Appendices

A. Figures of Optimal Observables

This appendix contains figures for the various optimal observables described in section 5.4.

(a)

(b)

(c)

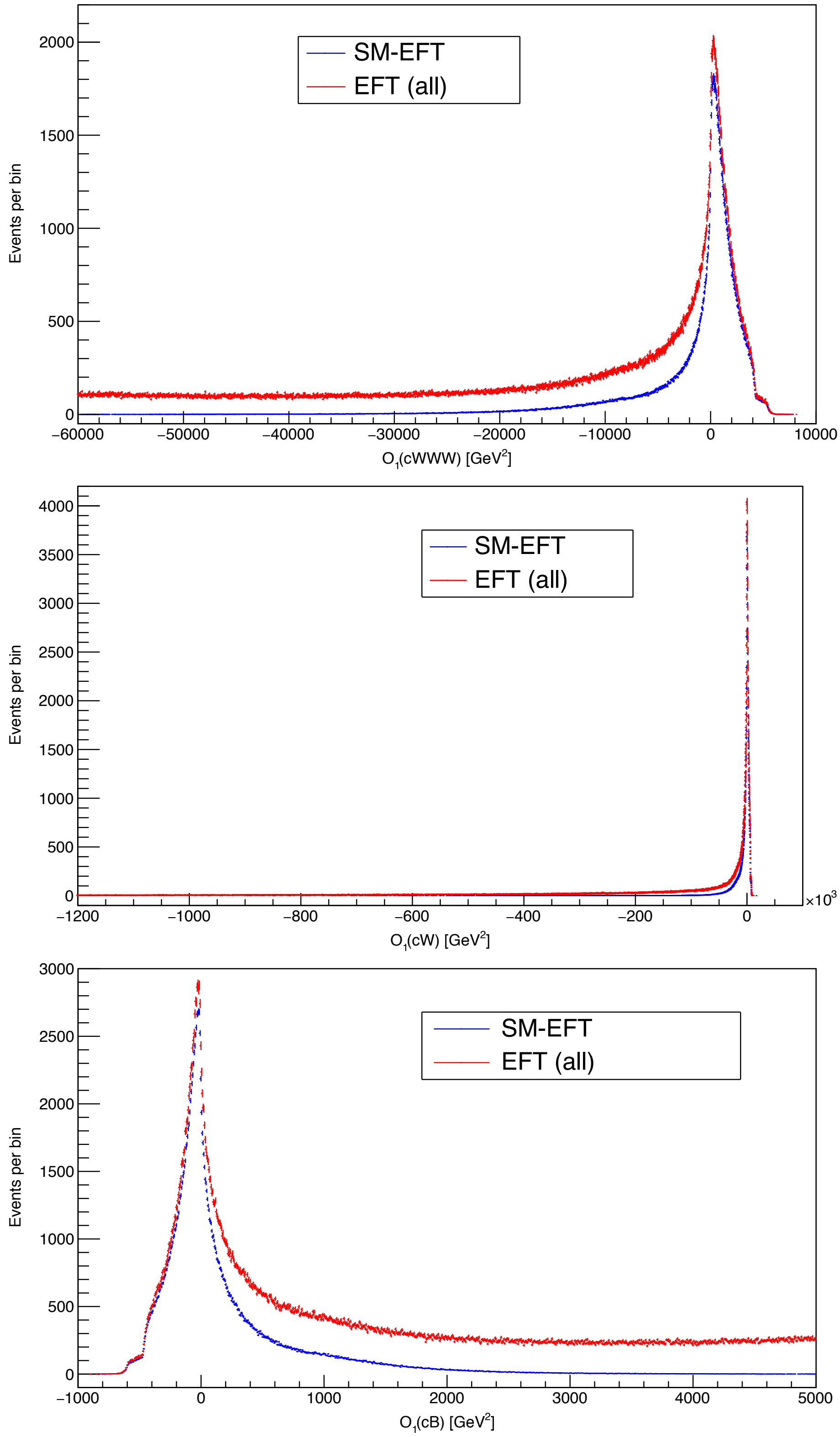
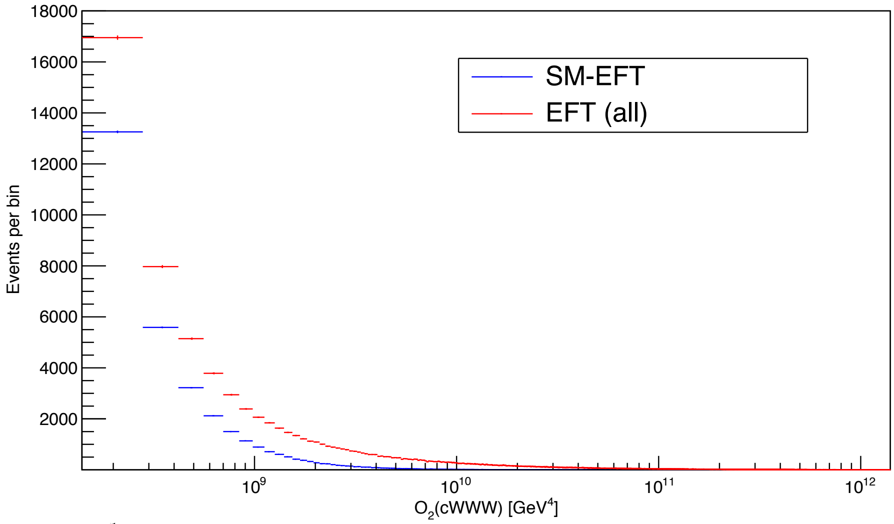
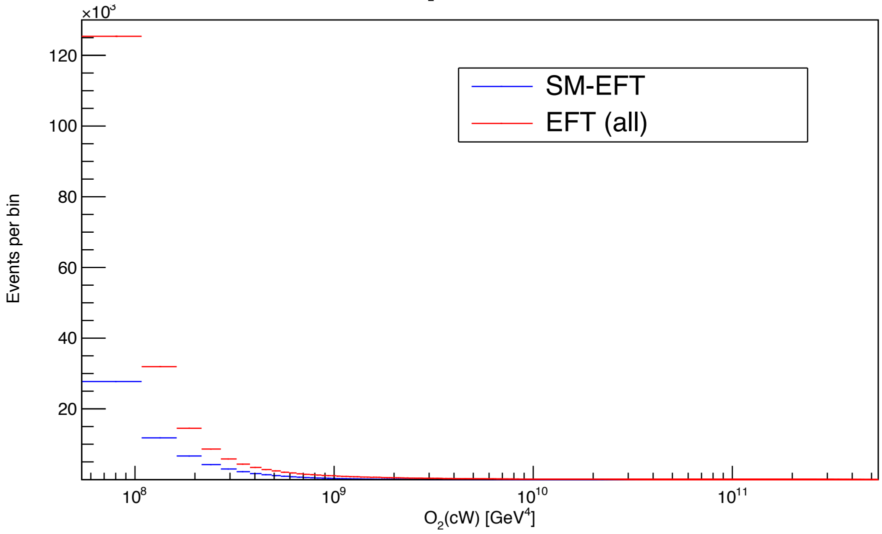


Figure A.1: First-order optimal observables, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, for EFT and SM parton-level WZ-production (luminosity 10 fb^{-1}).

(a)



(b)



(c)

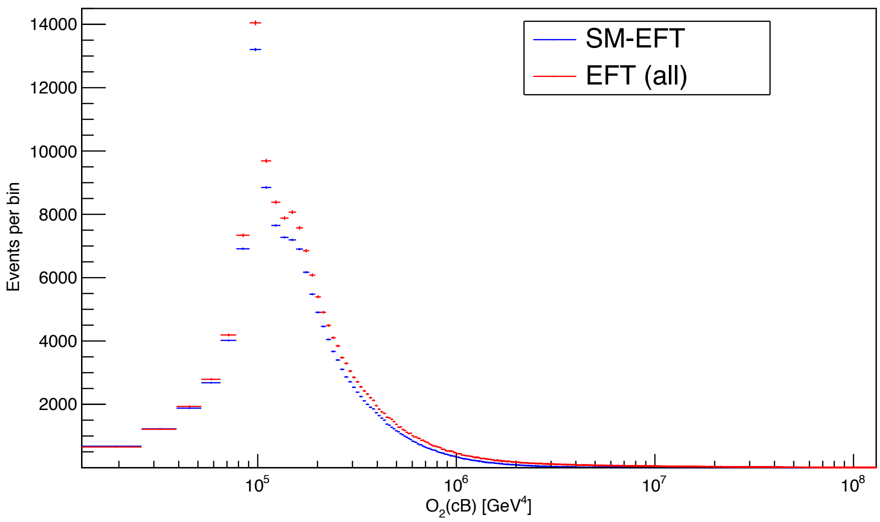


Figure A.2: Second-order optimal observables, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, for EFT and SM parton-level WZ-production (luminosity 10 fb^{-1}).

