Operational transparency: Showing we are different

Existing studies on operational transparency have stressed the many benefits of adopting transparent processes. But the benefits of transparency described in these studies largely apply equally to all competing firms in a given market. And yet, operational transparency is far from universal. In a food court of present-day malls, one will find open kitchens next door to closed ones. Our point of departure from the existing literature is to explore the impact of competition on transparency choice. Reasons why a firm might not go transparent primarily focus on the situation where "opening up" reveals something unsavory about the product or service. We show that even when both firms have "nothing to hide", they still might not go transparent. The reason? "Opening up" can diminish variance in perceived differences in offerings and intensify price competition, leading to lower profits. Conversely, this reveals a previously unexplored reason for going transparent. If operational transparency differentiates a firm's offering from competitors by "showing we are different", this avoids price competition and increases profits. Our insights derive from analyzing a two-player and three-period game-theoretic model of operational transparency where the transparency and pricing decisions of firms are endogenous. The model considers two impacts of operational transparency: (i) a mean-shifting effect that boosts customer valuations (as typically discussed in existing literature) and (ii) a heterogeneity-reducing effect that reduces the variability of customer perceptions of the quality of operational practices. With these two effects, we show how an equilibrium can arise among two nearly identical firms where one goes transparent and the other does not. This outcome realizes the food court phenomenon of an open kitchen next to a closed one arising from competitive concerns.

Key words: operational transparency, competition

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1. Introduction

Subway and Potbelly are two successful sandwich chain restaurants with one striking operational difference. Subway "sandwich artists" make your sandwich right before your eyes, while at Potbelly, you wait behind a tall counter, obscured from directly witnessing the sandwich-making process. What explains this difference?

Operations management researchers have recently been quite interested in the benefits of operational transparency. Buell and Norton (2011) show that customers value being aware of others' hard work. Buell et al. (2017) describe a connection between workers and customers that operational transparency reveals, improving the efficiency and appreciation of both parties. Buell et al. (2021) explores how operational transparency engenders trust, which attracts customers to engage with a service. But wouldn't these positive effects apply equally to Subway and Potbelly? Why the difference in transparency strategies?

Buell (2019) provides an insightful list of reasons why a firm might *not* pursue operational transparency, despite its apparent benefits:

- "it reveals things people don't want to see,"
- "it engenders anxiety,"
- "it shatters faith in the relationship,"
- "it destroys the magic,"
- "it exposes an ineffective process,"
- "it reveals a company's best efforts yield poor results,"
- "it shows the company's products are inferior to competitors,"
- "it highlights a lack of progress,"
- "it reveals the company's harm workers or the environment," and
- "it's deceptive."

It is, again, hard to see why these reasons would be more applicable to Potbelly than Subway. In our review of online customer feedback and comparisons between Potbelly and Subway, we saw fans on either side.[1] If anything, we see more comments about Potbelly's higher quality and happier employees than vice versa. Is there something more "magical" about a Potbelly sandwich than a Subway one that revealing the process would destroy? Not that we can tell.

With the existing literature not quenching our thirst for a satisfying explanation of the Subway-Potbelly difference, we went in search of other factors that may impact operational transparency decisions. An obvious one is that the cost of implementing a transparent design may outweigh its benefit. But, again, in the case of two sandwich shop chains, it is hard to see cost as a significant differentiator.

We propose a new explanation rooted in the nature of the competitive landscape. Subway and Potbelly might simply choose different operational transparency strategies to differentiate themselves from each other: to "show that we are different". By differentiating, they avoid more direct price competition and maintain higher profits. If Potbelly revealed its sandwich-making process, there be some gains (like those discussed in the literature), but there is now one less dimension that distinguishes the two chains. We formalize this reasoning with a game-theoretic model, described below.

We do not want to suggest that this competitive consideration is the only explanation of the Subway-Potbelly operational difference (it may be as simple as the founder of Potbelly doesn't

[1] There is a large array of websites that compare different subway sandwich shops in the United States. Here are a few examples that compare Subway and Potbelly directly: https://www.businessinsider.com/comparison-of-sandwiches-from-potbelly-and-subway-2016-9#i-ordered-my-usual-a-wreck-without-the-roast-beef-on-white-bread-thats-turkey-ham-salami-and-swiss-cheese-3, https://www.insider.com/taste-test-same-meal-subway-potbelly-sandwich-shop-2021-11, https://www.mashed.com/1088743/subway-vs-potbelly-which-is-better/.

like others to see how his sandwiches are made), but it does raise relevant questions about how competition impacts the operational transparency decision of firms. To our knowledge, adding the possibility of a competitive response has not been discussed in the operational transparency literature. We raise, and attempt to answer, two research questions in this regard:

(Q1) How does the nature of the competitive environment a firm impact its operational transparency decision?

(Q2) Why do we see a mix of strategies (transparent vs. nontransparent) among different competing firms?

By a "competitive environment" we mean that there is more than one firm selling differentiated products (or services) in roughly the same category to a common pool of consumers. For example, Subway and Potbelly both sell submarine sandwiches in a "fast food" type setting.

The most natural tool to explore our research questions is game theory. For simplicity, we studied a model with two competing firms, each selling a single product. Each firm has two decisions to make: their degree of operational transparency and price. The customer pool is broken down into two subsets, where each subset has a preference for the product of one of the firms but can be persuaded to choose the other if the offer is right. The degree of *brand preference heterogeneity*—that is, how strong is the preference of each customer segment for their preferred product—is one of the parameters of our model.

Next, we model the impact of operational transparency in two ways. First, by "going transparent", a firm can shift customer expectations of its product's value. We call this the *mean-shifting effect*. This effect is meant to capture the benefits of operational transparency typically discussed in the literature. For example, the mean-shifting effect could represent the potential for increased customer perception of value when observing the care taken by a worker when placing toppings on a sandwich at Subway.

Second, "going transparent" has an impact on the variability in how customers perceive the value added to a product from its production process. Consider sandwich-making at Potbelly. Because the process is obscured from the view of customers, some customers may believe extreme care is taken when making sandwiches, while others may believe the sandwich is assembled in an unsanitary workspace. By not being able to observe the process, imaginations have room to run wild. By contrast, there is far less diversity of opinions about the care by which Subway sandwiches are made: seeing is believing! Operational transparency does not eliminate differences in perception, but it certainly reduces variance in perception. The greater the operational transparency, the smaller this variance. We call this the *heterogeneity-reducing effect* of operational transparency. For an illustration of the two effects of operational transparency, see Figure 1 below.

Our model tries to keep differences between the two firms to a minimum to isolate attention on the impact of competitive considerations. Thus, we assume that both firms have an equal-sized following of preferred customers, with each group having an equally strong preference for their brand. Both firms are assumed to have an equal amount to gain from the mean-shifting effect of going transparent. Finally, we assume that customer perceptions of firm operations are identically distributed in the two different customer populations, and operational transparency reduces the variability of this distribution in an identical fashion across the two firms.

Our answers to (Q1) and (Q2) are phrased in light of the three factors described in the earlier three paragraphs—brand-preference heterogeneity, the mean-shifting effect of operational transparency, and the heterogeneity-reducing effect of operational transparency. We analyze a three-period game, with two decision epochs for the two firms—choosing their level of operational transparency first, followed by pricing—and then customers selecting the firm that maximizes their utility for consumption. For a detailed timeline of the game, see Figure 2.

The equilibria that result depend on our three factors. The easiest case to analyze is when the mean-shifting effect is zero (see Section 4.2), where we find that both firms take the same action, going transparent when brand preference heterogeneity is sufficiently high and otherwise staying closed. The intuition for this outcome has already been hinted at. Under high brand heterogeneity, engaging in operational transparency reduces the "noise" in customer valuation due to variability in operational perceptions that may otherwise cloud a customer's appreciation of the differences in the products, avoiding the downward spiral of price competition that results from selling nearly identical products. That is, both firms have the incentive to reveal the significance of their brand differences by showing more of their operational processes that might otherwise "wash out" brand effects with innuendo about how they run their operations.

On the other hand, if brand preference heterogeneity is low, going transparent reduces variability in operational perceptions leading to customer valuations that are more tightly clustered. In other words, as operations become more transparent, products that had little brand differentiation start to look even more similar to each other, inviting intensified price competition. As a result, firms "hide behind" varied opinions about how they operate to differentiate themselves from each other and avoid direct competition.

Roughly the same logic holds in the other setting we analyze; if there is a sufficiently high degree of brand preference heterogeneity, then both firms will go transparent in order to differentiate themselves. Of course, this competitive analysis *does not* provide a compelling answer to the Subway-Potbelly difference, where one firm goes transparent, and the other does not. But what about the case when there is little or no difference in brand preference but there is a mean-shifting effect from transparency? In this case, we find something that was unexpected to us *a priori*:

when the mean-shifting effect is relatively small, only one firm will go transparent, in part to get the added benefits of operational transparency but primarily to differentiate themselves from their competitor. The competing firm forgoes the benefits of going transparent because these benefits are outweighed by the cost of heightened competition. Of course, when the mean-shifting effect of operational transparency is large, transparency benefits can outweigh losses from heightened competition.

Applying this understanding to the Subway-Potbelly example, one way to view the situation is that consumers are somewhat indifferent in their allegiance to the two brands, so distinctions in operational transparency are a form of product differentiation. Potbelly might increase the average customer valuation of their offering by going transparent, but this gain might be small compared to the increased competition they face by making their offering less distinguishable from Subway. Thus, our model confirms the perspective that Potbelly stays less transparent because of the nature of the competitive environment.

Of course, our model has implications beyond the Subway-Potbelly example. Consider the following. Since 2014, the China Food and Drug Administration has deployed a "Bright Kitchen, Bright Stove" policy to ensure food safety in China's restaurants.[2] By 2018, twenty percent of restaurants in China had taken steps to become more transparent. Outlets of large restaurant chains were found to be more enthusiastic in implementing transparency than independent restaurants. Our analysis suggests that transparency will be more prevalent among firms whose pool of customers they compete over have high brand heterogeneity in their tastes. Thus, if the government hopes to induce restaurants to increase transparency, they may start by offering subsidies for transparent conversions to groups of restaurants that are nearby to one another and whose customers show somewhat strong preferences for one restaurant over the others. These restaurants might be induced to go transparent to further solidify their competitive differentiation. The government is less advised to focus its efforts on restaurants with very similar products with weak customer brand preferences who do not expect operational transparency to create a major positive shock in customer value. These firms are likely to stay nontransparent to avoid a more competitive environment that operational transparency might usher in.

Organization of the paper

The rest of the paper is organized as follows. Section 2 reviews relevant literature. Section 3 presents our game-theoretic model, which includes a careful description of its sequence of events. Section 4 contains our analysis across four subsections. The first subsection describes our overall analytical strategy using backward induction. The remaining three sections analyze our model in increasing

[2] http://www.gov.cn/xinwen/2019-04/09/content_5380855.htm

complexity, starting with special cases. These subsections contain our main findings, along with a discussion of intuition, insight, and application. Section 5 concludes and offers managerial insights that could be useful for decision-makers pondering a move toward transparency. Proofs of all results are found in an accompanying online appendix.

2. Literature review

Our work relates to several strands of literature in operations management and marketing. First, our work continues in the strong tradition of trying to understand how customer experience and engagement impact operations. One of the early contributions in this area was due to Chase (1978), who emphasized that minimizing direct customer interaction with the service system can maximize the system's potential to function at optimal efficiency. Differing from his opinion, there is a wider agreement that delivering exceptional customer experiences is crucial for attaining competitive advantage, customer satisfaction, differentiation, reputation, loyalty, and word-of-mouth (Jain et al. 2017, Manning and Bodine 2012, Shaw and Ivens 2002, Gentile et al. 2007, Verhoef et al. 2009, Kumar and Pansari 2016). Distinct from involving customers in specific activities, this paper examines how customer experiences of operating processes through the implementation of operational transparency can contribute to achieving a competitive advantage in a competitive environment.

