ω are known. Then the time of secondary eclipse could be predicted, but it will actually be observed a time Δt later due to the finite speed of light, where

$$\Delta t = \frac{2a}{c} \frac{m_\star^2 - m_p^2}{\left(m_\star + m_p\right)^2} \frac{1 - e^2}{1 - e^2 \sin^2 \omega} \tag{14}$$

This equation is derived by delaying the image of each body according to its line-of-sight distance, then solving for the time at which these images cross, i.e., the midtime of secondary eclipse as observed. This delay can be large for transiting planets, e.g., Δt ≈ 160 s for HD 80606b, a systematic effect that is comparable in magnitude to the measurement error of ~260 s for an individual secondary eclipse (*Laughlin et al.,* 2009). The reason this effect has an observable magnitude is because it originates within special relativity, and so it scales as 1/c. Effects of general relativity, such as those embodied in equation (2) or the effect of curvature on light propagation (*Shapiro,* 1964), scale as $1/c^2$, and are thus harder to detect.

A final step is needed to compare these theoretical event times to the observed data. The most important effect of a second body on transit times is to change the true period (as mentioned in section 2.2), so in practice the times are scaled to match the observed period and shifted to match the observed epoch of transit. Then variations on top of that simple linear ephemeris can be used, e.g., to detect or set limits on the presence of perturbing bodies (section 5.1). Care needs to be taken if radial velocities are being simultaneously fit, because such data specify a particular *velocity* of the star corresponding to a particular *time*, whereas transit data specify a particular *time* corresponding to a particular relative *position* of the planet to the star.

3. DYNAMICAL PHENOMENA

3.1. Astrophysical Two-Body Problem

3.1.1. Periastron advance. In section 2.1 we encountered several extra forces that modify realistic two-body motion from Keplerian ellipses. Let us take a perturbation theory approach, in which the force is calculated over a Keplerian orbit, to find how that orbit itself evolves (*Burns,* 1976). Now that the effective force of gravity is no longer of the form $1/r^2$, the ellipse does not close, and the periastron advances, which amounts to a reorientation of the orbit within its own plane. We shall calculate this precession rate due to the perturbing forces of relativity, tides, and rotational oblateness.

Periastron advance is the relativistic effect that changes orbits on the shortest timescale, and averaging equation (2) over an orbit, we determine its angular rate to be

$$\dot{\omega}_{GR} = \frac{3G^{3/2}\left(m_\star + m_p\right)^{3/2}}{a^{5/2} c^2 \left(1 - e^2\right)} \tag{15}$$

This effect causes an additional 43 arcsec per century of precession for Mercury (in addition to the precession caused by the other planets), which was the famous first hint that nature obeyed Einstein's equations. Higher-order corrections to this precession rate, precession due to the star's spin (the Lense-Thirring effect), orbital decay due to gravitational wave emission, and other relativistic effects are all negligible for exoplanets.

The force due to tidal distortion of the planet (equation (3)) causes apsidal motion at the rate

$$\dot{\omega}_T = \frac{15}{2} n k_L \frac{m_\star}{m_p} \frac{1 + (3/2)e^2 + (1/8)e^4}{\left(1 - e^2\right)^5} \left(\frac{R_p}{a}\right)^5 \tag{16}$$

where n ≡ 2π/P is the mean motion (*Sterne,* 1939). This effect is generally much bigger than that of the tide raised on the star by the planet, and for hot Jupiters with periods less than three days, it can dominate all other precessional effects (*Ragozzine and Wolf,* 2009). For physically smaller planets, and for Jupiter-type planets with periods ≥3 d, relativistic precession (equation (15)) typically dominates.

Rotational distortion gives rise to a force (equation (4)) that causes a periastron advance rate

$$\dot{\omega}_R = \frac{n k_{L,\star}}{2} \frac{1 + m_p/m_\star}{\left(1 - e^2\right)^2} \left(\frac{\Omega_\star}{n}\right)^2 \left(\frac{R_\star}{a}\right)^5 \tag{17}$$

Around fast-rotating and large stars (i.e., young or early type), this effect can dominate the others. If the stellar spin is misaligned with the orbit by an angle ψ, equation (17) requires an extra term ($5 \cos^2 \psi - 1)/4$; for spin-orbit angles satisfying 63.4° < ψ < 116.6°, the apsidal motion is retrograde. With spin-orbit misalignment, the nodal angle also precesses; equations for the coupled spin and orbital motion are given by *Eggleton and Kiseleva-Eggleton* (2001).

Of course, the star has a tidal bulge and the planet has a rotational bulge as well, but these never contribute substantially to the total precession.