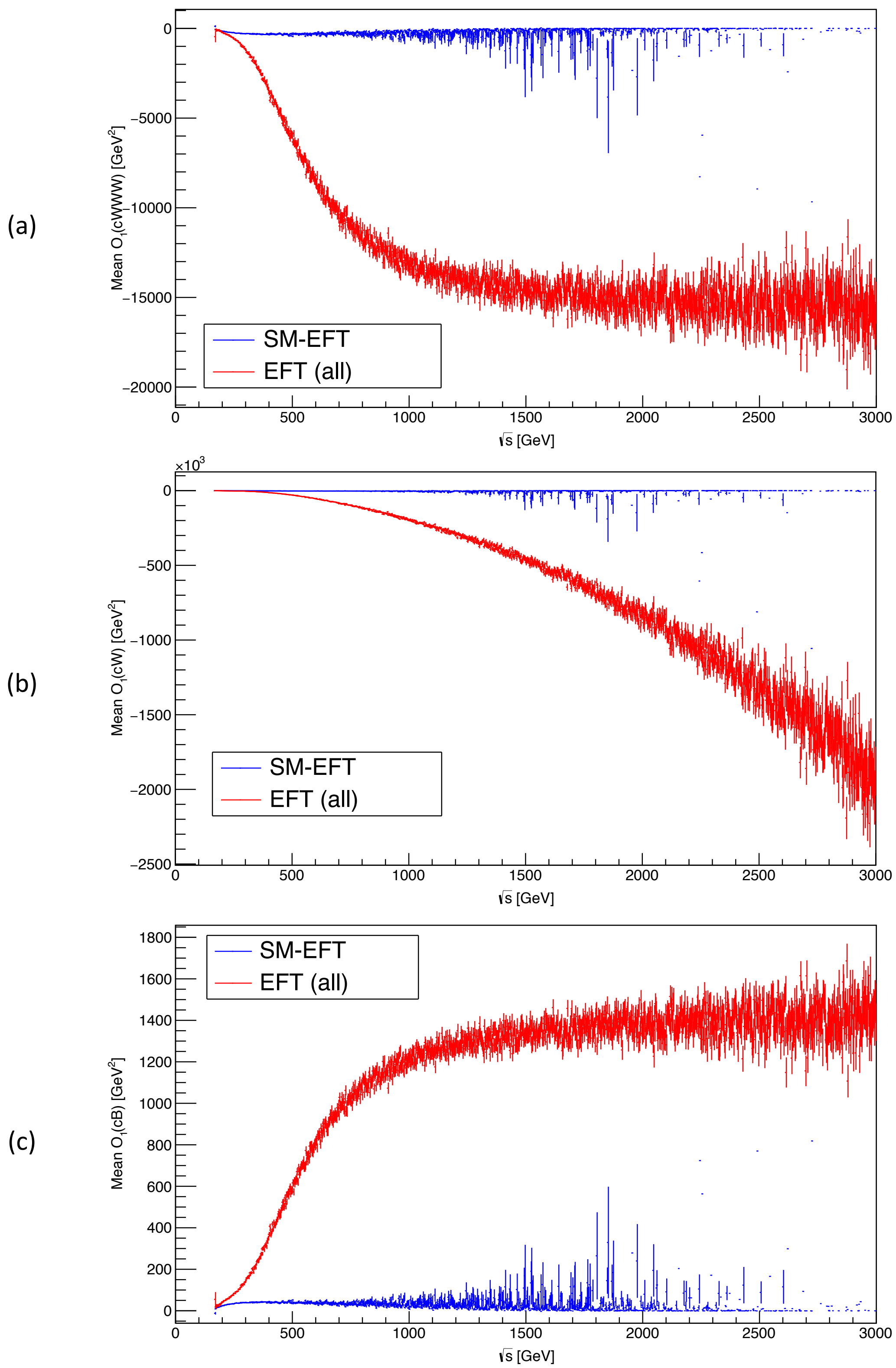


Figure A.3: First-order mean optimal observables, for EFT model parameters (a) c_{WWW}*, (b)* c_W *and (c)* c_B*, for EFT and SM parton-level WZ-production (luminosity 10* fb^{-1}*).*

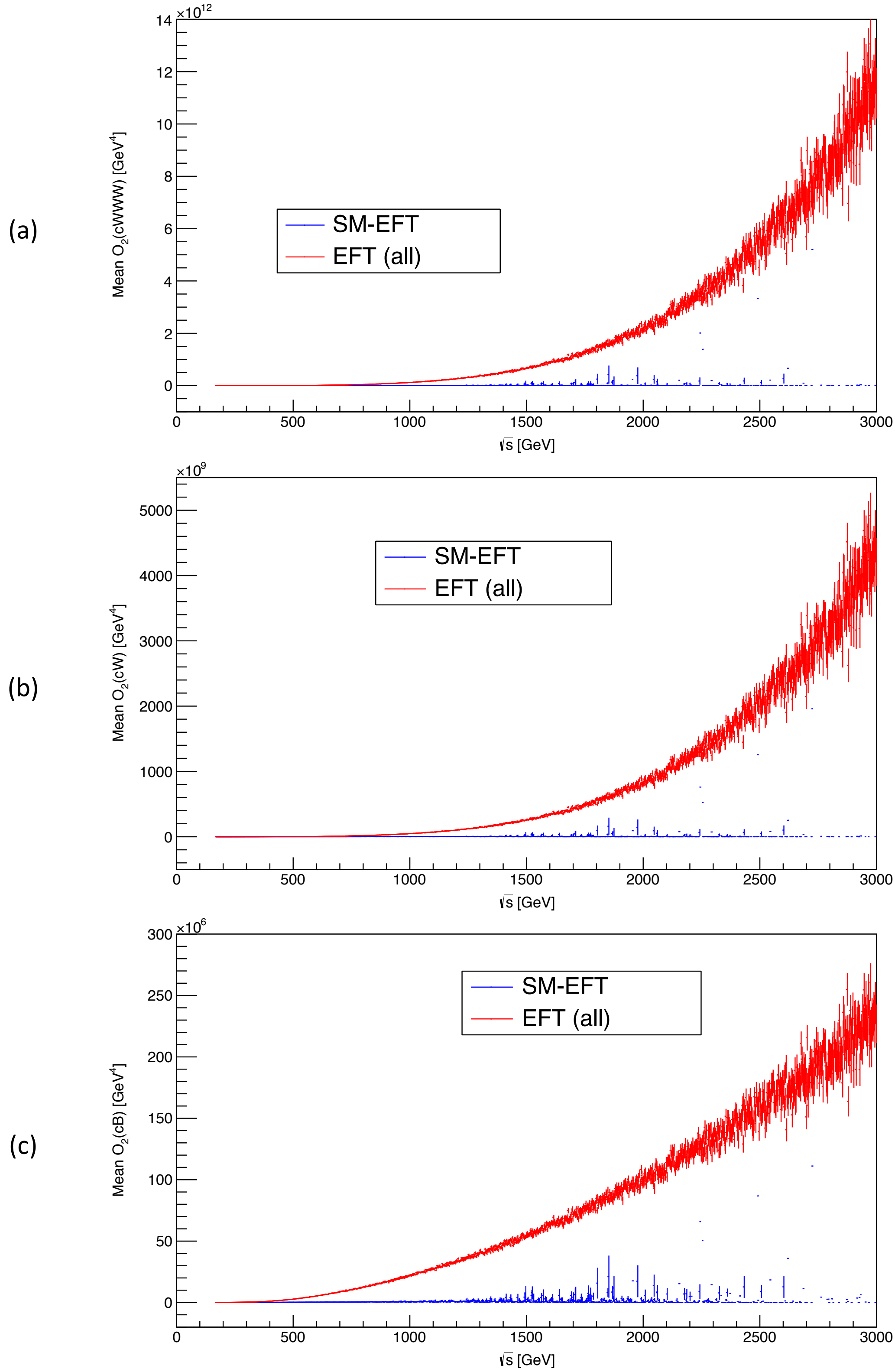


Figure A.4: Second-order mean optimal observables, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, for EFT and SM parton-level WZ-production (luminosity 10 fb^{-1}).

(a)

(b)

(c)