To engage customers, Buell and Norton (2011) laid the groundwork for operational transparency research by demonstrating that increased visibility into service processes enhances customer satisfaction and trust. Their work highlights the importance of providing customers with information about the efforts behind the delivery of goods and services. Operational transparency, the act of providing visibility into the inner workings of a process, service, or organization, has gained increasing attention in both the academic and business worlds. Thereafter, the role of operational transparency has been applied to multiple fields, including the crowdfunding industry (Mejia et al. 2019), healthcare (Saghafian and Hopp 2020, Lee et al. 2021), public sector organizations (Sørensen and Torfing 2011), logistics (Bray 2023), and government (Buell et al. 2021). These studies underline the broad applicability of transparency principles across various sectors. They show that operational transparency allowed potential backers to assess project quality better and reduced information asymmetry. The greater transparency, including shared decision-making and open communication, led to improved experiences and better outcomes. There are many mean-shifting effects when applying operational transparency. For example, Buell et al. (2017) suggested that when customers were given insight into the efforts undertaken by service providers, they perceived a higher level of value and were more likely to reciprocate through increased patronage, positive word-of-mouth, and a higher willingness to pay for services. Saghafian and Hopp (2020) proposed the use of public reporting of medical treatment outcomes as a tool for increasing quality transparency and improving alignment between patient choices. They considered the impact of different types of patients and competition among healthcare providers. However, they only considered the mean-shifting effect of transparency, and their takeaways focused on the healthcare industry, while our goal is to derive analytical results to reveal broadly applicable principles. Buell and Choi (2019) stressed that providing transparency into an offering's tradeoffs improves customer compatibility. Although they provided significant insights into the mean-shifting and heterogeneity-reducing effects that are discussed in our analysis, they examined operational transparency in an independent environment. Our work, on the other hand, differs from these papers as we account for competition and customer heterogeneity.

In our formulation, customers make choices based on maximizing their utility. In that sense, our work relates to work on choice models; see, e.g. McFadden and Train (2000), Revelt and Train (1998), see also the surveys by Train (2009), Hensher et al. (2005) for an overview. This paper diverges from the existing body of work by considering distinct customer segments. The consideration of customer heterogeneity and interpreting the heterogeneity from the degree of familiarity with the brand or prior experience, etc. Yoo and Sarin (2018) stressed that customers may behave in a boundedly rational way and rely on their initial preference or liking for a product to simplify the decision process. One of the early works that acknowledged consumer heterogeneity is Smith (1956), which considers this aspect of designing and managing products and services that appeal to different segments. Market segmentation involves dividing the market into distinct groups based on their needs, preferences, or characteristics, allowing firms to tailor their products, services, and marketing efforts to cater to these diverse customer groups. Prahalad and Ramaswamy (2004) introduced the idea of "customer co-creation", which encourages organizations to involve customers in the service creation process to address their diverse preferences better and enhance the overall service experience. Our analysis identifies a new consideration for operational transparency by considering customer heterogeneity and competition in the marketplace.

In the operations literature, Kwark et al. (2014) also discussed a tool with mean-shifting and heterogeneity-reducing effects. They emphasized the importance of online reviews in providing information about quality and fit dimensions. They studied the impact of such reviews in a competitive channel structure that included two manufacturers and a retailer. Our context of operational transparency primarily provides quality information, and customers tend to respond similarly about a firm's quality when presented with increased transparency for more understanding of the products. Also, we consider a simple competitive scenario to obtain insights instead of a supply chain setting.

3. Model

We begin with Section 3.1 by describing a basic setting of two firms selling differentiated products to a common pool of customers. The basic setting does not include the operational transparency decision. We start with this basic setting to set notation and introduce the reader to a (hopefully) somewhat familiar setting. Next, in Section 3.2, we describe the operational transparency decision and its impact on the decisions of customers. Finally, in Section 3.3, we provide a detailed description of the game we analyze, including a careful description of the sequence of events.

3.1. Basic setting

Two firms (labeled A and B and indexed by $i \in \{A, B\}$) sell differentiated products in the same product or service category to a common pool of customers. Each customer has unit demand and must choose to consume either the offering of firm A (simply called brand A) or the offering of firm B (brand B).[3]

Customers are partitioned into two segments defined by the brand they most prefer. That is, there is a segment of customers that prefer brand A (that we call segment A) and a segment of customers that prefer brand B (segment B). Each customer segment has a mass normalized to 1 for a total customer mass of 2. We use $j \in \{A, B\}$ to index the customer segments.

The value that a consumer from segment j has for product i has the following structure

$$V_{ij} \doteq \begin{cases} q + \alpha + \epsilon_{ij} & \text{if } i = j \\ q - \alpha + \epsilon_{ij} & \text{if } i \neq j \end{cases} \tag{1}$$

We now describe each of the components of V_{ij}. Every customer shares the same underlying assessment q of the inherent *quality* of the product category, regardless of brand or customer segment.[4]

The parameter α is the value a customer receives by consuming their preferred brand, while $-\alpha$ is the analogous penalty for consuming their nonpreferred brand.[5] Note that the larger the α is, the greater the difference in the preferences across customer segments for the two brands. Accordingly, we call α the *degree of brand preference heterogeneity*.

[3] The reader will note that we have not provided the consumers with an outside option. This is justified in certain constrained decision environments where individuals are required to select an option from a predetermined set of sellers where the concept of an "outside option" may not be applicable. In the context of restaurants, we were motivated by experiences of traveling to a new city and being dropped off in a food court on a tour where we had to select to eat from the available options.

[4] The reader might be curious about how firms with different inherent qualities might approach the operational transparency decision differently. While an interesting direction, as discussed in the introduction, our focus is developing a model to isolate the effects of how competition modulates the operational transparency decision. For this reason, we have endeavored to keep the two firms as nearly identical as possible but still yield a model with sufficient complexity to derive insights into the impact of competition. This is a principle the reader will see applied in later parts of our model development.

[5] The reader might find it more natural to have a positive reward only for consuming their preferred brand and a penalty normalized to zero for consuming their nonpreferred brand. There are analytical reasons, however, for introducing the reward and penalty in this way, as it adds some symmetry that makes analysis easier. Of course, one could simply change the quality values q to go back and forth between these two approaches.

The final component of (1) is the random variable ϵ_{ij} that captures the subjective perception of a customer in segment j towards the operational process that is used to produce and deliver a product or service with brand i. We assume that the ϵ_{ij} are independent and identically distributed normal random variables with mean zero and standard deviation σ. Let Φ and ϕ be the cumulative distribution function and probability density function of the standard normal distribution, respectively.

The contribution of ϵ_{ij} towards customer value in (1) is distinct from the product's inherent quality and a customer's brand preference. The variables ϵ_{ij} may represent, for instance, customer perceptions of a process's sustainability, fair practices, degree of automation, or cleanliness. In the basic setting, half of the customers have positive views about the operational process ($\epsilon_{ij} > 0$), while half of the customers have negative views ($\epsilon_{ij} < 0$). We call σ the *degree of operational perception heterogeneity* because it measures how varied customers are in their valuations of the operational processes of firms A and B.

In summary, there are *two* sources of customer heterogeneity in this model: brand preference (captured by α) and subjective operational perceptions (captured by σ). Because the focus of this paper is the impact of competition on operational transparency, the model has the most granularity when it comes to operational perceptions and keeps other differences between customers as parsimonious as possible.

Finally, each firm i decides a *selling price* p_i for product i. For simplicity, we normalize the production costs of both firms to zero. Thus, a customer from segment j receives net *utility* U_{ij} when consuming brand i of

$$U_{ij} = V_{ij} - p_i = \begin{cases} q - p_i + \alpha + \epsilon_{ij} & \text{if } i = j \\ q - p_i - \alpha + \epsilon_{ij} & \text{if } i \neq j. \end{cases} \tag{2}$$

We assume that all customers are utility maximizers and observe the quantity ϵ_{ij} before making their decision of which brand to consume. For more discussion of the sequence of events in the game, see Section 3.3.

3.2. Operational transparency decisions

In addition to choosing price, firms also select an *operational transparency level* $t_i \in [0, \bar{t}]$ where $\bar{t} < 1$. A larger t_i means that firm i reveals more about its operational process. The upper bound $\bar{t}$ is strictly less than 1, reflecting (as we shall see in more detail below) an assumption that complete transparency that removes all customer operational perception heterogeneity is impossible. We assume that there is no cost for a firm to change its operational transparency level.[6]

[6] This assumption suffices to describe scenarios where there is a fixed cost of setting $t_i > 0$, and that this fixed cost is small compared to the increase in revenue from increasing the transparency level. As we show in Lemma 3,

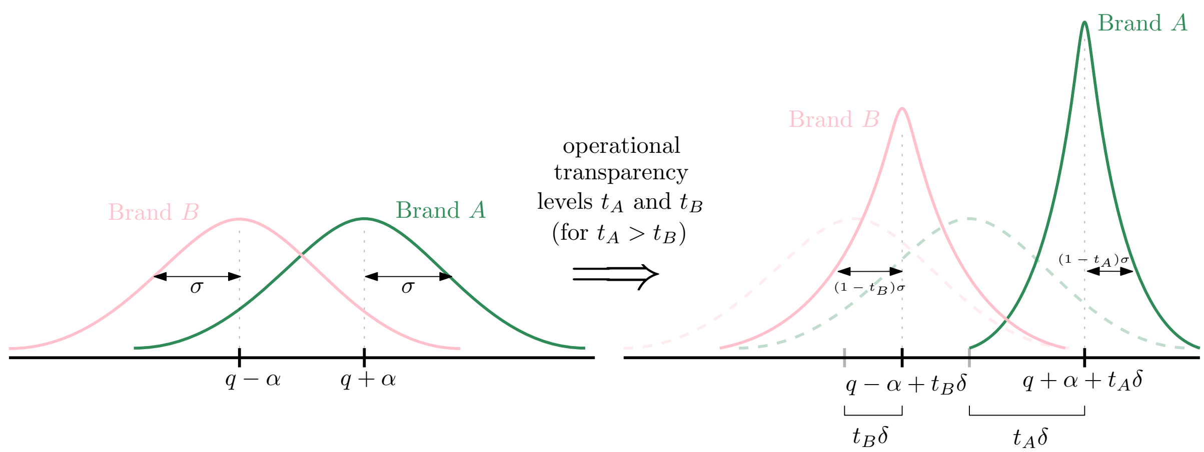

Figure 1 **An illustration of two effects of operational transparency. The curves in the figure represent the probability distribution functions (bell curves) for the customer valuation random variables V_{ij} defined in (3) for a customer in segment $i = A$.**

We model the impact of operational transparency in two ways. First, customers get a positive utility shock by having increased awareness of how the product is produced, possibly even by being entertained (for example, watching a skilled chef make handmade noodles or a machine making donuts). The amount of positive utility shock depends on the degree of transparency. The maximum available positive shock is the positive quantity δ (assumed uniform across both products). The effective increase in customer utility depends on t_i as a multiplicative factor. That is, if firm i chooses operational transparency level t_i, then any customer that consumes product i gets an additional utility shock of $t_i\delta$. We call this the *mean-shifting effect* of operational transparency since it acts as a shift in the product's observable quality from q to $q + t_i\delta$. We call δ the *mean-shifting effect parameter*.[7]

The second effect of operational transparency is to reduce customer operational perception heterogeneity. This is based on the idea that by revealing more of the operation, customers will base their assessments of the operation on more data and less on speculation, which reduces variability in their assessments.[8] We model this by the operational transparency level t_i reducing the impact of

the optimal choice for t_i is either 0 or $\bar{t}$, and so as long as the fixed cost of transparency is less than the benefit of taking transparency $\bar{t}$, we can take the fixed cost to be zero without loss. Another possible interpretation is that the cost of going transparent is partially (or completely) covered by a subsidy or grant from the government so as to not be relevant. The possibility of subsidies was part of the "Bright Kitchens, Bright Stoves" policy in China that was discussed in the introduction. The generalization to a fixed cost "with bite" or a variable cost of transparency complicated our analysis and "tips the balance" in a different way towards not going transparent outside of competitiveness concerns. In concert with our focus on competition, we did not analyze transparency costs further, but it would be interesting as an extension for further research.

[7] In our analysis, we have assumed that δ is nonnegative, which is consistent with the literature on the benefits of firms going transparent. Our analysis of how competitive behavior can keep a firm from going transparent is less compelling when δ is negative, and so we keep δ nonnegative as a more compelling and interesting case.

[8] This is consistent with the psychology literature on the synchronization of collective beliefs (see, for instance, Vlasceanu et al. (2020)).

