder **(b)** 8 bit polar decoder

Abbildung 0.1: Polar code encoding and decoding

The polar encoder is shown in Fig. **??**.

Inhaltsverzeichnis

1 Introduction

This is the introductory chapter. It is usually a page or two. Tell a story about the objectives, explain them briefly and outline the structure of your thesis.

1.1 Structuring your thesis

An example structure would be:

1. Introduction
2. Theoretical basis
3. Your work
4. Measurement results
5. Conclusion

This is just an example, choose a structure that fits the nature of your work.

1.2 References

Citing references is always good. Plagiarizing, however, is strictly forbidden!

1.3 Images

If possible use vector graphics. Only use pixel graphics for photos. TikZ is also an interesting option for creating all sorts of images.

And don't forget 10,0815 GB of data is quite a lot. This is an example how to use the siunitx package.

2 An example chapter

Polar codes are defined for a specific system model. The objective of this chapter is to introduce the key concepts. Notations are introduced and important terms are revisited in order to refer to them.

2.1 Key channel coding concepts

The system model used throughout this thesis follows the remarks in [?] and [?]. It is intended to define the domain for which polar codes are developed.

The objective of channel coding is to transmit information from a source to a sink over a point-to-point connection with as few errors as possible. A source wants to transmit binary data $u \in \mathcal{U} = \{0,1\}$ to a sink where u represents one draw of a binary uniformly distributed random variable. The source symbols are encoded, transmitted over a channel and decoded afterwards in order to pass an estimate $\hat{u}$ to a sink.

This thesis uses a common notation for vectors which is introduced here shortly. A variable x may assume any value in an alphabet $x \in \mathcal{X}$. Multiple variables are combined into a vector $x^N = (x_0, \ldots, x_{N-1})$ of size N with its alphabet $x^N \in \mathcal{X}^N$. A subvector of x^N is denoted $x_i^j = (x_i, \ldots, x_{j-1})$ where $0 \leq i \leq j \leq N$. A vector where $i = j$ is an empty vector. A vector x^N may be split into even and odd subvectors which are denoted $x_{0,e}^{2n} = (x_0, x_2, \ldots, x_{2n-2})$, $x_{0,o}^{2n} = (x_1, x_3, \ldots, x_{2n-1})$. This numbering convention is in accordance with [?], where the author makes a strong point for this exact notation and some papers on polar codes follow it too, e.g. [?].

2.1.1 Encoder

The encoder takes a frame u^k and maps it to a binary codeword x^N, where k and N denote the vector sizes of a frame and a codeword respectively with $k \leq N$. An ensemble of all valid codewords for an encoder is a code $\mathcal{C}$. It should be noted that $|\mathcal{C}| = |\mathcal{X}^N|$ must hold in order for the code to be able to represent every possible frame.

Not all possible symbols from $\mathcal{X}^N$ are used for transmission. The difference between all possible codewords 2^N and used codewords 2^k is called redundancy. With those two values, the code rate is defined as $R = \frac{k}{N}$. It is a measure of efficient channel usage.

The encoder is assumed to be linear and to perform a one-to-one mapping of frames to codewords. A code is linear if $\alpha x + \alpha' x' \in \mathcal{C}$ for $\forall x, x' \in \mathcal{C}$ and $\forall \alpha, \alpha' \in \mathbb{F}$ hold. It should be noted that all operations are done over the Galois field GF(2) or $\mathbb{F} = \{0,1\}$ if not stated otherwise. Then the expression can be simplified to

$$x + x' \in \mathcal{C} \quad \text{for} \quad \forall x, x' \in \mathcal{C}. \tag{2.1}$$

A linear combination of two codewords must yield a codeword again.

For linear codes it is possible to find a generator matrix $G \in \mathbb{F}^{k \times N}$ and obtain a codeword from a frame with $x^N = u^k G^{k \times N}$. All linear codes can be transformed into systematic form

$G = I_k P$. I_k is a $k \times k$ dimensional identity matrix. If G is systematic, all elements of a frame u^k are also elements of the codeword x^N. Also, a parity check matrix $H = -P^T I_{N-k}$ with dimensions $(N-k) \times N$ can be calculated from G. A parity check matrix satisfies $\forall x \in \mathcal{C} : Hx^T = 0^T$. Thus, a parity check matrix can be used to verify correct codeword reception and furthermore error correction may be performed. Error correction with H may be done, e.g. syndrome decoding.

A code can be characterized by the minimum distance between any two codewords. In order to obtain this value we use the Hamming distance. This distance $d(v^N, x^N)$ equals the number of positions in v^N that differ from x^N. Minimum distance of a code is than defined by $d(\mathcal{C}) = \min\{d(x,v) : x,v \in \mathcal{C}, x \neq v\}$. For linear codes this can be simplified to comparing all codewords to the zero codeword $d(\mathcal{C}) = \min\{d(x,0) : x \in \mathcal{C}, x \neq 0\}$ which is called Hamming weight.

2.1.2 Channel model

Channel coding relies on a generic channel model. Its input is $x \in \mathcal{X}$ and its distorted output is $y \in \mathcal{Y}$. A channel is denoted $W : \mathcal{X} \to \mathcal{Y}$ along with its transition probability $W(y|x), x \in \mathcal{X}, y \in \mathcal{Y}$. A **DMC!** (**DMC!**) does not have memory, thus every symbol transmission is independent from any other. Combined with a binary input alphabet it is called a **BDMC!** (**BDMC!**). For a symmetric channel model, $P(y|1) = P(-y|-1)$ must hold for an output alphabet $y \in \mathcal{Y}, \mathcal{Y} \subset \mathbb{R}$ [**?**]. Assuming symmetry for a **BDMC!** leads to a symmetric **BDMC!**. In Sec. **??** several examples of such channels are discussed.