3.1.2. Tidal dissipation. Tidal energy is converted to heat when a tidal bulge rotates through a body or varies in amplitude, due to the material's resistance to shearing motion. First, the dissipative torque changes the rotation of the planet to a rate at which the time average of that torque vanishes. At this spin rate the time average of the shear, and the energy dissipation rate, is minimized. In a fixed, circular orbit, the spin angular velocity equals the orbital angular velocity and the obliquity is zero, so in the frame corotating with the perturber, the tide is no longer time variable, stopping energy loss. In an eccentric orbit, the spin will either settle at a pseudosynchronous state (*Peale and Gold,* 1965; *Hut,* 1981; *Levrard et al.,* 2007), or be trapped in a spin-orbit resonance (of which Mercury is the prototype); the latter is only possible if the body has a permanent quadrupole moment due to its rigidity, and is therefore not expected for gas giants. For rocky planets

with dynamically important atmospheres (of which Venus is the prototype), the picture can be qualitatively different, including up to four stable rotation states (*Correia et al.,* 2008).

On a longer timescale, the eccentricity damps. The correlation between eccentricity and orbital distance (or period) is the main constraint on tidal theory for exoplanets (see chapter by Cumming). This damping can in principle be due to either dissipation in the planet or the star. If dissipation in the star is important, eventually the planetary orbit will decay into the star (e.g., *Rasio et al.,* 1996; *Jackson et al.,* 2009; *Barker and Ogilvie,* 2009). If dissipation in the planet is important, then it may have ingested more tidal energy than its own binding energy. In that case, gas giants could inflate or even disrupt (*Gu et al.,* 2003), and such heating on terrestrial planets would have significant geophysical consequences (*Wisdom,* 2008).

The physical causes of tidal damping for giant planets are still poorly known; as yet no first-principle theory is efficient enough to damp the eccentricity of hot Jupiters, or to generate the inferred histories of satellite systems around the four solar system giants. *Lin et al.* (2000) and *Ogilvie and Lin* (2004) discuss these matters and review the literature. In the absence of such a theory, a phenomenological approach has gained currency (*Goldreich and Soter,* 1966). A fraction 1/Q of the tidal energy is dissipated per tidal forcing cycle (or per orbit, depending on the author). This allows differential equations for tidal damping to be derived, in which damping times scale with Q (*Mardling and Lin,* 2002; *Matsumura et al.* 2008). Empirical constraints on Q for close-in gas giants have been worked out (*Wu,* 2003; *Jackson et al.,* 2008; *Matsumura et al.,* 2008).

3.1.3. Miscellaneous orbital evolution. There are numerous other effects that can modify a planet's orbit about its star. Here we simply list some of these effects, referring the reader to work that describes them in detail.

Close in to the star, the planet may be tidally stripped of mass. As the mass leaves the planet, it applies a torque on its orbit. The reaction of the orbit has been calculated for circular orbits as the planet finishes migration due to a gas disk (*Trilling et al.,* 1998), for moderate eccentricities as the planet tidally circularizes and perhaps inflates (*Gu et al.,* 2003), and for eccentricities near 1 when the planet is shot near the star by either a dynamical instability or a chance flyby (*Faber et al.,* 2005).

Once close to the star, the planet's atmosphere absorbs and reradiates photons in preferential directions, which can lead to at most a 5% change in semimajor axis — enough to influence resonant configurations with more distant planets (*Fabrycky,* 2008).

A planet or planets may scatter and eject a sea of small bodies (planetesimals) after the main formation phase, which leads to planetary migration. This effect was first worked out for the giant planets of the solar system (*Fernandez and Ip,* 1984; *Malhotra,* 1993a, 1995), and has since been applied to exoplanets (*Murray et al.,* 1998; *Morbidelli et al.,* 2007; *Thommes et al.,* 2008). For more on migration, particularly in a gas disk, see the chapter by Lubow and Ida.

Finally, far from the host star, passing stars may perturb planetary orbits (see, e.g., *Spurzem et al.,* 2009).

3.2. Short-Period, Secular, and Resonant Interactions

The interaction terms in the equations of motion (equation (5)) lead to all the interesting behavior of N-planet systems, and here we show heuristically how short-period, secular, and resonant behaviors arise. [For a traditional expansion in terms of orbital elements, using the so-called disturbing function, see *Murray and Dermott* (1999).] Figure 2 shows how the Jacobian orbital elements of two planets, initially on circular orbits, evolve as the planets move from opposition (being on opposite sides of the star), through conjunction (lined up with the star on the same side), then back to opposition. As the planets approach each other, the mutual gravity moves them onto slightly different orbits. At conjunction, the inner planet has been torqued forward, to a more distant, slower orbit; the outer planet has been torqued backward, to a closer-in, faster orbit. As the planets move through and recede from conjunction, these orbit changes are mostly reversed.