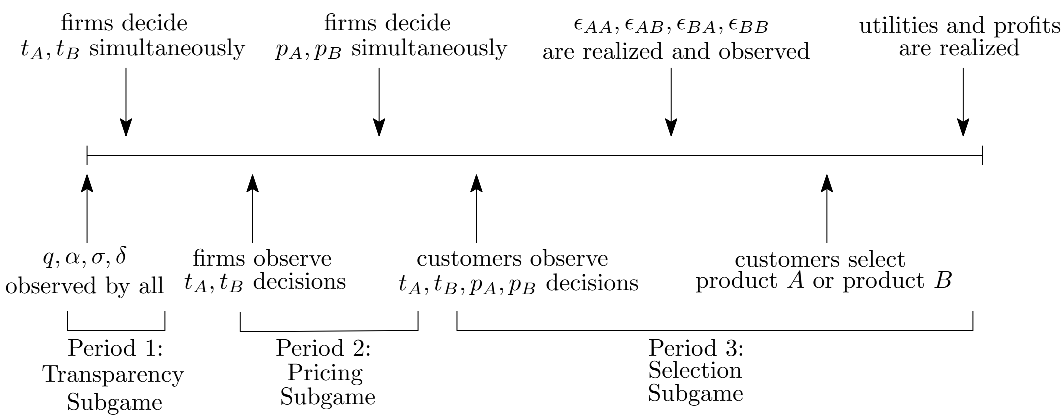

Figure 2 **The Sequence of Events.**

the random term ϵ_{ij}. Under operational transparency level t_i, the unobservable quality of segment j consuming product i changes from the random variable ϵ_{ij} to the random variable $(1-t_i)\epsilon_{ij}$. We call this the *heterogeneity-reducing effect* of operational transparency.

Combining the two effects of operational transparency—the mean-shifting effect and the heterogeneity-reducing effect—the value of a customer in segment j for consuming product i becomes:

$$V_{ij} = \begin{cases} q + \alpha + t_i\delta + (1-t_i)\epsilon_{ij} & \text{if } i = j \\ q - \alpha + t_i\delta + (1-t_i)\epsilon_{ij} & \text{if } i \neq j \end{cases} \tag{3}$$

This is the most general form of customer valuation we consider in the paper. For an illustration of the two effects of operational transparency, see Figure 1.

Finally, let's return to the assumption that $t_i \leq \bar{t} < 1$; that is, complete operational transparency is not feasible. This is a natural assumption in light of the ambiguity-reducing effect. Complete elimination of uncertainty in quality is not possible since there are idiosyncratic factors that impact quality that is not easy to recognize by a customer. A restaurant may open its kitchen to observers, but some uncertainty nonetheless remains: What should a customer be looking for? How should a customer interpret what they see?

3.3. Sequence of events

We now describe the sequence of events in the game, as summarized in Figure 2. The game has three periods. Each period has one type of decision. In Period 1, firms simultaneously decide their operational transparency levels t_A and t_B in what we call the *Transparency Subgame*. Their decision is based on public knowledge of the observable qualities q, the degree of brand preference heterogeneity α, the degree of operational perception heterogeneity σ, and the size of the mean-shifting effect of transparency δ. The equilibrium choices, of course, require anticipation of the downstream pricing actions and, ultimately, the decisions of customers. The firms are expected profit maximizers, where the expectation is taken over the distributions of the unobserved quality variables ϵ_{ij}.

Period 2 starts with firms observing each other's transparency level choices t_i. Then, the firms simultaneously select prices p_A and p_B in what we call the *Pricing Subgame*. We model the pricing decisions as occurring after the operational transparency decision for the following reason: The choice of operational transparency is more *fixed* (often involving careful design choices of the restaurant) while pricing is a more flexible, and thus reactive, decision.

Period 3 starts with customers observing the pricing and transparency decisions of the firm. Based on these observations, the operational perception components ϵ_{ij} are realized. After forming their operational perceptions, each customer selects to purchase brand A or brand B. We call this the *Selection Subgame*. That is, a customer in segment j solves the optimization problem:

$$\max\{U_{Aj}, U_{Bj}\} \tag{4}$$

where U_{ij} is defined in (6). This choice is, naturally, a function of the choices of t_A, t_B, p_A, and p_B by the firms in the first two periods. After all of the decisions are made, customer utilities and firm profits are realized. This ends the game.

4. Analysis

In this section, we analyze our model in order to derive insights into our two main research questions from the introduction: (Q1) and (Q2). For (Q1), our focus is on understanding the impact of the three key parameters in the model: the degree of brand preference heterogeneity α, the degree of operational perception heterogeneity σ, and the size of the mean-shifting effect of transparency δ.

4.1. Analytical approach and breakdown of cases

In order to answer (Q1), we need to ascertain structural insights into the equilibrium choices of t_A and t_B for the two firms. Ideally, this comes in some closed-form relationship between t_i and the key parameters of the model: α, σ, and δ. We derive results roughly along these lines but achieve more in the cases that are simpler to analyze (see Table 1). For example, in the simplest case we study (Special Case 1 in Section 4.2), we show in Lemma 3 that it is optimal for the firms to choose a transparency level at one of the two extremes: 0 and $\bar{t}$. This means there are exactly three possibilities in what firms will choose as transparency levels: (i) both choose $\bar{t}$ (what we will denote by YY, where "Y" denotes "yes" to transparency), (ii) both choose 0 (or NN, where "N" denotes "no" to transparency), and (iii) where the firms are split on their transparency decision (what we will denote by YN and NY). Understanding when a possibility (iii) occurs provides insight into research question (Q2). As the models become more complex (in Sections 4.3 and 4.4), we must compromise here and restrict attention to setting where we restrict the transparency choices to be Y or N. Either the firms fully commit to transparency or they do not.

The main results come in the form of describing regions for the parameters (α, σ, and δ) where the outcomes YY, NN, and YN/NY occur as equilibrium (see, Theorems 1 and 2). Interpreting these regions provides insight into the operational transparency of firms and helps us answer (Q1) and (Q2).

Of course, in order to describe these regions, we need a strategy for solving the game described in Section 3.3. This requires some sophisticated backward induction.

First, we need to solve for the optimal decisions of the customers in the Selection Subgame as a function of the t_i and p_i. We let $D_{ij}(p_A, p_B, t_A, t_B)$ denote the *demand* of customers in segment j who select product i. This demand is a mass of customers with a weight between 0 and 1. Solving for $D_{ij}(p_A, p_B, t_A, t_B)$ is a straightforward optimization problem; no equilibrium concepts are required here.

Second, the demand functions $D_{ij}(p_A, p_B, t_A, t_B)$ are input into Pricing Subgame in Period 2. The Pricing Subgame is solved using a Nash equilibrium solution concept, which yields equilibrium price choices $p_i(t_A, t_B)$ as functions of the operational transparency level decisions t_A and t_B. We abuse notation slightly to let $D_{ij}(t_A, t_B)$ denote the demand of customers in segment j for product i under equilibrium prices $p_i(t_A, t_B)$.

Finally, we return to Period 1 to solve the Transparency Subgame. We again use a Nash equilibrium solution concept to yield equilibrium operational transparency levels t_A^* and t_B^*. The final prices that prevail in the market are thus $p_i^* = p_i(t_A^*, t_B^*)$ with demands $D_{ij}^* = D_{ij}(t_A^*, t_B^*)$.

The objective functions of the firms in their two subgames are the profit functions:

$$
\begin{aligned}
\Pi_A(t_A, t_B, p_A, p_B) &:= p_A(D_{AA}(p_A, p_B, t_A, t_B) + D_{AB}(p_A, p_B, t_A, t_B)), \\
\Pi_B(t_A, t_B, p_A, p_B) &:= p_B(D_{BA}(p_A, p_B, t_A, t_B) + D_{BB}(p_A, p_B, t_A, t_B)).
\end{aligned}
\tag{5}
$$

Recall, for instance, that D_{AB} is the demand for brand A by customers in segment B. We abuse notation slightly to let $\Pi_i(t_A, t_B)$ denote the profit as a function of t_A and t_B at the equilibrium price levels $p_i(t_A, t_B)$. Thus, the overall equilibrium profits of the firm are $\Pi_i(t_A^*, t_B^*)$.

The analysis of the "full" scenario depicted in Figure 2 in its entirety is complex, so we approach it by solving two special cases first. Analyzing these two special cases gives the reader a sense of our overall approach but also provides insight into our research questions (Q1) and (Q2). The special cases provide simpler answers to these questions and highlight what features of the model drive certain outcomes.

A roadmap for our analysis of these special cases, and the general problem, is given in Table 1. Because of the centrality of customers' subjective perceptions of operational processes to our

	$\delta=0$	$\delta\neq 0$
$\alpha=0$	Yoo and Sarin (2018)	**Special Case 2** no brand preference heterogeneity (Section 4.3)
$\alpha\neq 0$	**Special Case 1** no mean-shifting effect (Section 4.2)	**General model** (Section 4.4)

Table 1 **An agenda for our analysis, broken down in simpler subcases**

research questions, no special case sets $\sigma=0$. It should be noted that the most constrained case ($\alpha=0,\delta=0$) was analyzed in Yoo and Sarin (2018), in a setting designed to study how consumers perceived quality ambiguity affects competition and market outcomes.

4.2. Special Case 1: No mean-shifting effect

Let's begin our analysis of the game in Figure 2 in the special case where the mean-shifting effect parameter δ is zero. In this case, the utility functions of the customers simplify to:

$$U_{ij}=\begin{cases} q-p_i+\alpha+(1-t_i)\epsilon_{ij} & \text{if } i=j \\ q-p_i-\alpha+(1-t_i)\epsilon_{ij} & \text{if } i\neq j \end{cases} \tag{6}$$

In this scenario, there is still brand preference heterogeneity, but now the only impact of operational transparency is to reduce the variance of operational perceptions without changing their mean.

We solve the resulting game by backward induction. Starting with the Selection Subgame of Period 3, the following result yields structure on the demand functions $D_{ij}(p_A,p_B,t_A,t_B)$ as a function of the firm decisions p_A,p_B,t_A, and t_B.

Lemma 1 (Solution to the Selection Subgame) *Suppose there is no mean-shifting effect of operational transparency (that is, $\delta=0$). Then, the demand functions that result when solving the Selection Subgame are:*

$$\begin{aligned}
D_{AA}(p_A,p_B,t_A,t_B)&=\Phi\left(\frac{2\alpha-\Delta p}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right),\\
D_{AB}(p_A,p_B,t_A,t_B)&=\Phi\left(\frac{-2\alpha-\Delta p}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right),\\
D_{BA}(p_A,p_B,t_A,t_B)&=\Phi\left(\frac{-2\alpha+\Delta p}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right),\\
D_{BB}(p_A,p_B,t_A,t_B)&=\Phi\left(\frac{2\alpha+\Delta p}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right),
\end{aligned}$$

where $\Delta p:=p_A-p_B$.

Take D_{AA} for example; customers from segment A will choose firm A only if the utility U_{AA} is higher than U_{BA}. According to the definition of utility function (6), we have $U_{AA} = q - p_A + \alpha + (1-t_A)\epsilon$ and $U_{BA} = q - p_B - \alpha + (1-t_B)\epsilon$. So the demand of D_{AA} is expressed as:

$$D_{AA} = \mathbb{P}(U_{AA} \geq U_{BA}) = \mathbb{P}((1-t_A)\epsilon_{AA} - (1-t_B)\epsilon_{BA} \geq p_A - p_B - 2\alpha).$$

As assumed, ϵ_{AA} and ϵ_{BA} are independent identical normal distributions with mean 0 and standard deviation σ. Then, the mean and the standard deviation of the new variable $(1-t_A)\epsilon_{AA} - (1-t_B)\epsilon_{BA}$ are 0 and $\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}$, respectively. Hence, $D_{AA} = \Phi\left(\frac{2\alpha - \Delta p}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)$. Note that $\Phi(\cdot)$ and $\phi(\cdot)$ represent the cumulative distribution function and probability density function of a standard normal distribution, respectively. Similarly, we can conclude the specific form of D_{BA}, D_{AB}, and D_{BB}.