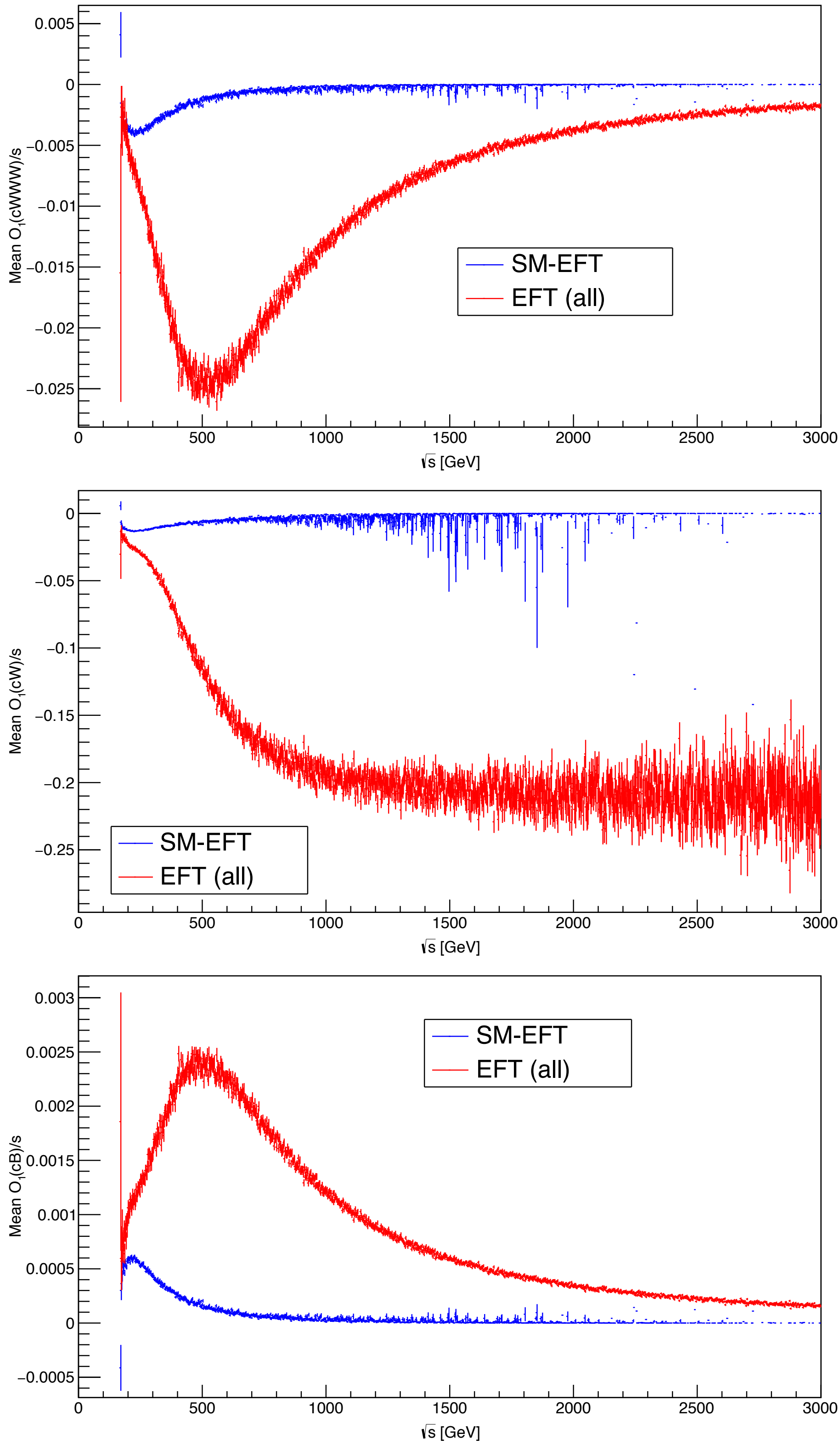


Figure A.5: First-order mean optimal observables divided by powers of invariant mass, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, for EFT and SM parton-level WZ-production (luminosity 10 fb^{-1}).

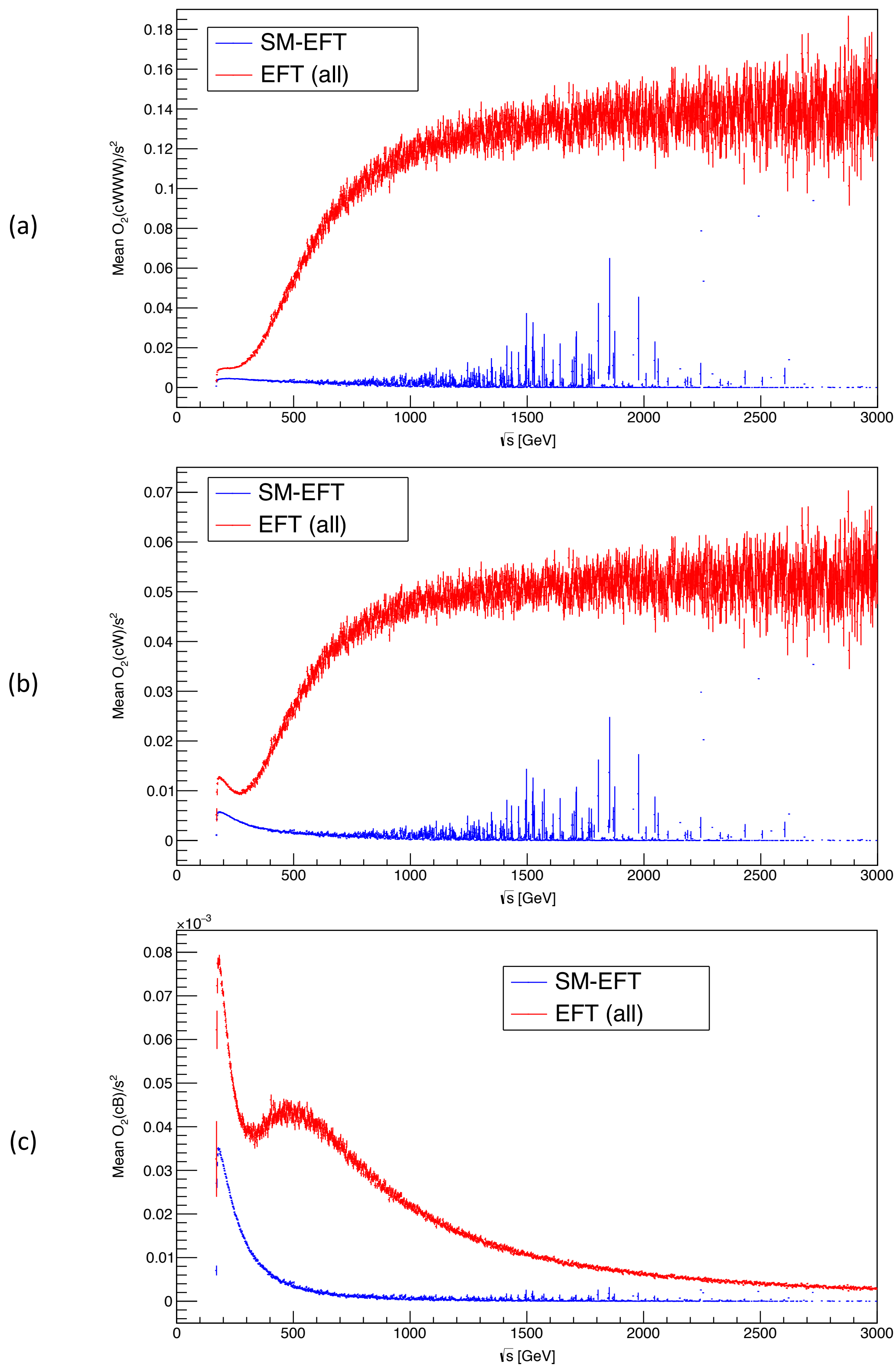


Figure A.6: First-order mean optimal observables divided by powers of invariant mass, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, for EFT and SM parton-level WZ-production (luminosity 10 fb^{-1}).

B. Figures of Cross-Validation Minimization for Bin Optimization

In this appendix, we include figures showing the various cross-validation minimization curves used to select the optimal bin sizes for our various observables in section 6.2.1.1.

