ESE-615 - Autonomous Racing ()

Scan Matching

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Problem Lab Assignment (points)

Theoretical Questions

1. B_i

   (a) $B_i = M_i^T M_i = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ p_{i0} & p_{i1} \\ -p_{i1} & p_{i0} \end{pmatrix} \begin{pmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \end{pmatrix} = \begin{pmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \\ p_{i0} & p_{i1} & p_{i0}^2 + p_{i1}^2 & 0 \\ -p_{i1} & p_{i0} & 0 & p_{i0}^2 + p_{i1}^2 \end{pmatrix}$

   (b) $\forall x\ ,\ x^T B_i x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix} \begin{pmatrix} 1 & 0 & p_{i0} & -p_{i1} \\ 0 & 1 & p_{i1} & p_{i0} \\ p_{i0} & p_{i1} & p_{i0}^2 + p_{i1}^2 & 0 \\ -p_{i1} & p_{i0} & 0 & p_{i0}^2 + p_{i1}^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = (x_1 - p_{i1}x_4 + p_{i0}x_3)^2 +$

   $(x_2 + p_{i0}x_4 + p_{i1}x_3)^2 \geq 0$

   Proof ends.

2. **Optimization Problem**

   (a) $W = \begin{bmatrix} 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & \mathrm{I}_{2\times 2} \end{bmatrix}$, $M = \sum_i M_i^T C_i M_i$ $g = \sum_i -2\pi_i^T C_i M_i$, where $C_i = n_i n_i^T$

   (b) $\forall x$, $x^T M x = \sum_i x^T M_i^T n_i n_i^T M_i x = \sum_i x^T M_i^T n_i (x^T M_i^T n_i)^T \geq 0$

   $\forall x$, we have $x^T W x = x_3^2 + x_4^2 = 1 \geq 0$