All of these demand functions have a similar structure (in particular, the same denominator) but with slightly different numerators. The numerators would be simplified if the prices of the products were equal (setting $\Delta p = 0$). Interestingly, this is exactly what transpires when we solve the Pricing Subgame.

Lemma 2 (Equilbria of the Pricing Subgame) *Suppose there is no mean-shifting effect of operational transparency (that is, $\delta = 0$). Then*

(i) there exist unique equilibrium prices as functions of t_A and t_B of the form:

$$p_A(t_A, t_B) = p_B(t_A, t_B) = \frac{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}{2\phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)}. \tag{7}$$

(ii) at the equilibrium prices in (7), customer demands (as a function of t_A and t_B) are

$$\begin{aligned} D_{AA}(t_A, t_B) = D_{BB}(t_A, t_B) &= \Phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right) \\ D_{AB}(t_A, t_B) = D_{BA}(t_A, t_B) &= \Phi\left(\frac{-2\alpha}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right), \end{aligned} \tag{8}$$

where, in particular, the mass of customers that select product i (for $i = 1, 2$) is one.

(iii) at the equilibrium prices in (7), the firms have the same profit functions, namely $\Pi_A(t_A, t_B) = \Pi_B(t_A, t_B) = p_A(t_A, t_B) = p_B(t_A, t_B)$ for all t_A and t_B.

We give the full proof of Lemma 2 in Section A.1. Lemma 2(ii) reveals an interesting structure, illustrated in Figure 3. This result shows that an identical proportion of brand A is sold to those who prefer its brand as those for brand B. This is reflected in the figure by the fact that the larger green region and the larger pink region are equal in size. This also implies that the total amount

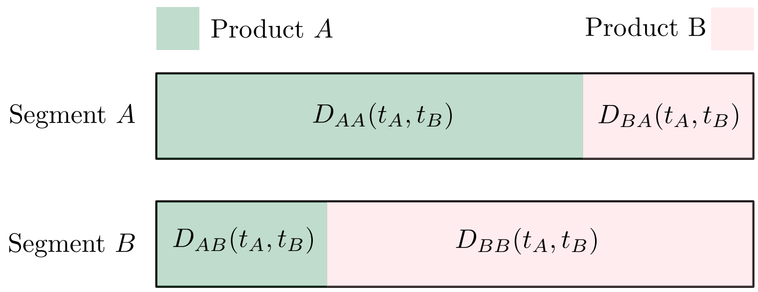

Figure 3 An illustration of Lemma 2. The sum of the two green-shaded regions has a unit mass. Similarly, for the sum of the two pink-shaded regions.

of brand A sold is equal to the total amount of brand B, each selling to a unit mass of customers. This simplifies the structure of the profit functions in part (iii) of the lemma.

The simple structure of the profit functions in Lemma 2(iii) shows that we can greatly simplify the Transparency Subgame. Indeed, it suffices to solve a symmetric game where the two firms choose actions $t_A, t_B \in [0, \bar{t}]$ with the common payoff function

$$\Pi(t_A,t_B) := \frac{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}{2\phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)}. \tag{9}$$

The next result reveals a special property of the payoff function Π that allows for a simple analysis of the Transparency Subgame.

Lemma 3 (Optimizing the payoff function Π) *The payoff function Π defined in (9) has the following property. For any given value $\hat{t}_B$ of t_B, the value of t_A that maximizes $\Pi(t_A,\hat{t}_B)$ is either $t_A = 0$ or $t_A = \bar{t}$. The converse is also true, for any given value $\hat{t}_A$ of t_A, the value of t_B that maximizes $\Pi(t_A,\hat{t}_B)$ is either $t_B = 0$ or $t_B = \bar{t}$.*

Lemma 3 is attributed to the monotonicity of the profit function Π defined in (9) with respect to the transparency degree. For more comprehensive details, please refer to Section A.2.

This lemma endows the Transparency Subgame with a simple structure. There are only four possible choices for the equilibrium prices $(\hat{p_A},\hat{p_B})$: (i) $(\hat{p_A},\hat{p_B}) = (\bar{t},\bar{t})$, (ii) $(\hat{p_A},\hat{p_B}) = (\bar{t},0)$, (iii) $(\hat{p_A},\hat{p_B}) = (0,\bar{t})$, and (iv) $(\hat{p_A},\hat{p_B}) = (0,0)$. As discussed in Section 4.1, we denote these cases YY, YN, NY, and NN, where "Y" denotes taking maximal transparency and "N" denotes not pursuing transparency. The following result shows that only two of these possible outcomes can occur in the current setting.

Theorem 1 (Equilibria in the Transparency Subgame) *Suppose there is no mean-shifting effect of operational transparency (that is, $\delta = 0$). Then the Transparency Subgame has equilibrium (t_A^*, t_B^*) whose form depends on the parameters α and σ in the following way:*

$$(t_A^*, t_B^*) = \begin{cases} (0,0) & \text{if } {}^{\alpha}/_{\sigma} \leq m_1 \\ (0,0) \text{ or } (\bar{t},\bar{t}) & \text{if } m_1 < {}^{\alpha}/_{\sigma} < m_2 \\ (\bar{t},\bar{t}) & \text{if } {}^{\alpha}/_{\sigma} \geq m_2 \end{cases} \tag{10}$$

where

$$m_1 = (1-\bar{t})\sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{1-(1-\bar{t})^2}} \quad \text{and} \quad m_2 = \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{2}{1+(1-\bar{t})^2}}}{1-(1-\bar{t})^2}}.$$

That is, NN is the unique equilibrium if ${}^{\alpha}/_{\sigma} \leq m_1$, YY is the unique equilibrium if ${}^{\alpha}/_{\sigma} \geq m_2$ and either YY or NN can be equilibria if $m_1 < {}^{\alpha}/_{\sigma} < m_2$. In addition, there exists a critical threshold denoted as

$$m_0 = (1-\bar{t})\sqrt{\frac{\ln(1-\bar{t})}{(1-\bar{t})^2-1}},$$

where equilibrium YY outperforms equilibrium NN when ${}^{\alpha}/_{\sigma}$ is above the threshold and underperforms NN when below the threshold.

The theorem reveals that the Transparency Subgame either has YY and NN as equilibria, and these are unique equilibria for extreme values of ${}^{\alpha}/_{\sigma}$. For non-extreme values of ${}^{\alpha}/_{\sigma}$, i.e., when ${}^{\alpha}/_{\sigma} \in (m_1, m_2)$, YY and NN can both be equilibria. Further, when ${}^{\alpha}/_{\sigma}$ is within the range of (m_0, m_2), the YY equilibrium generates greater profits compared to the NN equilibrium, and when ${}^{\alpha}/_{\sigma}$ is within the range of (m_1, m_0), the NN equilibrium generates greater profits compared to the YY equilibrium, as illustrated in Figure 4. The detailed analysis of Theorem 1 can be found in Section A.3.

To get an intuitive sense of why this result holds, we need to examine the meaning of the ratio ${}^{\alpha}/_{\sigma}$. This ratio is large if where brand preference heterogeneity is more acute than operational perception heterogeneity. A large value of α means that the products are quite differentiated from each other, and so by engaging in operational transparency, the "noise" coming from operational perceptions that may cloud a customer's appreciation of the differences in the two products is diminished. This differentiation allows the two firms to "show that we are different", avoiding the downward spiral of price competition that results from selling nearly identical products. That is, both firms have the incentive to reveal the significance of their brand differences by showing more of their operational processes that might otherwise "wash out" brand effects with innuendo about how they run their operations.

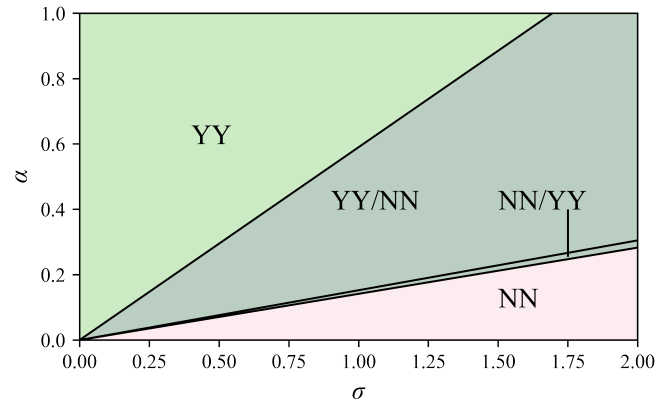

Figure 4 **An illustration of regions in the space of α and σ that support YY or NN as unique equilibria results when $\delta = 0$. YY/NN means both can be equilibria in the corresponding area. In addition, YY in the front means it is a better equilibrium. Similarly, NN/YY means both can be equilibria, and NN is the better equilibrium. The figure is generated for $\bar{t} = 0.9$.**

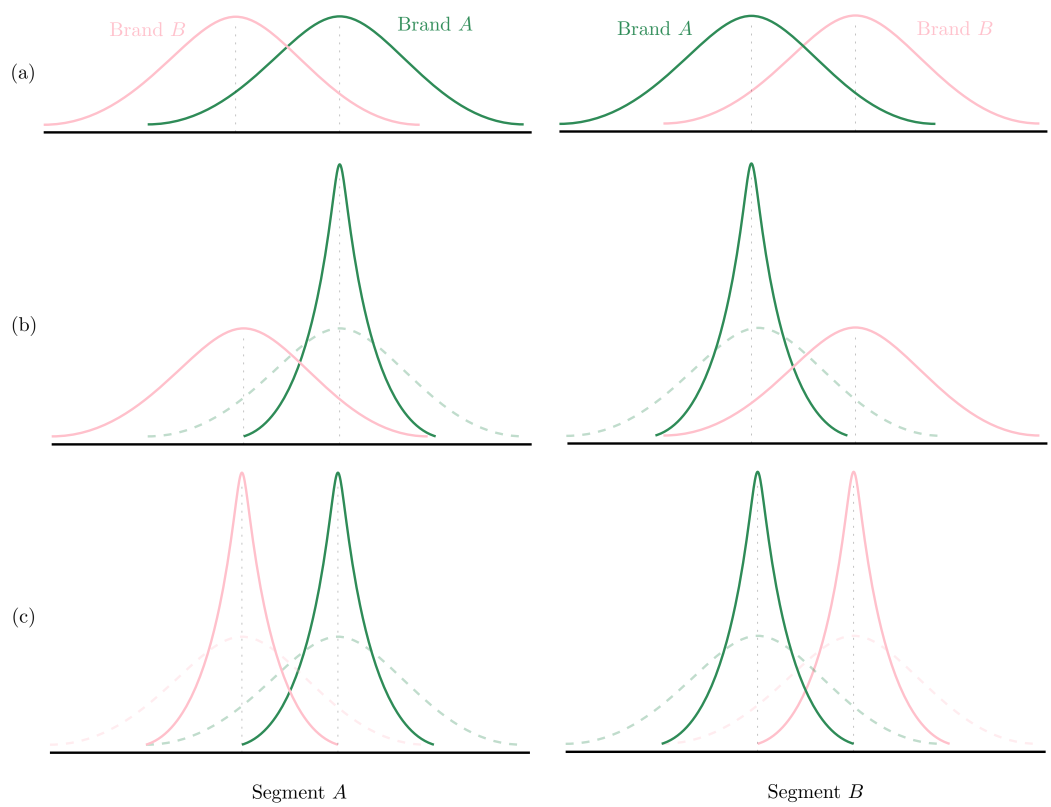

Figure 5 **An illustration of deviations when $\alpha/\sigma \geq m_2$. The left-hand side depicts the value distributions of segment A customers. The right-hand side is segment B customers. From top to bottom, we see a deviation through the thought process of both firms; in (a) we start with an NN outcome, then in (b) firm A deviates by going transparent, and finally, in (c), firm B best responds by also going transparent.**

To get at the intuition of the case where α is large, consider Figure 5, which focuses on the thought process of firm A. Suppose firm A ponders deviating from the NN outcome illustrated in Figure 5(a). It considers moving to customer distribution like in Figure 5(b) by going transparent. When firm A goes transparent, operational perceptions narrow about brand A. That is, customers have fewer extremely positive views and fewer extremely negative views. Because of the large separation provided by a large α, the loss of extremely positive views does not hurt firm A that much. Among segment A customers, as we see in the left panel of Figure 5(b), the right tail of brand A's distribution curve is still predominantly above the right tail of brand B's distribution curve. While losing those positive reviewers among segment B customers hurts firm A among those customers, all of that lost is gained back in A customers because the total mass of customers who purchase brand A remains constant (as we saw in Figure 3). However, we have shifted the distribution to customers with a stronger initial preference for firm A, which allows for higher pricing of brand A. Indeed, the gain among segment A customers by "tightening" the lower tail in the left panel of Figure 5(b) can be significant, as a much large proportion of customers will have higher valuations for brand A than brand B, reflected in a much large area under the green curve that is above the pink curve at higher valuations.