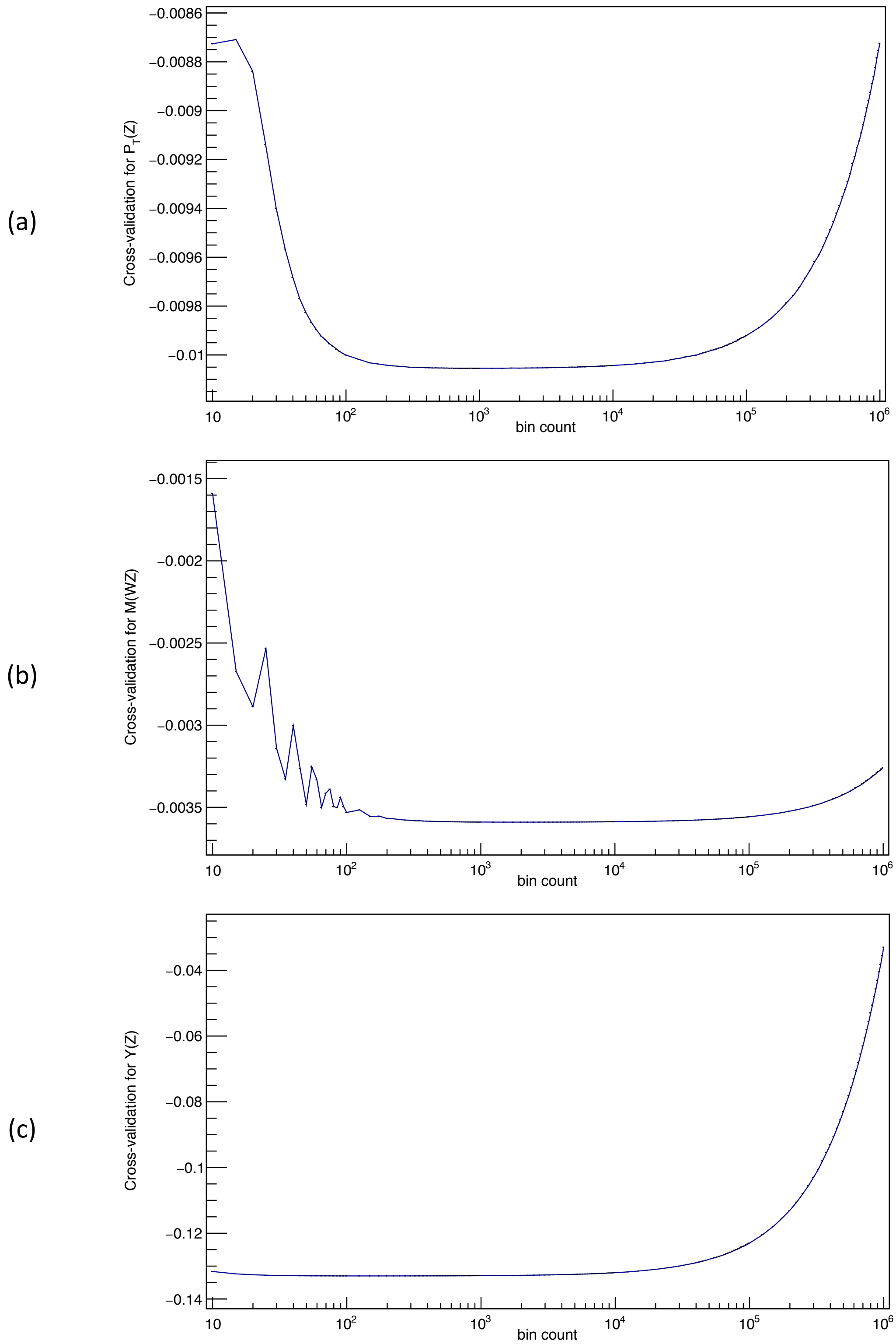


Figure B.1: Cross-validation minimization for (a) $P_T(Z)$, (b) M(WZ) and (c) y(Z), using SM parton-level WZ-production (luminosity 10 fb^{-1}).

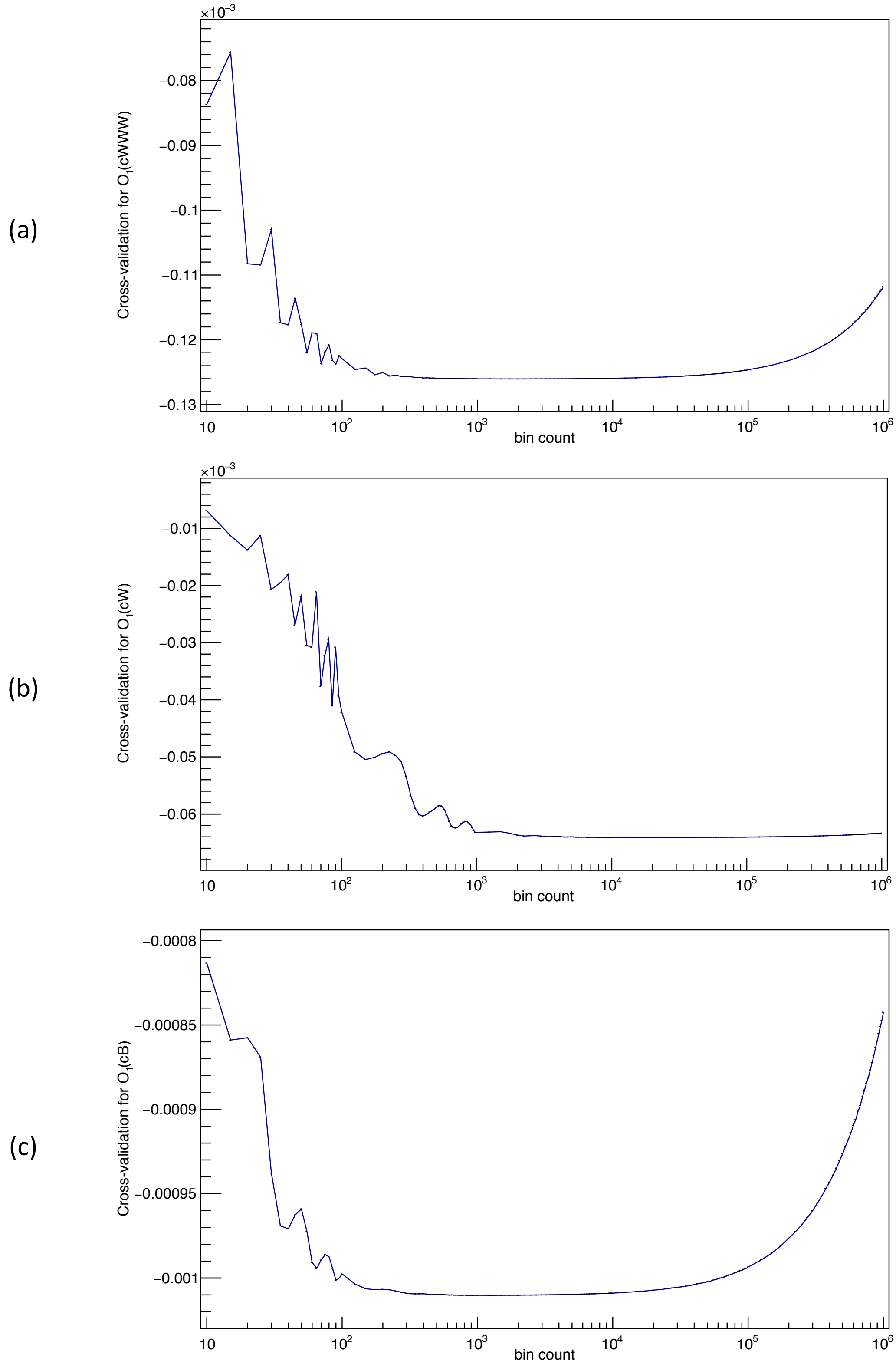


Figure B.2: Cross-validation minimization for first-order optimal observables, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, using SM parton-level WZ-production (luminosity 10 fb^{-1}).

(a)

(b)

(c)

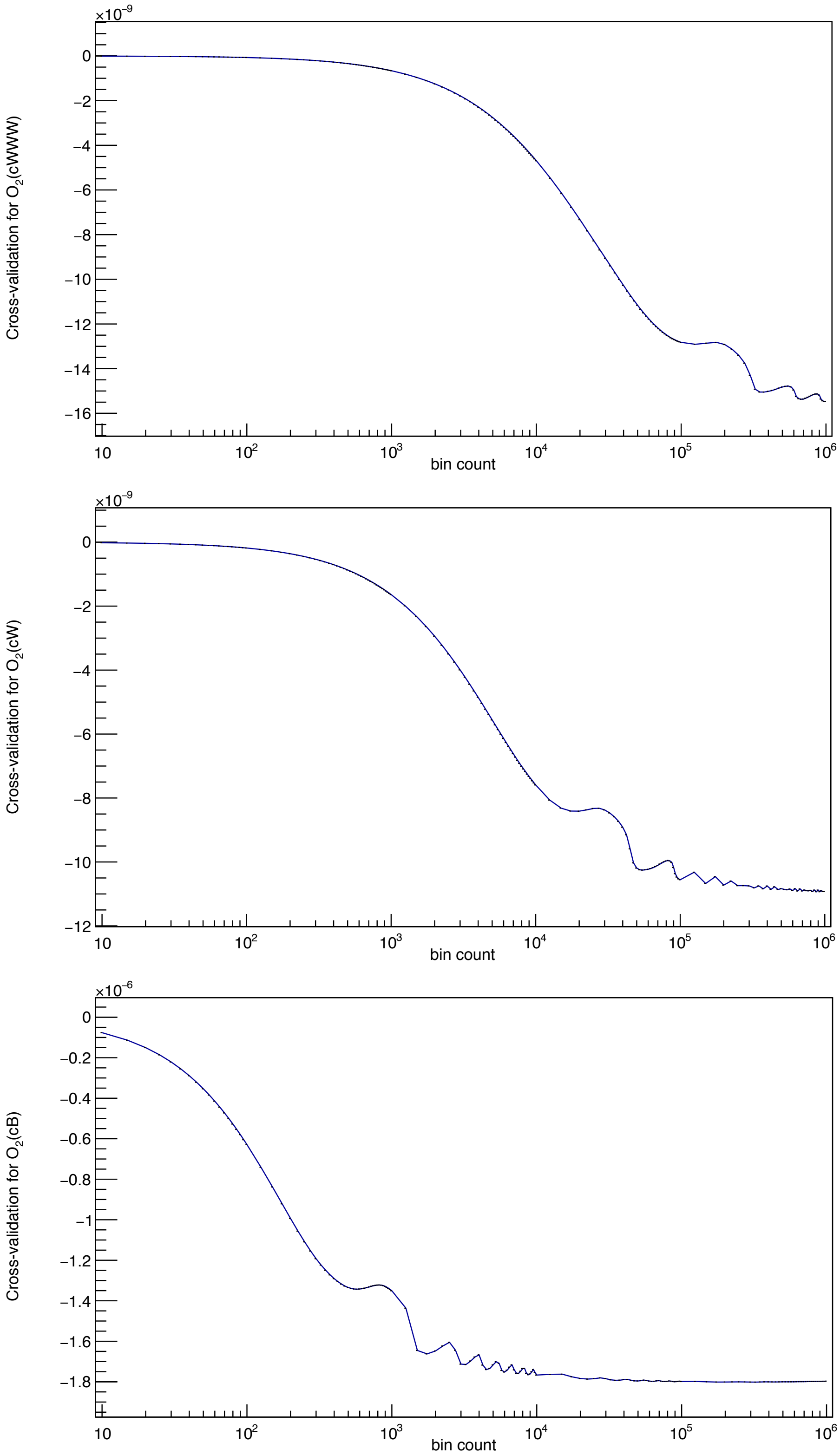


Figure B.3: Cross-validation minimization for second-order optimal observables, for EFT model parameters (a) c_{WWW}, (b) c_W and (c) c_B, using SM parton-level WZ-production (luminosity 10 fb^{-1}).

