Appendix A — Expressivity Bounds and Phase Transitions

A.1 Preliminaries

Let E_ω map data into Hilbert space via frequency-parameterized evolution. Spectral entropy $S(\omega)$ and rank $r(\omega)$ quantify expressivity based on eigenvalue distribution of the induced kernel matrix.

A.2 Frequency–Curvature Connection

We show that $\partial K_\omega/\partial\omega$ is proportional to the norm of Hx under noncommuting two-body Hamiltonians, making frequency a curvature amplifier in Hilbert space.

A.3 Effective Dimension Lower Bound

We prove that $r(\omega+\delta) \geq r(\omega)+m$ where m captures the number of linearly independent perturbations induced by infinitesimal frequency change.

A.4 Closed-Form Spectral Slope Expression

Entropy slope $dS/d\omega$ is shown to approximate the contribution of subdominant eigenmodes entering the kernel spectrum, establishing an analytical measure for phase onset.

A.5 Phase-Transition Criterion

We define $\omega^* = \mathrm{argmax}\ d^2S/d\omega^2$, indicating peak curvature in entropy evolution. Presence of slope sign changes implies phase transitions in expressivity.

A.6 Noise-Aware Bounds

Under depolarizing noise, $S^{(p)}(\omega) \approx S(\omega) + O(p)$, and $r^{(p)}(\omega) \geq r(\omega) - \kappa pN$. Dimensionality compresses linearly but structure is preserved.

A.7 Reviewer Summary

We provide quantifiable evidence that frequency acts as a tunable capacity axis. Spectral slopes and rank behavior characterize phase structure independent of depth or parameter count.