From this figure, we can also see why YN is not a stable outcome. Firm B clearly has an incentive to "tighten" its distribution for similar reasons as firm A, as it can consolidate in its market and shed segment A customers it had to price aggressively to attract. This, again, reinforces the benefit of the firms to "show that they are different" and avoid pricing competition, particularly in their weaker market.

On the other hand, if α is low, the brand preference effects are weak, and if operational perception heterogeneity is diminished through a firm going transparent, then customer valuations become even more tightly clustered around their similar averages. In other words, as operations become more transparent, products that had little brand differentiation start to look even more similar to each other, inviting intensified price competition. In other words, both firms "hide behind" varied opinions about how they operate to differentiate themselves from each other and thus avoid direct competition.

The analysis in this subsection provides a few insights into our research questions (Q1) and (Q2). Regarding (Q1), we see a critical role here for α (in comparison to σ). If the two firms have different distinct brands, and these distinctions are highly differentially valued by customers, it can be to each firm's advantage to go transparent in order to further differentiate their offerings and avoid competition. On the other hand, when α is small, opaque operations are a better tool to avoid direct competition.

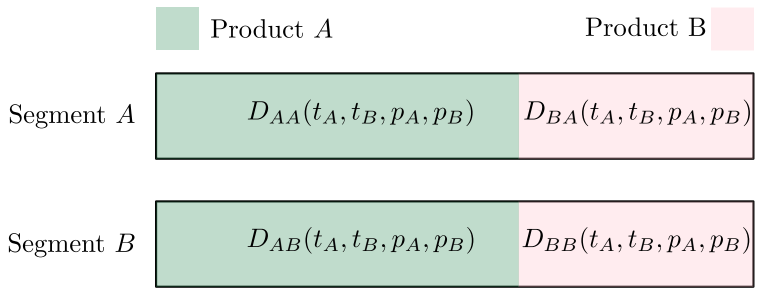

Figure 6 **An illustration of Lemma 4. The illustration assumes that firm A chooses more operational transparency than firm B (so that $\Delta t > 0$), yielding firm A a larger market share than firm B, in accordance with (17).**

However, this Special Case offers little insight into (Q2). A key fact here is from Lemma 2, which shows that under optimal pricing, both firms have identical profit functions, making the Transparency Subgame symmetric. It stands to reason, then, that a symmetric outcome is expected in this scenario. Thus, our analysis shows that it is necessary to include the mean-shifting effect of transparency to derive non-symmetric equilibria in the game. As the next subsection illustrates, this is indeed the case, even when we set $\alpha = 0$.

4.3. Special Case 2: No brand preference heterogeneity

Let us now consider the case where there is a mean-shifting effect of operational transparency ($\delta > 0$), but there is no brand preference heterogeneity ($\alpha = 0$). This simplifies customer utilities to:

$$U_{ij} = q - p_i + t_i\delta + (1 - t_i)\epsilon_{ij} \text{ for all } i, j \in \{A, B\} \tag{11}$$

One might think that this scenario will be as easy to analyze as Special Case 1, but this turns out not to be the case. The fact that the t_i impacts two terms in the expression of U_{ij}—$t_i\delta$ and $(1 - t_i)\epsilon_{ij}$—adds much complication. Luckily, we are still able to derive the forms for the expression of the demand functions $D_{ij}(t_A, t_B, p_A, p_B)$.

Lemma 4 (Solution to the Selection Subgame) *Suppose there is no customer heterogeneity (that is, $\alpha = 0$). Then, the demand functions that result when solving the Selection Subgame are:*

$$\begin{aligned} D_{AA}(p_A, p_B, t_A, t_B) = D_{AB}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{\delta\Delta t - \Delta p}{\sigma\sqrt{(1 - t_A)^2 + (1 - t_B)^2}}\right) \\ D_{BB}(p_A, p_B, t_A, t_B) = D_{BA}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{-\delta\Delta t + \Delta p}{\sigma\sqrt{(1 - t_A)^2 + (1 - t_B)^2}}\right) \end{aligned} \tag{12}$$

where $\Delta t := t_A - t_B$ and $\Delta p := p_A - p_B$.

Firm A \ Firm B	Y	N
Y	$\Pi_A(\bar{t},\bar{t})$, $\Pi_B(\bar{t},\bar{t})$	$\Pi_A(\bar{t},0)$, $\Pi_B(\bar{t},0)$
N	$\Pi_A(0,\bar{t})$, $\Pi_B(0,\bar{t})$	$\Pi_A(0,0)$, $\Pi_B(0,0)$

Table 2 A bimatrix game representation of the Transparency Subgame

The result here is intuitive because the valuations of the two segments are identically distributed; the mass of customers that demand brand A is the same from each of the two segments, similarly for brand B. For an illustration, see Figure 6. This means that, unlike in Special Case 1, one firm may sell more product than the other, depending on the value of numerators $\delta\Delta t - \Delta p$ and $-\delta\Delta t + \Delta p$ in (12).

Based on Lemma 4, the profits earned by firm A and firm B can be expressed as follows:

$$\Pi_A = p_A(D_{AA} + D_{AB}) = 2p_A\Phi\left(\frac{\delta\Delta t - \Delta p}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right),$$
$$\Pi_B = p_B(D_{BA} + D_{BB}) = 2p_B\Phi\left(\frac{-\delta\Delta t + \Delta p}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right).$$

This is where things get more difficult. Whereas in Special Case 1, the two firms were symmetric at the optimal prices and profits (Lemma 2(i)-(iii)), and we were able to solve the Transparency Subgame as a symmetric game, this is no longer the case in Special Case 2. Indeed, we were unable to derive closed forms expression for the optimal prices of the Pricing Subgame, and so we could only work with implicit formulations in our analysis of the Transparency Subgame.

In order to derive meaningful results in this more complicated analytical setting, we needed to simplify the decision sets in the Transparency Subgame. Whereas Lemma 3 allowed us to restrict attention to $t_i = 0$ or $t_i = \bar{t}$ without loss in Special Case 1, here we must make an assumption that the *choice* of t_i is restricted to the set $\{0, \bar{t}\}$ for $i \in \{A, B\}$. In other words, the firms must be fully committal in their transparency decision, either eschew transparency ($t_i = 0$) or fully embrace it ($t_i = \bar{t}$).

This assumption makes the Transparency Subgame a bimatrix game involving two players (firm A and B) and two actions per player: "Y" (i.e., $t_i = \bar{t}$) and "N" (i.e., $t_i = 0$). Table 2 provides the bimatrix description of the game.

This game is challenging to analyze because of the implicit nature of the optimal decision of the Pricing Subproblem, but we are nonetheless able to derive the following structural results.

Lemma 5 (Common payoffs under common actions) *Suppose there is no brand preference heterogeneity (that is, $\alpha = 0$). Then the profits of the two firms are equal under the outcomes YY and NN. That is,*

$$\begin{aligned}\Pi(\bar{t},\bar{t}) &:= \Pi_A(\bar{t},\bar{t}) = \Pi_B(\bar{t},\bar{t}) \\ \Pi(0,0) &:= \Pi_A(0,0) = \Pi_B(0,0)\end{aligned} \tag{13}$$

where Π denote the common profit function for the two firms when they take identical actions.

Under common actions, we get $\Delta t = 0$. Based on it, we can further show that $\Delta p = 0$. In this case, $D_A = D_B = 1$. Hence, $\Pi_A(\bar{t},\bar{t}) = \Pi_B(\bar{t},\bar{t}) = p(\bar{t},\bar{t})$ and $\Pi_A(0,0) = \Pi_B(0,0) = p(0,0)$. For detailed analysis, see Section A.4. Unlike Special Case 1, here $\Pi_A(0,\bar{t})$ may not equal $\Pi_B(0,\bar{t})$, but we still have the following symmetric property for the Transparency Subgame when $\alpha = 0$.

Lemma 6 (Common payoffs under symmetric actions) *Suppose there is no brand preference heterogeneity (that is, $\alpha = 0$). Then the profit of one firm under YN is the same as another firm under NY. That is,*

$$\begin{aligned}\Pi_A(\bar{t},0) &= \Pi_B(0,\bar{t}) \\ \Pi_B(\bar{t},0) &= \Pi_A(0,\bar{t})\end{aligned} \tag{14}$$

The lemma shows that the profits of the two firms are equal under symmetric actions (both act in the opposite transparency strategy). For detailed analysis, see Section A.5.

Lemma 7 (Conditions for equilibria in the Transparency Subgame) *Suppose there is no brand preference heterogeneity (that is, $\alpha = 0$). Then, the payoffs in the Transparency Subgame have the following properties:*

(i) There exists a n_1 such that $\Pi(0,0) > \Pi_A(\bar{t},0)$ and $\Pi(0,0) > \Pi_B(0,\bar{t})$ if and only if $\delta/\sigma < n_1$.

(ii) There exists a n_2 such that $\Pi(\bar{t},\bar{t}) > \Pi_A(0,\bar{t})$ and $\Pi(\bar{t},\bar{t}) > \Pi_B(\bar{t},0)$ if and only if $\delta/\sigma > n_2$.

where $\Pi(\cdot,\cdot)$ is as defined in (13).

This lemma gives conditions for when NN (part (i)) and YY (part (ii)) are equilibria of the bimatrix game in Table 2, in terms of the ratio δ/σ. See Section A.6 for a more detailed statement of this result. We do not have closed-form expressions for the quantities n_1 and n_2. These values, however, can be obtained numerically by solving a system of equations.

The conditions in Lemma 7 leave open the possibility that YN and NY may also be equilibria, contrary to what we saw in Special Case 1. This possibility is confirmed in the following theorem, which characterizes what equilibria are possibly in the Transparency Subgame under different values of the ratio δ/σ.

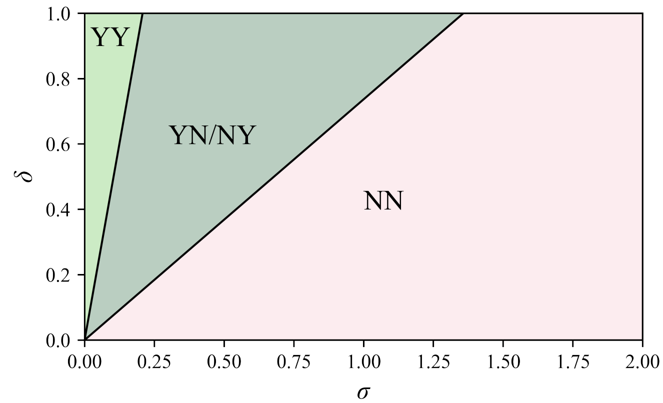

Figure 7 **An illustration of regions in the space of δ and σ that support YY or NN or YN/NY as equilibria results, when $\alpha = 0$. The figure is generated for $\bar{t} = 0.9$.**

Theorem 2 (Equilibria in the Transparency Subgame) *Suppose there is no brand preference heterogeneity (that is, $\alpha = 0$). Then the Transparency Subgame has equilibrium (t_A^*, t_B^*) whose form depends on the parameters δ and σ in the following way:*

$$(t_A^*, t_B^*) = \begin{cases} (0,0) & \text{if } \delta/\sigma \le n_1 \\ (0,\bar{t}) \text{ or } (\bar{t},0) & \text{if } n_1 < \delta/\sigma < n_2 \\ (\bar{t},\bar{t}) & \text{if } \delta/\sigma \ge n_2 \end{cases} \tag{15}$$

where n_1 and n_2 are defined in Lemma 7. That is, NN is the unique equilibrium if $\delta/\sigma < n_1$, YY is the unique equilibrium if $\delta/\sigma > n_2$ and either YN or NY can be equilibria if $n_1 < \delta/\sigma < n_2$.