C. Figures of Fit Objective Minimization

This appendix contains figures showing the fit objective minimization curves for a selection of the fit results given in section 7.

C.1 Binned fits

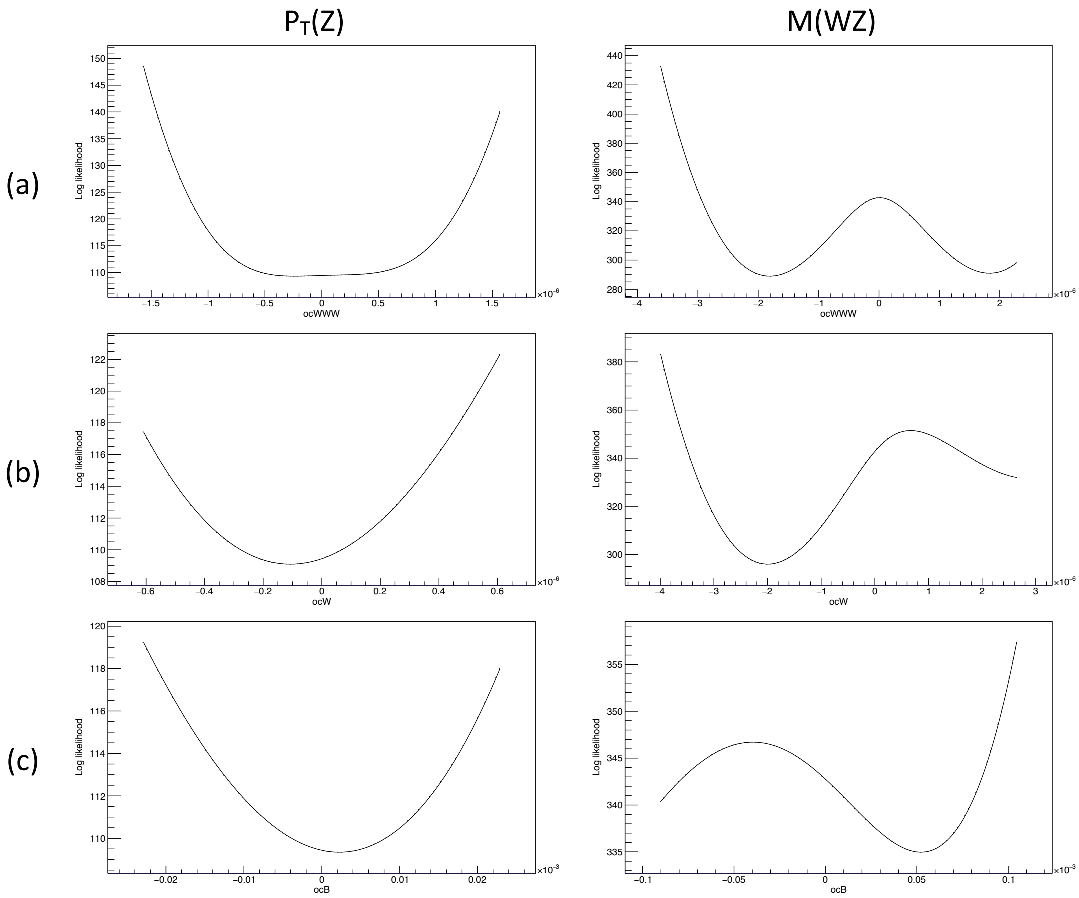


*Figure C.1: Log-likelihood minimization curves for **binned** fit of EFT to SM, for parton-level WZ-production, for observables P_T(Z) and M(WZ) and parameters (a) c_{WWW}, (b) c_W and (c) c_B.*

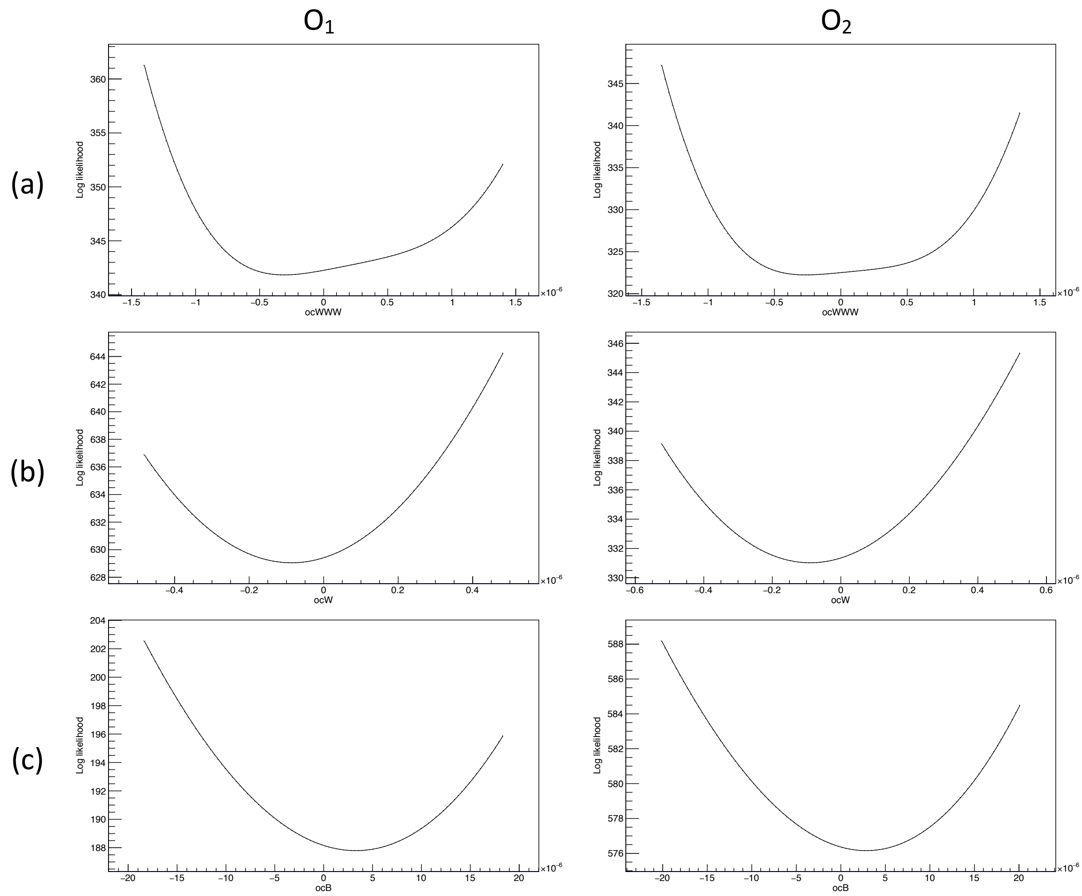


*Figure C.2: Log-likelihood minimization curves for **binned** fit of EFT to SM, for parton-level WZ-production, for first- and second-order optimal observables O_1 and O_2 for parameters (a)* c_{WWW}*, (b)* c_W *and (c)* c_B*.*