This channel concept may be extended to vector channels. A vector channel W^N corresponds to N independent uses of a channel W which is denoted as $W^N : \mathcal{X}^N \to \mathcal{Y}^N$. Also, vector transition probabilities are denoted $W^N(y^N|x^N) = \prod_{i=0}^{N-1} W(y_i|x_i)$.

2.1.3 Decoder

A decoder receives a possibly erroneous codeword y and checks its validity by asserting $Hy^T = 0^T$, thus performing error detection. A more sophisticated decoder tries to correct errors by using redundant information transmitted in a codeword. An optimal decoder strategy is to maximize the a-posteriori probability. Given the probability of each codeword $P(x)$ and the channel transition probability $P(y|x)$, the task at hand is to find the most likely transmitted codeword x under the observation y, $P(x|y)$. This is denoted

$$\hat{x}^{MAP} = \operatorname*{argmax}_{x \in \mathcal{C}} p(x|y) = \operatorname*{argmax}_{x \in \mathcal{C}} p(y|x)\frac{p(x)}{p(y)} = \operatorname*{argmax}_{x \in \mathcal{C}} p(y|x)p(x) \tag{2.2}$$

with Bayes' rule. Assume every codeword is transmitted with same probability $P(x^{(i)}) = P(x^{(j)}), \forall x^{(i)}, x^{(j)} \in \mathcal{C}$. This simplifies the equation and yields a **ML!** (**ML!**) decoder

$$\hat{x} = \operatorname*{argmax}_{x \in \mathcal{C}} p(y|x) \tag{2.3}$$

which estimates the most likely codeword to be transmitted given a received possibly erroneous codeword [**?**]. This decoding principle could be employed in conjunction with the Hamming distance and thus yield $\hat{x} = \operatorname{argmin}_{x \in \mathcal{C}} d(x,y)$. In conclusion the task at hand is to find a code which inserts redundancy intelligently, so a decoder can use this information to detect and correct transmission errors.

2.1.4 Asymptotically good codes

A repetition code is a very simple code which helps clarify certain key concepts in the channel coding domain. Assume the encoder and decoder use a repetition code. For example a repetition code with $k = 1$ and $N = 3$ has two codewords $\mathcal{C} = \{000{,}111\}$. Thus in this example $R = \frac{1}{3}$. We can also obtain its generator and parity check matrices.

$$G = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}, \qquad H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{2.4}$$

H can be used to detect if a transmission error occurred by verifying if $Hx^T = 0^T$. In case an error occurred, a **ML!** decoder does a majority decision to estimate the most likely codeword.

Repetition codes shed light on a problem common to a lot of codes. If reliability of a code needs to be improved, it comes at the expense of a lower code rate. Increasing N comes at the expense of decreasing $R = \frac{1}{N}$ because $k = 1$ for all repetition codes. Thus for a very reliable repetition code $\lim_{N\to\infty} R$ tends towards 0.

The above results leads to the definition of asymptotically good codes $\mathcal{C}(N_s{,}k_s{,}d_s)$ [**?**]. Two properties must hold for this class of codes,

$$R = \lim_{s\to\infty} \frac{k_s}{N_s} > 0 \quad \text{and} \quad \lim_{s\to\infty} \frac{d_s}{N_s} > 0. \tag{2.5}$$

The code rate must be > 0 for all codes which repetition codes do not satisfy. And the distance between codewords must grow proportionally to the code block size.

2.2 Channels

Several common channel models exist to describe the characteristics of a physical transmission. Common properties were discussed in Section **??** whereas in this Section the differences are targeted. The three most important channel models for polar codes are presented, namely the **BSC!** (**BSC!**), the **BEC!** (**BEC!**) and the **AWGN!** (**AWGN!**) channel.

2.2.1 AWGN channel

An **AWGN!** channel as used in this thesis has a binary input alphabet and a continuous output alphabet $\mathcal{Y} = \mathbb{R}$. Each input symbol is affected by Gaussian noise to derive an output symbol. Its average corresponds to the input symbol value and the variance can be interpreted as a measure of noise. Often the input is **NRZ!** (**NRZ!**) encoded which turns a **ML!** decision for a symbol into a sign decision.

2.2.2 Capacity and reliability

Channels are often characterized by two important measures, capacity and reliability. These measures are introduced in this Section. Channel capacity for symmetric **BDMC!** can be calculated by

$$I(W) = \frac{1}{2} \sum_{y\in\mathcal{Y}} \sum_{x\in\mathcal{X}} W(y|x) \log_2 \frac{W(y|x)}{\frac{1}{2}(W(y|0) + W(y|1))}. \tag{2.6}$$

It defines the highest rate at which a reliable transmission over a channel W can be conducted while the error probability may still tend towards 0. It is also called the Shannon capacity [**?**]

for symmetric channels. The Bhattacharyya parameter

$$Z(W) = \sum_{y \in \mathcal{Y}} \sqrt{W(y|0)W(y|1)} \tag{2.7}$$

is used to quantify a channel's reliability where a lower value for $Z(W)$ indicates higher reliability. It is also referred to as Z-parameter for obvious reasons. Also, an upper **ML!** decision error bound is given by $Z(W)$ [**?**].

3 Conclusion

So you made it! This is the last part of your thesis. Tell everyone what happened. You did something... and you could show that ... followed.

In the end make a personal statement. Why would one consider this thesis to be useful?

A Abbreviations

AWGN Additive White Gaussian Noise

BEC Binary Erasure Channel

BDMC binary **DMC!**

BSC Binary Symmetric Channel

DMC Discrete Memoryless Channel

LDPC Low-Density Parity-Check

ML Maximum Likelihood

NRZ Non-Return-to-Zero