The detailed analysis of Theorem 2 can be found in Section A.7. The theorem reveals that all the results are possible to be equilibria, depending on the ratio of δ/σ, illustrated in Figure 7.

We plot the following Figure 8 to show how the transparency strategies change with increasing δ. Figure 8(a) describes the initial condition NN when there is no mean-shifting effect about the firms. In Figure 8(b), δ turns from 0 to δ_1 (a small level of δ), and the equilibrium turns from NN to YN. We take the YN case to illustrate, NY is a similar logic. The incentive for firm A to accept operational transparency is that the expected value that consumers perceive will increase (from q to $q+\delta_1$) from consuming product A. However, Firm B has no incentive to follow the operational transparency strategy. Because, in this case, there is no protection from brand heterogeneity, showing the operation process will make consumers treat the two firms more similarly. At this time, keeping at least one firm non-transparency can create consumer heterogeneity of operational perception (consumers' imagination of the "difference" between the two firms). This will avoid intense competition and protect firms' profits. As we can see, YN/NY and NN occupy most of the area in Figure 7. Of course, YY is achievable if there is a huge benefit that comes with operational transparency (consumer perceived expected value from q to $q+\delta_2$), see Figure 8(c).

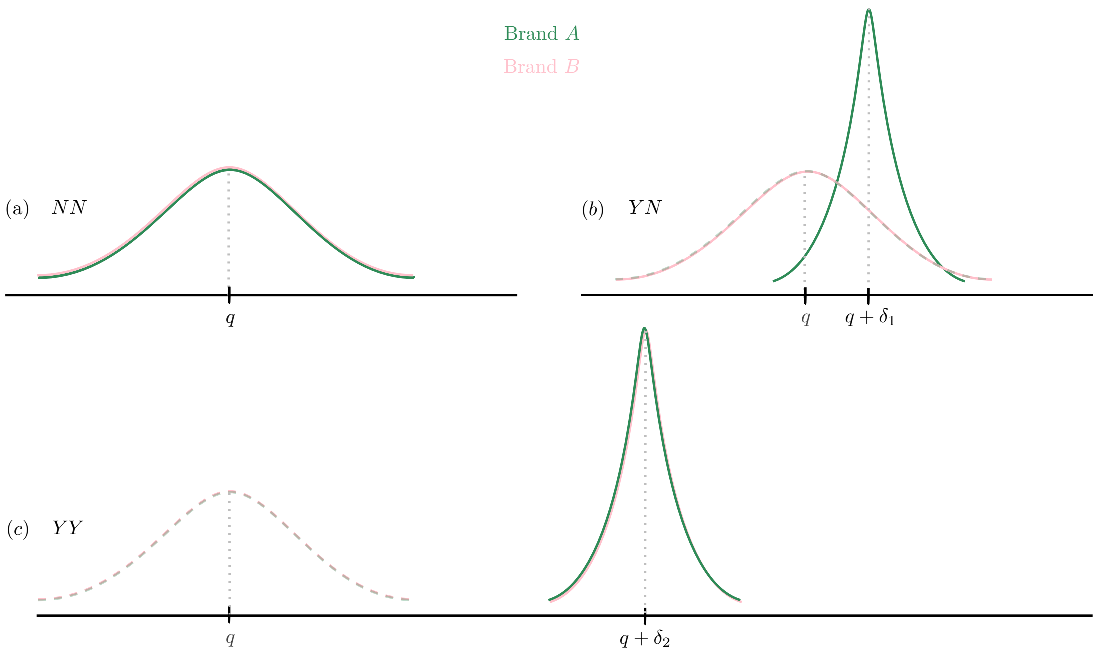

Figure 8 **An illustration of deviations depends on the value of δ. (a) illustrates that when $\delta = 0$, both firms choose N. The figure with the color green (pink) depicts the value distribution of segment A (B) customers. Figures are in the same shape because customers are homogeneous; in (b), $\delta = \delta_1$ (a slight increase in expected quality perception), only firm A chooses Y, and firm B best respond by staying N. The figure with the color green looks narrower than in (a) for decreased perception heterogeneity towards firm A; in (c), $\delta = \delta_2$ (a significant increase in expected quality perception), both firms choose Y.**

This Special Case offers insight into (Q2) that a mix of strategies (transparent vs. nontransparent) is often observed in various industries and hints why operational transparency is far from universal. We can see that the mean-shifting effect (typically discussed in existing literature) will drive both firms to do operational transparency but only when δ is quite large. Hence, it's hard for competing firms to embrace operational transparency simultaneously from the angle of the mean-shifting effect alone.

At last, we want to compare the two special cases. Theorem 1 shows that equilibrium YY is relatively easy to obtain with a slight increase of α, i.e., heterogeneity of brand preference. From Figure 4 (in Special Case 1), we can see that the slopes are quite gentle. We calculate that when $\alpha/\sigma > 0.14$, it is possible to achieve YY and when $\alpha/\sigma > 0.59$, YY is the unique equilibirum, under $\bar{t} = 0.9$. While Figure 7 (in Special Case 2) shows the steep slope (between area YY and YN/NY). We calculate that under $\bar{t} = 0.9$, only when $\delta/\sigma > 4.81$, YY is the equilibrium. Hence, the difference between the power of the two parameters (α vs. δ) to achieve YY is more than a factor of 8 times (compared with pure YY area, i.e., $\alpha/\sigma > 0.59$) and 34 times (compared with the YY/NN area, i.e., $\alpha/\sigma > 0.14$). It demonstrates that brand preference heterogeneity, i.e., α, is vital to both firms' operational transparency compared with the mean-shifting effect, i.e., δ.

4.4. General model

Let us consider all factors. That is, there is brand preference heterogeneity ($\alpha > 0$), a mean-shifting effect of operational transparency ($t_i\delta$ increases in t_i), and a variance-reduction effect of operational transparency ($(1-t_i)\sigma$ decreases in t_i). Then the utility functions of the customers are formalized as

$$U_{ij} = \begin{cases} q - p_i + \alpha + t_i\delta + (1-t_i)\epsilon_{ij} & \text{if } i = j \\ q - p_i - \alpha + t_i\delta + (1-t_i)\epsilon_{ij} & \text{if } i \neq j \end{cases} \tag{16}$$

As in the previous subsection, we will restrict the transparency choices to be 0 and $\bar{t}$ for analytical traceability, and so we analyze the bimatrix game Table 2 in the Transparency Subgame.

We are able to derive the forms for the expression of the demand functions $D_{ij}(t_A, t_B, p_A, p_B)$ in an implicit expression. We present it in the following Lemma 8 with detailed analysis in Section A.8.

Lemma 8 (Solution to the Selection Subgame) *With customer heterogeneity (that is, $\alpha > 0$) and the mean-shifting effect of operational transparency (that is, $\delta > 0$), the demand functions that result when solving the Selection Subgame are:*

$$\begin{aligned} D_{AA}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{2\alpha - \Delta p + \delta\Delta t}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right) \\ D_{AB}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{-2\alpha - \Delta p + \delta\Delta t}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right) \\ D_{BA}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{-2\alpha + \Delta p - \delta\Delta t}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right) \\ D_{BB}(p_A, p_B, t_A, t_B) &= \Phi\left(\frac{2\alpha + \Delta p - \delta\Delta t}{\sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}}\right) \end{aligned} \tag{17}$$

where $\Delta t := t_A - t_B$ and $\Delta p := p_A - p_B$.

Similar to Lemma 5, which concludes the common payoffs under common actions under Special Case 2. We have the same property under this general case. It is summarized in the following lemma.

Lemma 9 (Common payoffs under common actions) *With brand preference heterogeneity (that is, $\alpha > 0$) and the mean-shifting effect of operational transparency (that is, $\delta \neq 0$), the profits of the two firms are equal under the outcomes YY and NN. That is,*

$$\begin{aligned} \Pi(\bar{t}, \bar{t}) &:= \Pi_A(\bar{t}, \bar{t}) = \Pi_B(\bar{t}, \bar{t}) \\ \Pi(0,0) &:= \Pi_A(0,0) = \Pi_B(0,0) \end{aligned} \tag{18}$$

where Π denote the common profit function for the two firms when they take identical actions.

The detailed analysis can be found in Section A.9. Further, although the general model increases the difficulty of analysis, we can still obtain the following symmetric property.

Lemma 10 (Common payoffs under symmetric actions) *With brand preference heterogeneity (that is, $\alpha > 0$) and the mean-shifting effect of operational transparency (that is, $\delta > 0$), the profit of one firm under YN is the same as another firm under NY. That is,*

$$\begin{aligned} \Pi_A(\bar{t}, 0) &= \Pi_B(0, \bar{t}) \\ \Pi_B(\bar{t}, 0) &= \Pi_A(0, \bar{t}) \end{aligned} \tag{19}$$

The detailed analysis can be found in Section A.10.

Lemma 11 (Conditions for equilibria in the Transparency Subgame) *Under the general case, the payoffs in the Transparency Subgame have the following properties:*

(i) When σ tends to zero, we have $\lim_{\sigma \to 0} \Pi_A(\bar{t}, \bar{t}) / \Pi_A(0, \bar{t}) = +\infty$.

(ii) When σ tends to $+\infty$, we have $\lim_{\sigma \to +\infty} \Pi_A(0, 0) / \Pi_A(\bar{t}, 0) > 1$.

where $\Pi(\cdot, \cdot)$ is as defined in (18) and (19).

Lemma 11 illustrates that for firm A, when consumers almost have no perception heterogeneity of the operational process, i.e., σ closes to zero, there is an enormous benefit to adopting operational transparency (i.e., $t_A = \bar{t}$), given firm B chooses the transparency strategy. In this case, both firms occupy separate markets as a monopoly firm when they choose operational transparency. Hence, they can set a high price in this case. Also, for firm A, when consumers have tremendous perception heterogeneity of the operational process, i.e., σ closes to $+\infty$, it is better to choose non-transparency (i.e., $t_A = 0$) given firm B chooses the non-transparency strategy. Because, in this case, there is a massive variance in consumers' cognition of product differences, and keeping non-transparency equals keeping the "difference" between the two products, which can bring higher market profits. The detailed analysis of Lemma 11 can be found in Section A.11.

Theorem 3 (Equilibria in the Transparency Subgame at the Extreme Cases) *With any brand preference heterogeneity (that is, $\alpha > 0$) and the mean-shifting effect of operational transparency (that is, $\delta > 0$), the Transparency Subgame has equilibrium (t_A^*, t_B^*) whose form depends on the parameter σ in the following way:*

$$(t_A^*, t_B^*) = \begin{cases} (0, 0) & \textit{if } \sigma \textit{ tends to } +\infty \\ (\bar{t}, \bar{t}) & \textit{if } \sigma \textit{ tends to zero} \end{cases} \tag{20}$$

The theorem reveals the equilibrium results under the extreme situations of the parameter σ. The above Special Cases 1 and 2 have hinted that the increase in the variance of the subjective perception, i.e., σ, will weaken firms' willingness to adopt operational transparency (see Theorem 1 and Theorem 2). Specifically, σ indicates the ambiguity level of consumers' perception of the operation process, and the more significant the value, the greater the perception variance. In this regard, when σ tends to $+\infty$, consumers feel quite vague about the product operation process (thus producing different quality perceptions). Competing firms can use this large imaginary space to achieve the purpose of showing product "differences" between each other and easing competition. Conversely, σ tends to zero means that consumers have almost no perception difference in the operation process of the product. For brand heterogeneity exists, i.e., $\alpha > 0$, consumers only need to pay attention to the brand differences (which clearly show the difference between products) and choose their preferred products. At this point, operational transparency will bring greater benefits. The detailed analysis of Theorem 3 can be found in Section A.12.

5. Conclusion: Summary, managerial insights, and future directions

This paper has focused on how the nature of the competitive environment impacts the operational transparency of firms. Our game-theoretic model highlights three key parameters—the brand preference heterogeneity parameter α, the degree of operational perception heterogeneity σ, and the mean-shifting effect parameter δ—influence the transparency decisions of firms. We highlight how different combinations of these parameters support equilibria where both firms go transparent, stay nontransparent, or make opposite decisions. These different combinations highlight the importance of operational transparency in shielding firms from direct price competition by either going transparent to highly differentiate their offerings or staying nontransparent and allowing customers to perceive operational differences that may not be there.