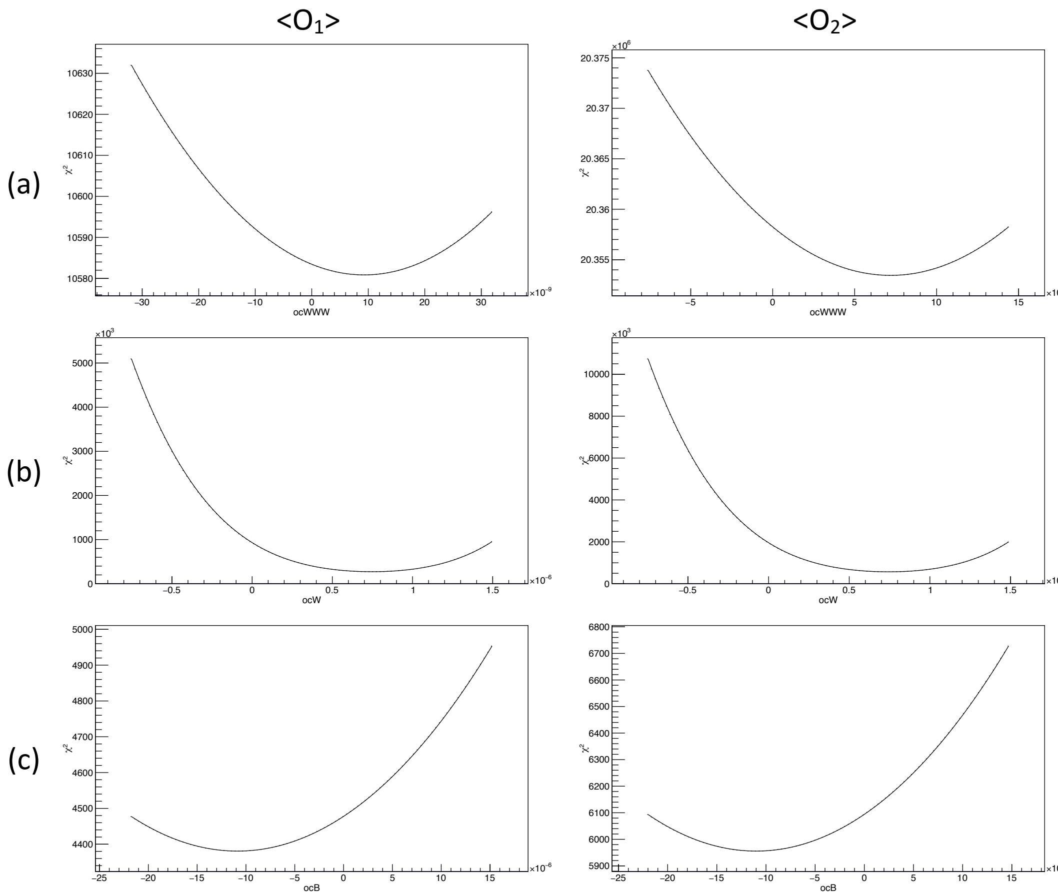


*Figure C.3: χ^2 minimization curves for **binned** fit of EFT to SM, for parton-level WZ-production, for mean first- and second-order optimal observables $\langle O_1 \rangle$ and $\langle O_2 \rangle$ for parameters (a) c_{WWW}, (b) c_W and (c) c_B.*

<O₁> <O₂>

(a) (b) (c)

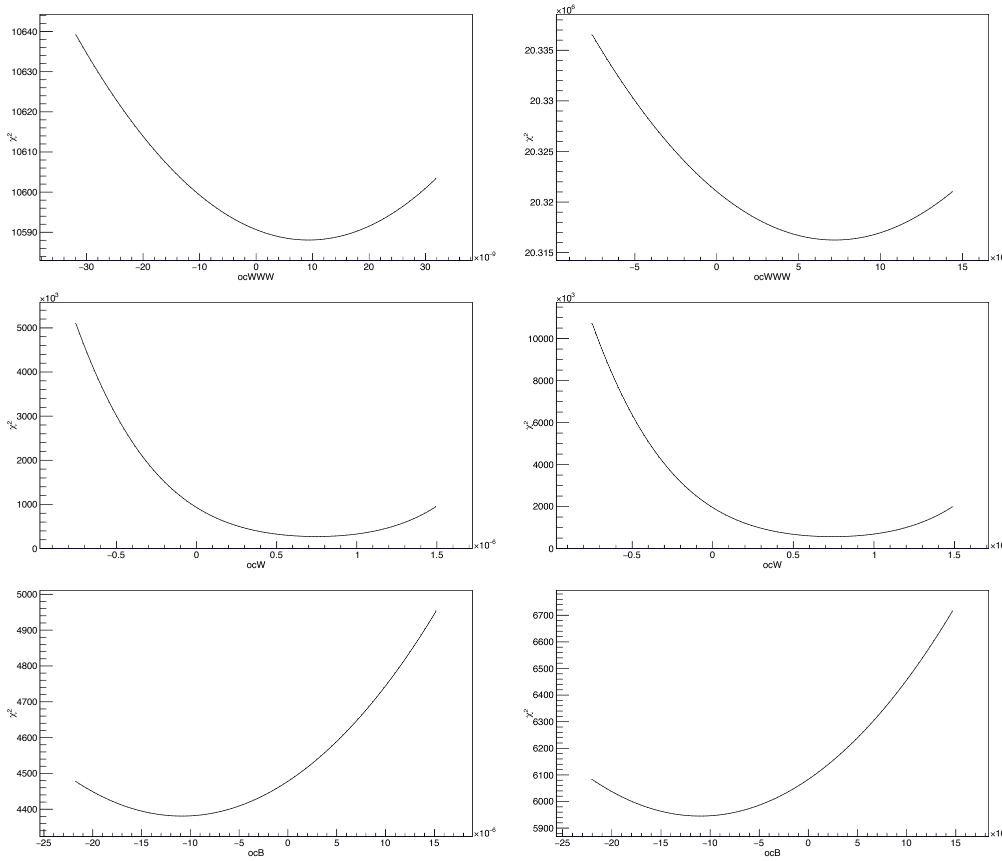


*Figure C.4: χ^2 minimization curves for **binned** fit of EFT to SM, for parton-level WZ-production, for mean first- and second-order optimal observables divided by powers of invariant mass <O_1/s> and <O_2/s^2> for parameters (a) $\mathrm{c_{WWW}}$, (b) $\mathrm{c_W}$ and (c) $\mathrm{c_B}$.*

C.2 Unbinned fits

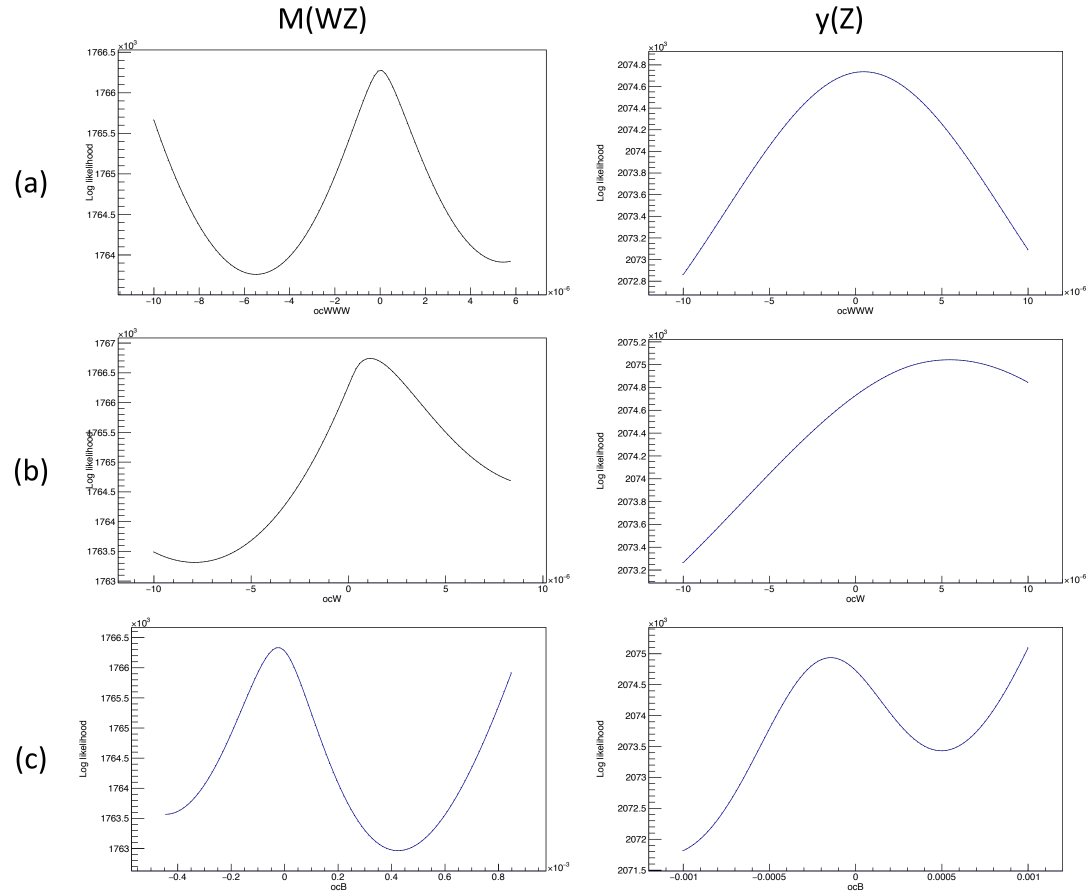


Figure C.5: Log-likelihood minimization curves for ***unbinned*** *fit of EFT to SM, for parton-level WZ-production, for observables M(WZ) and y(Z) and parameters (a)* c_{WWW}*, (b)* c_W *and (c)* c_B*.*