Now let us introduce moderate eccentricity to the orbits, and follow the system for many conjunctions. Due to the eccentricity, the paths of the planets at various conjunctions are either converging or diverging, and the changes in orbital elements on either side of conjunction do not cancel as completely. At a single conjunction, this causes the orbits to transfer energy (the semimajor axes change) and angular momentum (the eccentricities and orbit orientations change). After multiple conjunctions, the behavior of the system depends on whether the periods are near a ratio of small integers.

First, consider Fig. 3, which shows a hypothetical system with a period ratio far from any ratio of small integers. Because of this property, the conjunctions sample all parts of both orbits rather equally. The semimajor axes exhibit no long-term changes, which means the energy of each orbit is conserved. However, the angular momentum of each orbit is exchanged on long timescales, resulting in eccentricity variations. One way of seeing why this happens is by considering the time average of a planet over its orbit, so that its gravitational effect is that of an elliptical wire weighted inversely to the Keplerian velocity at each position. Each of the planets respond to the other planets as if they were such rings. Because the potential from such a ring is not time dependent, it produces a conservative force, and no energy can be exchanged: Semimajor axes and periods may not change. However, the lopsided rings do torque each other, and this corresponds to angular momentum (and thus eccentricity and inclination) changes. For instance, two eccentric, coplanar planets will undergo periodic oscillations in eccentricity that are 180° out of phase from each other. For more on this topic, called secular evolution, see section 3.3 and section 4.3.

Next, consider Fig. 4, which is a hypothetical system with periods very close to a ratio of small integers (2.01:1). In this situation, called a mean-motion resonance, conjunc-

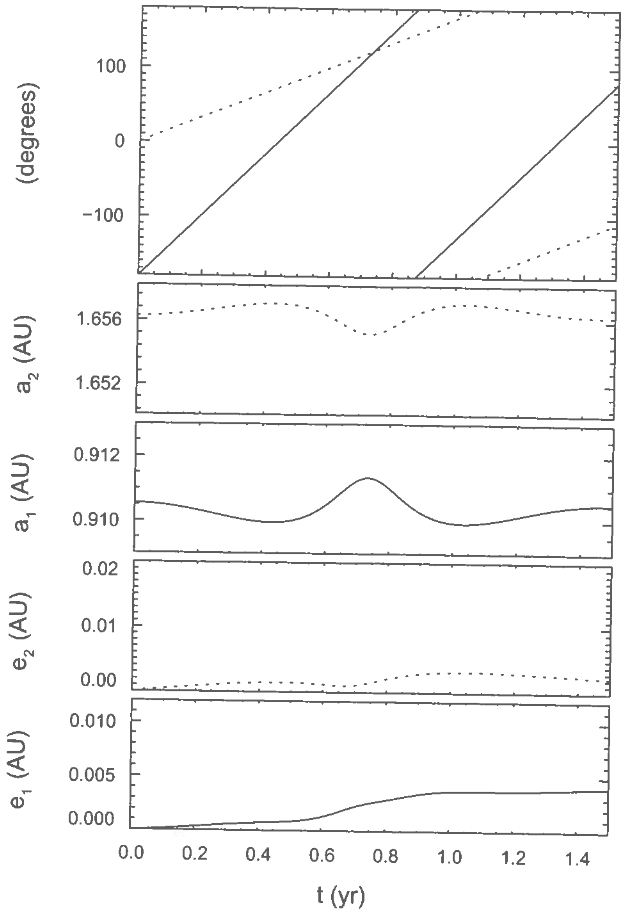

Fig. 2. Orbital element changes on the timescale of conjunctions, in a hypothetical system. The stellar mass is $m_\odot$, both planets have mass 10^{-3} $m_\odot$, their orbits are coplanar and initially circular, and they start on opposite sides of the star. Here and elsewhere, planets are numbered by increasing semimajor axis. *Top panel*: The mean longitudes of each planet (the two lines cross at conjunction); *second and third panels*: the planets' semimajor axes, which vary symmetrically about the conjunction; *fourth and fifth panels*: the planetary eccentricities, which receive a small kick.