Although our model and results are theoretical, our analysis nonetheless inspires some potentially useful advice. If you are a decision-maker at a firm pondering a move toward greater transparency, you might consider the following:

- *How your competitors react matters.* You should consider how your potential competitors will react to a move towards transparency. Some of the benefits of transparency discussed in the literature may be outweighed by the cost of enhanced competition.
- *How special are you to your customers?* The benefits of transparency are enhanced when transparency shows your loyal customers exactly what makes you different, solidifying their loyalty. However, if you are worried that customers can easily be persuaded to try other brands, efforts to "stand out" by going transparent may inadvertently reveal you as being more similar than different from your competitors, hurting your position in the marketplace.
- *How much of a benefit will going transparent provide us, knowing that customers will have more information about us to adjust their perceptions?* Operational transparency can have clear

benefits for the *average* perception of how customers value what your operational processes bring to your product or service, but going transparent may also serve to reduce heterogeneity in perceptions. The "boost" in average perception can be outweighed by reduced heterogeneity when this makes you look more like your competitors. Try to be sure you are getting a big "bump" by going transparent, otherwise, it might be better to let your customers hold a wider variety of beliefs about your operational processes. Maybe this variability in beliefs is what differentiates your offering in the marketplace.

The model and results we present can provide a foundation for further studies of the implications of competition for operational transparency. We discuss briefly here a few of the potential directions.

First, although this paper mostly focuses on the comparison between two simple policies (i.e., full and no transparency), partial transparency may be considered in practice. Our focus on full or no transparency was without loss in Special Case 1, but in the other two cases, it was taken as an assumption. Moreover, in practice, operational transparency takes on more than one dimension. A restaurant may reveal the process by which they make sandwiches but not where they source their ingredients. A more sophisticated model would take a multi-dimensional approach to operational transparency, which could yield fresh insights.

Second, this study explored how firms can utilize operational transparency to enhance their revenue. Our findings indicate that in highly competitive markets, prices tend to decrease. Therefore, it raises the question of whether operational transparency is always beneficial to customers. Additionally, what policies should the government adopt to achieve social efficiency with regard to operational transparency?

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Online appendix for "Operational Transparency: Showing we are different"

In the following proofs, we define $\kappa = \sigma\sqrt{(1-t_A)^2+(1-t_B)^2}$, and $\kappa > 0$.

A.1. Proof of Lemma 2

According to Lemma 1, we can calculate the profit functions as follows:

$$\begin{aligned}
\Pi_A &= p_A(D_{AA}+D_{AB}) \\
&= p_A\left[\Phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\Phi\left(\frac{-2\alpha-\Delta p}{\kappa}\right)\right], \\
\Pi_B &= p_B(D_{BA}+D_{BB}) - c\cdot t_B \\
&= p_B\left[\Phi\left(\frac{-2\alpha+\Delta p}{\kappa}\right)+\Phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right].
\end{aligned}$$

First, we examine the equilibrium prices/profits for each firm. We obtain the best-response prices by applying the first-order conditions, i.e., $\frac{\partial \Pi_A}{\partial p_A}(p_A(t_A,t_B),p_B(t_A,t_B)) = 0$ and $\frac{\partial \Pi_B}{\partial p_B}(p_A(t_A,t_B),p_B(t_A,t_B)) = 0$. Hence, we conclude that

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A} &= p_A\left(\frac{\partial D_{AA}}{\partial p_A}+\frac{\partial D_{AB}}{\partial p_A}\right)+D_{AA}+D_{AB} \\
\frac{\partial \Pi_B}{\partial p_B} &= p_B\left(\frac{\partial D_{BA}}{\partial p_B}+\frac{\partial D_{BB}}{\partial p_B}\right)+D_{BA}+D_{BB}
\end{aligned}$$

Given any t_A and t_B, $p_A(t_A,t_B)$ and $p_B(t_A,t_B)$ are defined as the equilibrium prices of firm A and firm B, respectively, and $\Delta p(t_A,t_B) = p_A(t_A,t_B) - p_B(t_A,t_B)$. For simplicity, we abuse the notations $p_A(t_A,t_B), p_B(t_A,t_B), \Delta p(t_A,t_B)$ as $\hat{p_A}, \hat{p_B}, \Delta\hat{p}$, respectively. Specifically,

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A}(\hat{p_A},\hat{p_B}) &= -\frac{\hat{p_A}}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta\hat{p}}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta\hat{p}}{\kappa}\right)\right]+\Phi\left(\frac{2\alpha-\Delta\hat{p}}{\kappa}\right)+1-\Phi\left(\frac{2\alpha+\Delta\hat{p}}{\kappa}\right)=0, \\
\frac{\partial \Pi_B}{\partial p_B}(\hat{p_A},\hat{p_B}) &= -\frac{\hat{p_B}}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta\hat{p}}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta\hat{p}}{\kappa}\right)\right]+1-\Phi\left(\frac{2\alpha-\Delta\hat{p}}{\kappa}\right)+\Phi\left(\frac{2\alpha+\Delta\hat{p}}{\kappa}\right)=0.
\end{aligned}$$

We further simplify the above two equations and get the following ones.

$$\frac{1-\Phi(x_1)+\Phi(x_2)}{\phi(x_1)+\phi(x_2)} = \frac{\hat{p_A}}{\kappa}, \tag{A.1}$$

$$\frac{1+\Phi(x_1)-\Phi(x_2)}{\phi(x_1)+\phi(x_2)} = \frac{\hat{p_B}}{\kappa}. \tag{A.2}$$

where the variables x_1 and x_2 are defined as

$$x_1 = \frac{2\alpha+\Delta\hat{p}}{\kappa}, \quad x_2 = \frac{2\alpha-\Delta\hat{p}}{\kappa}. \tag{A.3}$$

Calculating (A.1)-(A.2), we obtain

$$\frac{2\left(\Phi(x_2)-\Phi(x_1)\right)}{\phi\left(x_1\right)+\phi\left(x_2\right)}=\frac{\Delta\hat{p}}{\kappa}. \tag{A.4}$$

In order to prove that $\Delta\hat{p}=0$ in this case, we will use a proof by contradiction. Assuming $\Delta\hat{p}\neq 0$, based on equations (A.3), we can establish the following argument: (x_2-x_1) has the opposite sign of $\Delta\hat{p}$ due to the relationship

$$(x_2-x_1)\Delta\hat{p}=-\frac{2(\Delta\hat{p})^2}{\kappa}<0. \tag{A.5}$$

For $\Phi(x)$ is an increasing function of x, we can get that when $x_1\neq x_2$,

$$(x_2-x_1)\left(\Phi(x_2)-\Phi(x_1)\right)>0. \tag{A.6}$$

Combining (A.5) and (A.6), if $\Delta\hat{p}\neq 0$, we have

$$\left(\Phi(x_2)-\Phi(x_1)\right)\Delta\hat{p}<0.$$

This implies that the sign of the left-hand side (LHS) of Equation (A.4) is opposite to that of the right-hand side (RHS) of Equation (A.4). Consequently, Equation (A.4) holds only if $\Delta\hat{p}=0$, indicating that $\hat{p_A}=\hat{p_B}$. By substituting $\Delta\hat{p}=0$ into Equations (A.1) and (A.2), we obtain

$$\hat{p_A}=\hat{p_B}=\frac{\kappa}{2\phi\left(\frac{2\alpha}{\kappa}\right)}.$$

Namely,

$$p_A(t_A,t_B)=p_B(t_A,t_B)=\frac{\kappa}{2\phi\left(\frac{2\alpha}{\kappa}\right)}=\frac{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}{2\phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)}.$$

The uniqueness of price equilibrium is placed later, and combined with this, the proof of Lemma 2(i) is complete.

Based on Lemma 1 and the condition $\Delta\hat{p}=0$, the equilibrium demand in each segment can be expressed as follows:

$$D_{AA}(t_A,t_B)=D_{BB}(t_A,t_B)=\Phi\left(\frac{2\alpha}{\kappa}\right)=\Phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right),$$

$$D_{AB}(t_A,t_B)=D_{BA}(t_A,t_B)=\Phi\left(\frac{-2\alpha}{\kappa}\right)=\Phi\left(\frac{-2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right).$$

Hence, the equilibrium demand for firm A and firm B are given by:

$$D_A(t_A,t_B)=D_{AA}(t_A,t_B)+D_{AB}(t_A,t_B)=1$$

$$D_B(t_A,t_B)=D_{BB}(t_A,t_B)+D_{BA}(t_A,t_B)=1.$$

Then, the proof of Lemma 2(ii) is complete.

Finally, in the equilibrium, both firm A and firm B experience the same demand of one and achieve the same profit. The profit of firm A and firm B is the same as the price. That is,

$$\Pi_A(t_A,t_B)=\Pi_B(t_A,t_B)=\frac{\kappa}{2\phi\left(\frac{2\alpha}{\kappa}\right)}=\frac{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}{2\phi\left(\frac{2\alpha}{\sigma\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)}.$$

The proof of Lemma 2(iii) is complete.

At last, we establish the uniqueness of equilibrium prices $\hat{p_A}$ and $\hat{p_B}$ to complete the proof Lemma 2(i). According to Milgrom and Roberts (1990), a unique Nash Equilibrium $\hat{p_A},\hat{p_B}$ can be guaranteed if the following conditions hold:

$$\begin{aligned}\frac{\partial\Pi_A^2}{\partial^2 p_A}+\frac{\partial\Pi_A^2}{\partial p_A\partial p_B}<0,\\ \frac{\partial\Pi_B^2}{\partial^2 p_B}+\frac{\partial\Pi_B^2}{\partial p_B\partial p_A}<0.\end{aligned}$$

The first derivative of Π_A is given by the following expression:

$$\frac{\partial\Pi_A}{\partial p_A}=-\frac{p_A}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right]+\Phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+1-\Phi\left(\frac{2\alpha+\Delta p}{\kappa}\right).$$

Additionally, the second-order derivative of Π_A is as follows:

$$\begin{aligned}\frac{\partial\Pi_A^2}{\partial^2 p_A}=&-\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right]-\frac{p_A}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)\frac{2\alpha-\Delta p}{\kappa^2}-\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\frac{2\alpha+\Delta p}{\kappa^2}\right]\\&-\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right],\end{aligned}$$

and the cross-partial derivative with respect to p_A and p_B is given by:

$$\frac{\partial\Pi_A^2}{\partial p_A\partial p_B}=-\frac{p_A}{\kappa}\left[-\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)\frac{2\alpha-\Delta p}{\kappa^2}+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\frac{2\alpha+\Delta p}{\kappa^2}\right]+\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right].$$

Hence, we find

$$\frac{\partial\Pi_A^2}{\partial^2 p_A}+\frac{\partial\Pi_A^2}{\partial p_A\partial p_B}=-\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right]<0.$$

Likewise, we proceed to analyze the first and second derivatives of p_B. The first derivative of p_B is expressed as:

$$\frac{\partial\Pi_B}{\partial p_B}=-\frac{p_B}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right]+1-\Phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\Phi\left(\frac{2\alpha+\Delta p}{\kappa}\right).$$

Furthermore, the second-order derivative of Π_B is given by:

$$\begin{aligned}\frac{\partial\Pi_B^2}{\partial^2 p_B}=&-\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right]-\frac{p_B}{\kappa}\left[-\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)\frac{2\alpha-\Delta p}{\kappa^2}+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\frac{2\alpha+\Delta p}{\kappa^2}\right]\\&-\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)+\phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right],\end{aligned}$$

while the cross-partial derivative with respect to p_B and p_A is denoted as:

$$\frac{\partial \Pi_A^2}{\partial p_B \partial p_A} = -\frac{p_B}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right)\frac{2\alpha-\Delta p}{\kappa^2} - \phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\frac{2\alpha+\Delta p}{\kappa^2}\right] + \frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right) + \phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right].$$

Thus, we can determine that

$$\frac{\partial \Pi_B^2}{\partial^2 p_B} + \frac{\partial \Pi_B^2}{\partial p_B \partial p_A} = -\frac{1}{\kappa}\left[\phi\left(\frac{2\alpha-\Delta p}{\kappa}\right) + \phi\left(\frac{2\alpha+\Delta p}{\kappa}\right)\right] < 0.$$

