

Math 320 Final Notes - Week 2

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1.5 - Cardinality

Definition 1 (one-to-one, onto). A function $f : A \rightarrow B$ is one-to-one (1-1) if $a_1 \neq a_2$ in A implies that $f(a_1) \neq f(a_2)$ in B . The function f is onto if, given any $b \in B$, it is possible to find an element $a \in A$ for which $f(a) = b$.

Definition 2 (bijection). The set A has the same cardinality as B if there exists $f : A \rightarrow B$ that is 1-1 and onto. In this case, we write $A \sim B$.

Definition 3 (countable). A set A is countable if $\mathbb{N} \sim A$. An infinite set that is not countable is called an uncountable set.

Theorem 1 (1.5.6). (i) The set \mathbb{Q} is countable.

(ii) The set \mathbb{R} is uncountable.

Theorem 2 (1.5.7). If $A \subseteq B$ and B is countable, then A is either countable or finite.

Theorem 3 (1.5.8). (i) If A_1, A_2, \dots, A_m are each countable sets, then the union $A_1 \cup A_2 \cup \dots \cup A_m$ is countable.

(ii) If A_n is a countable set for each $n \in \mathbb{N}$, then $\bigcup_{n=1}^{\infty} A_n$ is countable.