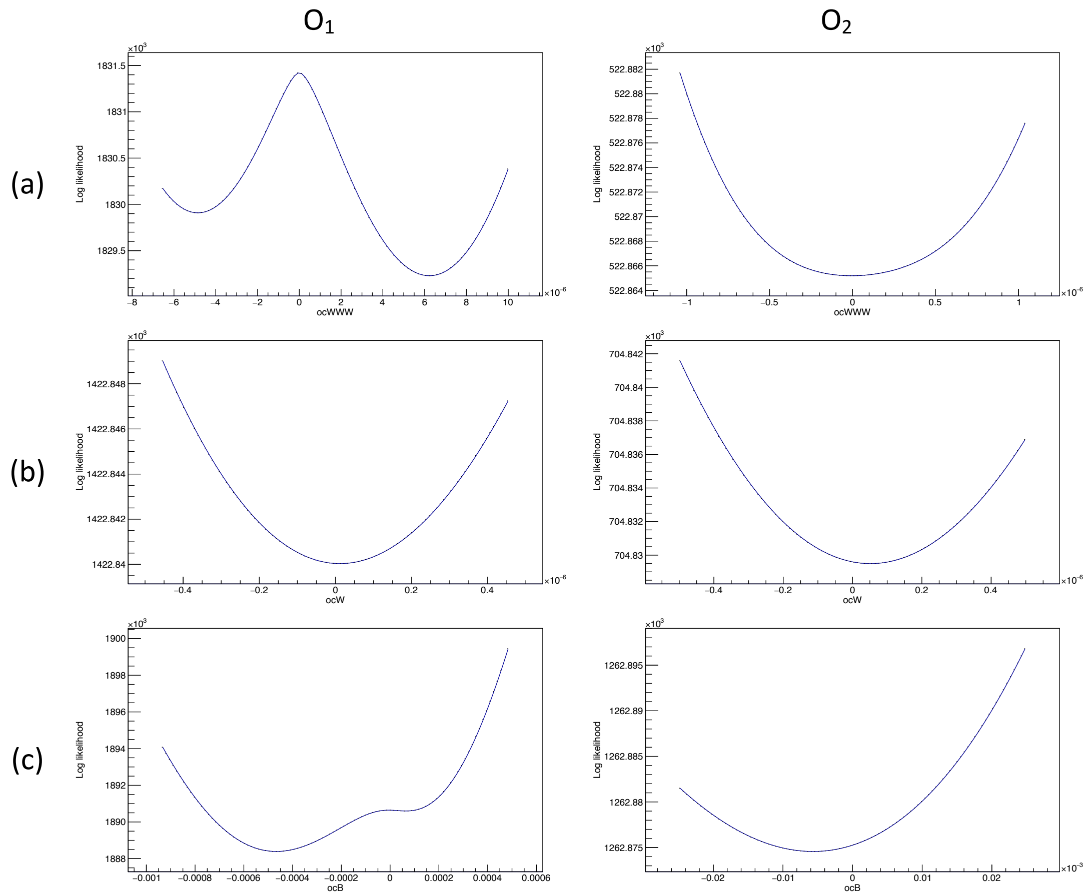


*Figure C.6: Log-likelihood minimization curves for **unbinned** fit of EFT to SM, for parton-level WZ-production, for first- and second-order optimal observables O_1 and O_2 for parameters (a) c_{WWW}, (b) c_W and (c) c_B.*

D. Analysis of Fit Systematic Errors from Binning

Although a large effort has been made to find the optimal number of bins, the choice of binning still affects the fit results. To estimate how much the fit results vary with different bin counts, all the fits were performed again with half and double the number of bins used for the results in 7.1 and 7.2. Here we focused only on the fits for a single parameter. Then using the asymmetric 95% confidence-limits, we used the largest upper and lower bounds of all three fit-results to define a total 95% confidence region. Assuming this total to be a squared sum of the 95% confidence value for the chosen nominal bin counts and a 95% confidence value for systematic errors, we could then calculate the systematic error estimates.

The results are shown in the tables below.

For **binned** fits, the systematic error confidence limits are small for non-mean observables, showing a good fit stability. However, for mean optimal observables, the result of changing bin sizes is more prominent, and generally larger than the given fit error, especially for parameter c_B, where the systematic errors due to binning were very large compared to the fit error. Overall, though, the mean optimal observables continue to have the smallest total confidence limit (though for c_B it is a marginal difference).

For **unbinned** fits, the systematic errors due to binning are generally a bit larger that their given fit errors, for c_{WWW} and c_W, but very much larger for cB. Even though the data is not binned, the fit-dependency on the binning of the model appears to be large.

Further, **unbinned** fits to y(Z) often resulted in a fit-value that was at the preset limits for the fit parameter (±10 or ±1000). The minimization curves shown in figure C.5 show that no minimum exists near zero for unbinned y(Z). In our case, y(Z) is thus not a good observable to do unbinned fits with.

Finally, even with estimated systematic errors due to binning, the “true value” of zero for our parameters is not within the 95% total confidence limits for several fit results. This indicates that there are yet other systematic errors within the system.

For **unbinned** fits, one possible source of this systematic error is that unbinned fits only compare the relative shape of the model, normalizing it to a constant with every change in model parameters. The binned fits, on the other hand, compare the absolute value of the model to the data. This means that binned fits have one more degree of information than unbinned fits. Unbinned fits could be improved by adding a Poisson term taking into consideration the difference in number of events between the model and data. However, this is for further investigation.

*Table D.1: Binning systematic errors for **binned** fits to* c_{WWW}.
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.

Fit only c_{WWW} with Observable:		**Fit Value [10^{-6} GeV^2]**	**Fit 95% CL**		**Systematic (Binning) 95% CL**		**Total 95% CL**	
			Lower	**Upper**	**Lower**	**Upper**	**Lower**	**Upper**
1	$P_T(Z)$	−0.24	−0.44	+0.94	0	0	−0.44	+0.94
2	M(WZ) = $\sqrt{s}$	−1.81	−0.23	+0.24	0	+4.2	−0.23	+4.2
3	y(Z)	−0.91	−1.1	+4.5	−0.45	+0.30	−1.2	+4.5
4	$O_1(c_{WWW})$	−0.31	−0.43	+0.89	−0.19	+0.36	−0.47	+0.96
5	$O_2(c_{WWW})$	−0.26	−0.40	+0.85	−0.16	0	−0.43	+0.85
6	$O_1(c_W)$	−0.30	−0.38	+0.91	−0.12	+0.36	−0.40	+0.98
7	$O_2(c_W)$	−0.27	−0.40	+0.83	0	+0.35	−0.40	+0.90
8	$O_1(c_B)$	−0.27	−0.44	+0.85	−0.17	+0.26	−0.47	+0.89
9	$O_2(c_B)$	−0.28	−0.40	+0.95	0	+0.31	−0.40	+1.0
10	$\langle O_1(c_{WWW})\rangle$ vs $\sqrt{s}$	0.0093	−0.011	+0.011	−0.014	+0.041	−0.018	+0.043
11	$\langle O_2(c_{WWW})\rangle$ vs $\sqrt{s}$	**0.00719**	−0.00020	+0.00020	−0.00089	+0.028	−0.00091	+0.028
12	$\langle O_1(c_W)\rangle$ vs $\sqrt{s}$	**0.00770**	−0.00099	+0.00099	−0.0023	+0.021	−0.0025	+0.021
13	$\langle O_2(c_W)\rangle$ vs $\sqrt{s}$	**0.00764**	−0.00068	+0.00068	−0.0019	+0.027	−0.0020	+0.027
14	$\langle O_1(c_B)\rangle$ vs $\sqrt{s}$	0.0097	−0.020	+0.020	−0.022	+0.060	−0.030	+0.063
15	$\langle O_2(c_B)\rangle$ vs $\sqrt{s}$	0.0096	−0.017	+0.017	−0.019	+0.045	−0.026	+0.048
16	$\langle O_1(c_{WWW})/s\rangle$ vs $\sqrt{s}$	0.0093	−0.011	+0.011	−0.014	+0.041	−0.018	+0.043
17	$\langle O_2(c_{WWW})/s^2\rangle$ vs $\sqrt{s}$	**0.00719**	−0.00020	+0.00020	−0.00090	+0.028	−0.00092	+0.028
18	$\langle O_1(c_W)/s\rangle$ vs $\sqrt{s}$	**0.00770**	−0.00099	+0.00099	−0.0023	+0.021	−0.0025	+0.021
19	$\langle O_2(c_W)/s^2\rangle$ vs $\sqrt{s}$	**0.00764**	−0.00068	+0.00068	−0.0019	+0.027	−0.0020	+0.027
20	$\langle O_1(c_B)/s\rangle$ vs $\sqrt{s}$	0.0097	−0.020	+0.020	−0.022	+0.060	−0.030	+0.063
21	$\langle O_2(c_B)/s^2\rangle$ vs $\sqrt{s}$	0.0096	−0.017	+0.017	−0.019	+0.045	−0.026	+0.048

*Table D.2: Binning systematic errors for **binned** fits to c_W.*
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.