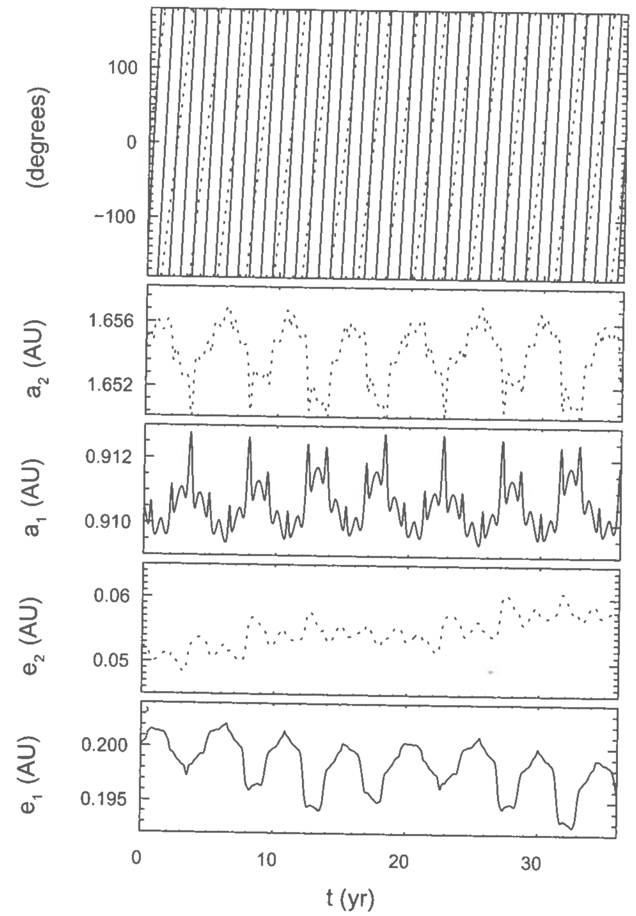

Fig. 3. Orbital element changes after many conjunctions. The hypothetical system is the same as in Fig. 2, except both planets start with eccentricity, with the inner planet's argument of pericenter 45° ahead of the outer planet's. The vertical axis on each panel has the same scaling as the corresponding panel in Fig. 2, emphasizing that the semimajor axes experience no net drift, but the eccentricities do.

tions occur at the same part of the orbit many times in a row, and the change in orbital elements builds. One may consider a changing period the hallmark of a mean-motion resonance. Along with period changes come eccentricity oscillations, which can be rather large over only tens of orbits. For noncoplanar planets, the distance between the location of conjunctions and the intersection of the orbital planes affects their dynamics, so resonances can also involve inclinations and not only eccentricities. In general, the angles that dictate the behavior of the resonance are called critical angles, and they have the form

$$\phi = j_1\lambda_1 + j_2\lambda_2 + j_3\varpi_1 + j_4\varpi_2 + j_5\Omega_1 + j_6\Omega_2 \tag{18}$$

where each planet has a mean motion of λ, a longitude of ascending node Ω, and a longitude of periastron ϖ. The j values are integers obeying $\Sigma j_i = 0$ (called the d'Alembert relation), which is required by the invariance of the system's behavior to the arbitrary reference direction from which angles are measured. At the very center of each resonance, where the planets come to conjunction at exactly the same point in their orbits, the periods are constant and precession rate is constant. In this case, there exists a slowly rotating frame in which the motion of each planet is perfectly periodic, yet not perfectly elliptical.

Individual resonances can help keep a system stable. For instance, when the critical argument for the interior 2:1 resonance has zero libration amplitude ($\theta = 2\lambda_2-\lambda_1-\varpi_1 = 0$, $\dot\theta =$