Fit only c_W with Observable:		Fit Value [10^{-6} GeV^2]	Fit 95% CL		Systematic (Binning) 95% CL		Total 95% CL	
			Lower	Upper	Lower	Upper	Lower	Upper
1	$P_T(Z)$	−0.11	−0.25	+0.26	0	0	−0.25	+0.26
2	$M(WZ) = \sqrt{s}$	**−2.00**	−0.32	+0.33	−0.67	+0.71	−0.74	+0.78
3	$y(Z)$	−0.37	−0.70	+0.77	−0.24	0	−0.74	+0.77
4	$O_1(c_{WWW})$	−0.11	−0.21	+0.21	−0.094	+0.078	−0.23	+0.22
5	$O_2(c_{WWW})$	−0.095	−0.22	+0.22	−0.083	+0.056	−0.24	+0.23
6	$O_1(c_W)$	−0.086	−0.20	+0.20	0	0	−0.20	+0.20
7	$O_2(c_W)$	−0.089	−0.21	+0.22	0	+0.070	−0.21	+0.23
8	$O_1(c_B)$	−0.091	−0.21	+0.21	−0.062	+0.046	−0.22	+0.22
9	$O_2(c_B)$	−0.086	−0.21	+0.21	−0.046	+0.059	−0.22	+0.22
10	$\langle O_1(c_{WWW})\rangle$ vs $\sqrt{s}$	**0.798**	−0.022	+0.022	−0.11	+0.56	−0.11	+0.56
11	$\langle O_2(c_{WWW})\rangle$ vs $\sqrt{s}$	**0.72301**	−0.00044	+0.00044	−0.0043	+0.89	−0.0043	+0.89
12	$\langle O_1(c_W)\rangle$ vs $\sqrt{s}$	**0.7478**	−0.0021	+0.0021	−0.035	+0.56	−0.035	+0.56
13	$\langle O_2(c_W)\rangle$ vs $\sqrt{s}$	**0.7439**	−0.0015	+0.0015	−0.028	+0.82	−0.028	+0.82
14	$\langle O_1(c_B)\rangle$ vs $\sqrt{s}$	**0.791**	−0.037	+0.037	−0.13	+0.56	−0.14	+0.56
15	$\langle O_2(c_B)\rangle$ vs $\sqrt{s}$	**0.796**	−0.031	+0.031	−0.13	+0.73	−0.13	+0.74
16	$\langle O_1(c_{WWW})/s\rangle$ vs $\sqrt{s}$	**0.798**	−0.022	+0.022	−0.11	+0.56	−0.11	+0.56
17	$\langle O_2(c_{WWW})/s^2\rangle$ vs $\sqrt{s}$	**0.72300**	−0.00044	+0.00044	−0.0043	+0.89	−0.0043	+0.89
18	$\langle O_1(c_W)/s\rangle$ vs $\sqrt{s}$	**0.7478**	−0.0021	+0.0021	−0.035	+0.56	−0.035	+0.56
19	$\langle O_2(c_W)/s^2\rangle$ vs $\sqrt{s}$	**0.7439**	−0.0015	+0.0015	−0.028	+0.82	−0.028	+0.82
20	$\langle O_1(c_B)/s\rangle$ vs $\sqrt{s}$	**0.790**	−0.037	+0.037	−0.13	+0.56	−0.14	+0.56
21	$\langle O_2(c_B)/s^2\rangle$ vs $\sqrt{s}$	**0.796**	−0.031	+0.031	−0.13	+0.73	−0.13	+0.74

*Table D.3: Binning systematic errors for **binned** fits to $\boldsymbol{c_B}$.*
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.

Fit only c_B with Observable:		Fit Value $[10^{-6}\ \mathrm{GeV}^2]$	Fit 95% CL		Systematic (Binning) 95% CL		Total 95% CL	
			Lower	Upper	Lower	Upper	Lower	Upper
1	$P_T(Z)$	2.3	−11	+10	−1.5	0	−11	+10
2	$M(WZ) = \sqrt{s}$	**52**	−21	+17	−36	+35	−42	+39
3	$y(Z)$	10	−23	+20	0	+9.2	−23	+22
4	$O_1(c_{WWW})$	3.7	−7.6	+7.4	−1.2	+1.7	−7.7	+7.6
5	$O_2(c_{WWW})$	4.1	−9.5	+9.1	0	+6.7	−9.5	+11
6	$O_1(c_W)$	3.1	−7.8	+7.5	−2.2	+1.7	−8.1	+7.7
7	$O_2(c_W)$	3.9	−8.9	+8.5	−5.2	0	−10	+8.5
8	$O_1(c_B)$	3.3	−7.6	+7.4	0	+1.2	−7.6	+7.5
9	$O_2(c_B)$	2.9	−8.8	+8.5	−3.6	+1.3	−9.5	+8.6
10	$\langle O_1(c_{WWW})\rangle$ vs $\sqrt{s}$	**−11.2**	−1.3	+1.3	−9.2	+3.1	−9.3	+3.4
11	$\langle O_2(c_{WWW})\rangle$ vs $\sqrt{s}$	**−9.9604**	−0.025	+0.025	−12	0.15	−12	+0.15
12	$\langle O_1(c_W)\rangle$ vs $\sqrt{s}$	**−10.35**	−0.12	+0.12	−8.2	0.66	−8.2	+0.68
13	$\langle O_2(c_W)\rangle$ vs $\sqrt{s}$	**−10.286**	−0.083	+0.083	−11	0.54	−11	+0.55
14	$\langle O_1(c_B)\rangle$ vs $\sqrt{s}$	**−10.9**	−2.2	+2.2	−9.3	+4.5	−9.6	+5.0
15	$\langle O_2(c_B)\rangle$ vs $\sqrt{s}$	**−11.0**	−1.8	+1.8	−11	+4.1	−11	+4.5
16	$\langle O_1(c_{WWW})/s\rangle$ vs $\sqrt{s}$	**−11.2**	−1.3	+1.3	−9.2	+3.1	−9.3	+3.4
17	$\langle O_2(c_{WWW})/s^2\rangle$ vs $\sqrt{s}$	**−9.9603**	−0.025	+0.025	−12	0.15	−12	+0.15
18	$\langle O_1(c_W)/s\rangle$ vs $\sqrt{s}$	**−10.35**	−0.12	+0.12	−8.2	0.66	−8.2	+0.68
19	$\langle O_2(c_W)/s^2\rangle$ vs $\sqrt{s}$	**−10.286**	−0.083	+0.083	−11	0.54	−11	+0.55
20	$\langle O_1(c_B)/s\rangle$ vs $\sqrt{s}$	**−10.9**	−2.2	+2.2	−9.3	+4.5	−9.6	+5.0
21	$\langle O_2(c_B)/s^2\rangle$ vs $\sqrt{s}$	**−11.0**	−1.8	+1.8	−11	+4.1	−11	+4.5

*Table D.4: Binning systematic errors for **unbinned** fits to c_{WWW}.*
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.
If the fit-value is at the limit set for the fit (−10) it is colored red.

Fit only c_{WWW} with Observable:		Fit Value [10^{-6} GeV^2]	Fit 95% CL	Systematic (Binning) 95% CL		Total 95% CL	
				Lower	Upper	Lower	Upper
1	$P_T(Z)$	**−0.69**	±0.30	0	+0.47	−0.30	+0.56
2	M(WZ) = $\sqrt{s}$	−5.49	±0.14	−1.4	+10	−1.4	+10
3	y(Z)	−10	±0.0085	−0.044	+20	−0.045	+20
4	$O_1(c_{WWW})$	**6.23**	±0.16	−2.5	+2.4	−2.5	+2.4
5	$O_2(c_{WWW})$	−0.0057	±0.52	−0.46	+0.42	−0.69	+0.67
6	$O_1(c_W)$	**0.84**	±0.33	−0.68	+0.57	−0.76	+0.66
7	$O_2(c_W)$	−0.0026	±0.50	−0.60	+0.37	−0.78	+0.62
8	$O_1(c_B)$	6.05	±0.16	−2.5	+2.6	−2.5	+2.6
9	$O_2(c_B)$	0.33	±0.52	−0.15	+0.37	−0.54	+0.64

*Table D.5: Binning systematic errors for **unbinned** fits to c_W.*
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.
If the fit-value is at the limit set for the fit (−10) it is colored red.

Fit only c_W with Observable:		Fit Value [10^{-6} GeV^2]	Fit 95% CL	Systematic (Binning) 95% CL		Total 95% CL	
				Lower	Upper	Lower	Upper
1	$P_T(Z)$	−0.52	±0.22	−0.17	+0.40	−0.28	+0.46
2	M(WZ) = $\sqrt{s}$	**−7.91**	±0.22	−2.0	+1.7	−2.0	+1.7
3	y(Z)	**−10**	±0.013	−0.053	+0.053	−0.055	+0.055
4	$O_1(c_{WWW})$	10	±0.0097	−12	0	−12	+0.010
5	$O_2(c_{WWW})$	0.041	±0.22	−0.16	+0.24	−0.27	+0.33
6	$O_1(c_W)$	0.012	±0.22	−0.13	+0.49	−0.26	+0.54
7	$O_2(c_W)$	0.052	±0.22	−0.19	+0.31	−0.29	+0.38
8	$O_1(c_B)$	10	±0.011	−11	0	−11	+0.011
9	$O_2(c_B)$	0.13	±0.23	−0.26	+0.53	−0.35	+0.58

*Table D.6: Binning systematic errors for **unbinned** fits to c_B.*
If zero (the true value) is outside the total 95% CL for a fit-value, the fit-value is marked in bold.
If the fit-value is at the limit set for the fit (−1000) it is colored red.

Fit only c_B with Observable:		Fit Value [10^{-6} GeV^2]	Fit 95% CL	Systematic (Binning) 95% CL		Total 95% CL	
				Lower	Upper	Lower	Upper
1	$P_T(Z)$	14	±10	−16	+7.5	−19	+13
2	M(WZ) = $\sqrt{s}$	**424**	±9.5	−70	+75	−71	+76
3	y(Z)	−1000	±1.5	0	+1800	−1.5	+1800
4	$O_1(c_{WWW})$	−474.3	±8.9	−95	+490	−96	+490
5	$O_2(c_{WWW})$	0.21	±9.2	−4.6	+1.6	−10	+9.3
6	$O_1(c_W)$	−3.7	±8.6	−22	+8.0	−23	+12
7	$O_2(c_W)$	0.12	±9.2	−5.9	+4.0	−11	+10
8	$O_1(c_B)$	−466.4	±8.9	−100	+480	−100	+480
9	$O_2(c_B)$	−5.6	±9.6	−15	+9.1	−18	+13