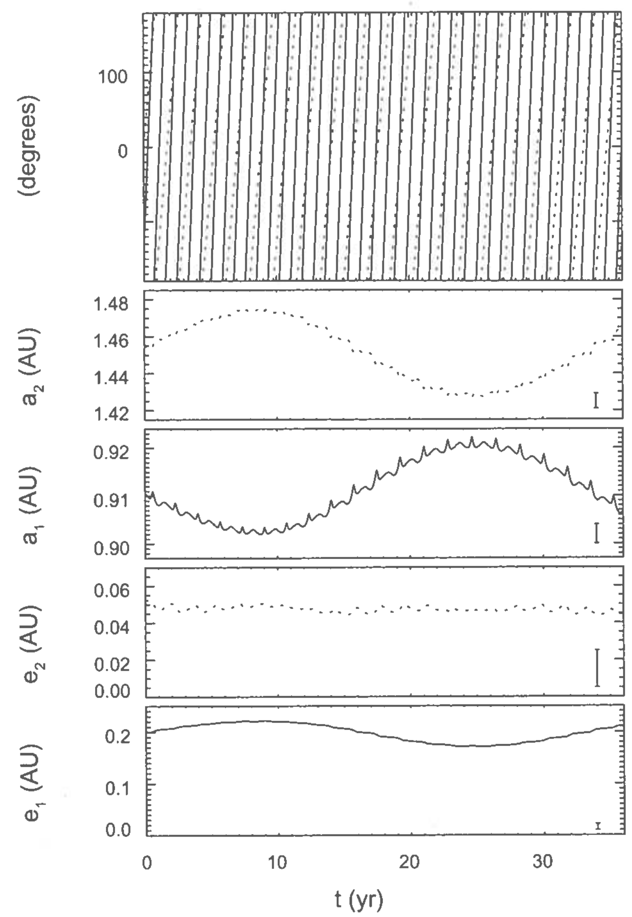

Fig. 4. Orbital element changes induced by a mean-motion resonance. The system is the same as in Fig. 3, except the outer planet starts at a different period, with a ratio of osculating periods of 2.01:1. Note that successive conjunctions (where the lines intersect in the top panel) occur at nearly the same longitude, which causes the period change to grow. The bar shows the vertical scale of each corresponding panel in Figs. 2 and 3, emphasizing that the semimajor axes and eccentricities are experiencing a large oscillation.

0), we may rearrange the equation to read $\lambda_1-\lambda_2 = \lambda_2-\varpi_1$, which shows that when the two planets are at conjunction ($\lambda_1-\lambda_2 = 0$), then the inner body is also at pericenter ($\lambda_1-\varpi_1 = 0$), so a close approach is avoided. Conversely, whenever the outer body is at the azimuthal location of the inner body's apocenter ($\lambda_2-\varpi_1 = \pi$), the two bodies are farthest apart ($\lambda_2-\lambda_1 = \pi$). This argument, applied generally to mean-motion resonances, is called a resonance protection mechanism for otherwise unstable systems.

The role and behavior of resonances during planetary migration is beyond the scope of this chapter. However, the reader is referred to *Peale* (1976) and *Malhotra* (1998) for Hamiltonian descriptions of resonances, which can cleanly treat migration. For a recent applications to exoplanets, see *Lee and Peale* (2002) for how two planets can capture into a resonance if their migration converges and *Chiang* (2003) for how two planets can excite each other's eccentricities as they pass through a resonance while their orbits diverge.

3.3. Advanced Interactions

Having surveyed the basic interactions between two planets in section 3.2, we now introduce several more advanced topics.

We previously saw that the eccentricities of planets outside mean-motion resonance can change on a long timescale. We now extend that concept to systems with three or more planets, systems in which the orbital elements of each planet vary on many different timescales. Resonances between these timescales can excite eccentricities to very high values [see *Moro-Martín et al.* (2007) for an example in an exoplanetary system]. For the three-planet system Upsilon Andromeda (υ And) in the Newtonian approximation, the innermost planet's eccentricity periodically reaches ~0.4, compared to ~0.06 in the absence of the outer planet or ~0.025 in the absence of the middle planet (*Barnes,* 2008). Also important in determining the qualitative behavior of the secular dynamics is extra precession, e.g., that supplied by relativistic or tidal effects (section 2.1); see *Wu and Goldreich* (2002) and *Migaszewski and Goździewski* (2009b). In Fig. 5 we plot the long-term behavior of the eccentricities in the υ And system considering (1) only Newtonian point masses, and (2) an extra force modeling the tidal bulge raised on the inner planet. The behavior of the outer planets is not much different, but the effect of the extra precession on the inner planet is to detune its pericenter precession rate away from an eccentricity-exciting secular resonance. Thus, the low current value of e_b argues an additional precession is active (*Adams and Laughlin,* 2006); both relativity and tides probably contribute with roughly equal precession rates of $\dot\omega \approx 10^{-11}$ s^{-1} each.

An extension of secular evolution theory can be made into the regime of high inclination and eccentricity, in which they are strongly coupled (*Kozai,* 1962; *Lidov,* 1962). In a system with a planet on an initially circular orbit, a third body on a distant exterior orbit will periodically pump the planetary eccentricity to a maximum of

$$e_{max} \approx \sqrt{1-(5/3)\cos^2 i} \tag{19}$$

where i is the mutual inclination, which must initially be in the range 39.2°–140.8°. [For a detailed description and a derivation of this behavior, called Kozai oscillations, see *Fabrycky and Tremaine* (2007).] Note that an initially perpendicular orbit (i = 90°) leads to an eccentricity of unity. In some systems, relativistic precession would suppress this behavior, but in others, tidal dissipation would take hold at the eccentricity maximum and circularize the planet at a period of a few days (*Fabrycky and Tremaine,* 2007; *Wu et al.,* 2007; *Nagasawa et al.,* 2008). An important