These inequalities confirm the satisfaction of the required condition for the uniqueness of the Nash equilibrium $\hat{p_A}, \hat{p_B}$ within the context of the examined scenario. Therefore, we have completed the proof. □

A.2. Proof of Lemma 3

We analyze the monotonicity of the profit function with respect to the transparency degree. Recall that $\Pi_i(t_A, t_B) = \frac{\kappa}{2\phi\left(\frac{2\alpha}{\kappa}\right)}, i = \{A, B\}$. Taking the derivative of $\Pi_A(t_A, t_B)$ and $\Pi_B(t_A, t_B)$ with respect to t_A and t_B, respectively. We get:

$$\begin{aligned}\frac{\partial \Pi_A(t_A, t_B)}{\partial t_A} &= \frac{(1-t_A)\sigma^2(4\alpha^2-\kappa^2)}{2\kappa^3\phi\left(\frac{2\alpha}{\kappa}\right)},\\ \frac{\partial \Pi_B(t_A, t_B)}{\partial t_B} &= \frac{(1-t_B)\sigma^2(4\alpha^2-\kappa^2)}{2\kappa^3\phi\left(\frac{2\alpha}{\kappa}\right)}.\end{aligned}$$

Therefore, the signs of $\frac{\partial \Pi_A(t_A,t_B)}{\partial t_A}$ and $\frac{\partial \Pi_B(t_A,t_B)}{\partial t_B}$ are consistent with the sign of $4\alpha^2 - \kappa^2$, where $\kappa = \sigma\sqrt{(1-t_A)^2 + (1-t_B)^2}$. Note that $t_A, t_B \in [0, \bar{t}]$. We conclude that:

- If $\alpha/\sigma \geq \frac{\sqrt{2}}{2}$, $\Pi_A(\Pi_B)$ is increasing in $t_A(t_B) \in [0, \bar{t}]$.
- If $\alpha/\sigma \leq \frac{\sqrt{2}}{2}(1-\bar{t})$, $\Pi_A(\Pi_B)$ is decreasing in $t_A(t_B) \in [0, \bar{t}]$.
- If $\alpha/\sigma \in \left(\frac{\sqrt{2}}{2}(1-\bar{t}), \frac{\sqrt{2}}{2}\right)$, we have a more nuanced result. Specifically, Π_A is decreasing in t_A for $t_A \in \left[0, 1-\sqrt{\frac{4\alpha^2}{\sigma^2} - (1-t_B)^2}\right]$ and increasing in t_A for $t_A \in \left(1-\sqrt{\frac{4\alpha^2}{\sigma^2} - (1-t_B)^2}, \bar{t}\right]$. Similarly, Π_B is decreasing in t_B for $t_B \in \left[0, 1-\sqrt{\frac{4\alpha^2}{\sigma^2} - (1-t_A)^2}\right]$ and increasing in t_B for $t_B \in \left(1-\sqrt{\frac{4\alpha^2}{\sigma^2} - (1-t_A)^2}, \bar{t}\right]$.

Depending on the value of α/σ, the payoff function $\Pi_A(\Pi_B)$ exhibits monotonicity properties concerning $t_A(t_B)$. Hence, the optimal value will be obtained at the endpoint 0 or $\bar{t}$. Finally, the proof is complete. □

A.3. Proof of Theorem 1

Denote $m = \alpha/\sigma$. The profit function of the firm A and firm B can be expressed as:

$$\Pi_A(t_A, t_B) = \Pi_B(t_A, t_B) = \frac{\kappa}{2\phi\left(\frac{2m}{\sqrt{(1-t_A)^2+(1-t_B)^2}}\right)} = \frac{\kappa\sqrt{2\pi}}{2} e^{\frac{2m^2}{(1-t_A)^2+(1-t_B)^2}}.$$

Since t_A and t_B are interchangeable, the best response of firm B will be the same as that of firm A. Without loss of generality, we can focus on the best response of firm A. Let's first consider the case when firm B takes maximal transparency, denoted as "Y", i.e., $t_B = \bar{t}$. We can derive the best response of firm A by comparing $\Pi_A(\bar{t},\bar{t})$ and $\Pi_A(0,\bar{t})$. Namely, the profits of firm A under strategy "Y" (taking maximal transparency) and "N" (taking minimal transparency):

$$\Pi_A(\bar{t},\bar{t}) = \frac{\sqrt{2\pi}}{2}\left(\sigma\sqrt{2}(1-\bar{t})\right)e^{\frac{m^2}{(1-\bar{t})^2}},$$

$$\Pi_A(0,\bar{t}) = \frac{\sqrt{2\pi}}{2}\left(\sigma\sqrt{1+(1-\bar{t})^2}\right)e^{\frac{2m^2}{1+(1-\bar{t})^2}}.$$

Introducing $k_1 = \frac{\sqrt{2}(1-\bar{t})}{\sqrt{1+(1-\bar{t})^2}}$, where $\bar{t} \in (0,1)$. We establish that $k_1 < 1$. Then, we get

$$\frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})} = k_1 e^{\frac{m^2(1-k_1^2)}{(1-\bar{t})^2}}.$$

Hence, $\Pi_A(\bar{t},\bar{t}) \geq \Pi_A(0,\bar{t})$ holds if and only if:

$$m > (1-\bar{t})\sqrt{\frac{\ln k_1}{k_1^2-1}} = (1-\bar{t})\sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{1-(1-\bar{t})^2}}.$$

Denote $m_1 = (1-\bar{t})\sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{1-(1-\bar{t})^2}}$. We get

$$\Pi_A(\bar{t},\bar{t}) > \Pi_A(0,\bar{t}) \quad \text{iff} \quad m > m_1. \tag{A.7}$$

Here, m_1 serves as the critical threshold between the value $\Pi_A(\bar{t},\bar{t})$ and $\Pi_A(0,\bar{t})$.

Next, we consider the case when firm B has chosen "N" (taking minimal transparency), i.e., $t_B = 0$. Then, we can derive the best response of firm A by comparing $\Pi_A(0,0)$ and $\Pi_A(\bar{t},0)$. We have

$$\Pi_A(0,0) = \frac{\sqrt{2\pi}}{2}\left(\sigma\sqrt{2}\right)e^{m^2},$$

$$\Pi_A(\bar{t},0) = \frac{\sqrt{2\pi}}{2}\left(\sigma\sqrt{1+(1-\bar{t})^2}\right)e^{\frac{2m^2}{1+(1-\bar{t})^2}}.$$

Introducing $k_2 = \sqrt{\frac{2}{1+(1-\bar{t})^2}}$, where $\bar{t} \in (0,1)$, we establish that $k_2 > 1$. Then, we conclude that

$$\frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)} = k_2 e^{m^2(1-k_2^2)}.$$

Therefore, $\Pi_A(0,0) > \Pi_A(\bar{t},0)$ holds if and only if:

$$m < \sqrt{\frac{\ln k_2}{k_2^2-1}} = \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{2}{1+(1-\bar{t})^2}}}{1-(1-\bar{t})^2}}.$$

Denote $m_2 = \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{2}{1+(1-\bar{t})^2}}}{1-(1-\bar{t})^2}}$. We conclude that:

$$\Pi_A(0,0) > \Pi_A(\bar{t},0) \quad \text{iff} \quad m < m_2. \tag{A.8}$$

Here, m_2 represents the critical threshold between the value $\Pi_A(0,0)$ and $\Pi_A(\bar{t},0)$.

To determine the equilibrium, we need to compare m_1 and m_2. Specifically,

$$\begin{aligned} m_1 &= (1-\bar{t})\sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{1-(1-\bar{t})^2}} \\ &= \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{\frac{1-(1-\bar{t})^2}{(1-\bar{t})^2}}} \\ &= \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}}}{2\left[\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2}-1\right]}} \end{aligned} \tag{A.9}$$

$$\begin{aligned} m_2 &= \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{2}{1+(1-\bar{t})^2}}}{1-(1-\bar{t})^2}} \\ &= \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2}}}{(1-\bar{t})^2-1}} \\ &= \sqrt{\frac{\left[1+(1-\bar{t})^2\right]\ln\sqrt{\frac{1+(1-\bar{t})^2}{2}}}{2\left[\frac{1+(1-\bar{t})^2}{2}-1\right]}} \end{aligned} \tag{A.10}$$

It is equal to compare (A.9) and (A.10). For function $f(x) = \frac{\ln x}{(x^2-1)}$ is decreasing in x, for any $x>0$, and $\frac{1+(1-\bar{t})^2}{2(1-\bar{t})^2} > \frac{1+(1-\bar{t})^2}{2}$, we conclude that

$$m_1 < m_2. \tag{A.11}$$

Then, we can derive the equilibrium results of firms' operational transparency strategies with respect to the value $m = \alpha/\sigma$. Combining inequalities (A.7), (A.8), and the relationship of $m_1 < m_2$, we conclude the following results.

- When $m \le m_1$, we get that

$$\Pi_A(0,\bar{t}) > \Pi_A(\bar{t},\bar{t}) \ \text{ and } \ \Pi_A(0,0) > \Pi_A(\bar{t},0).$$

  In other words, irrespective of firm B opting for "Y" (the maximum transparency) or "N" (the minimum transparency), firm A will choose "N" as its optimal strategy. Since t_A and t_B are interchangeable, the best response of firm B will be the same as that of firm A. The equilibrium turns out to be NN.

- When $m_1 < m < m_2$, we get that

$$\Pi_A(\bar{t},\bar{t}) > \Pi_A(0,\bar{t}) \quad \text{and} \quad \Pi_A(0,0) > \Pi_A(\bar{t},0).$$

Namely, if firm B selects "Y", firm A will also choose "Y", and if firm B selects "N", firm A will choose "N". Finally, it leads to the equilibrium strategies of YY or NN. To further analyze the equilibrium, we consider the difference in profits between YY and NN. That is,

$$\Pi(0,0) - \Pi(\bar{t},\bar{t}) = \frac{\sigma}{\sqrt{2}}\left(\frac{1}{\phi(\sqrt{2}m)} - \frac{1-\bar{t}}{\phi(\frac{\sqrt{2}m}{1-\bar{t}})}\right).$$

We observe that this expression is an increasing function of m. Moreover, we can identify the root of $\Pi(0,0) - \Pi(\bar{t},\bar{t})$ as $m_0 = (1-\bar{t})\sqrt{\frac{\ln(1-\bar{t})}{(1-\bar{t})^2-1}}$. We can calculate that $m_1 < m_0 < m_2$. If $m_0 < m < m_2$, the equilibrium operational transparency strategy YY is favored by both firms. On the other hand, if $m_1 < m < m_0$, the equilibrium strategies NN will be preferred.

- When $m \geq m_2$, we get that

$$\Pi_A(\bar{t},\bar{t}) > \Pi_A(0,\bar{t}) \quad \text{and} \quad \Pi_A(\bar{t},0) > \Pi_A(0,0).$$

That is, regardless of whether firm B chooses "Y" (the maximum transparency) or "N" (the minimum transparency), firm A will choose "Y" as its optimal strategy. The equilibrium turns out to be YY.

Hence, the proof is complete. $\square$

A.4. Proof of Lemma 5

When there is no brand heterogeneity, i.e., $\alpha = 0$, the profits are given by

$$\begin{aligned}\Pi_A &= p_A(D_{AA} + D_{AB}) = 2p_A\Phi\left(\frac{\delta\Delta t - \Delta p}{\kappa}\right),\\ \Pi_B &= p_B(D_{BA} + D_{BB}) = 2p_B\Phi\left(\frac{-\delta\Delta t + \Delta p}{\kappa}\right).\end{aligned}$$

By taking the derivative of Π_i with respect to p_i, $i = \{A, B\}$, we obtain the following expressions:

$$\begin{aligned}\frac{\partial \Pi_A}{\partial p_A} &= 2\left[\Phi\left(\frac{\delta\Delta t - \Delta p}{\kappa}\right) - \frac{p_A}{\kappa}\phi\left(\frac{\delta\Delta t - \Delta p}{\kappa}\right)\right],\\ \frac{\partial \Pi_B}{\partial p_B} &= 2\left[\Phi\left(\frac{-\delta\Delta t + \Delta p}{\kappa}\right) - \frac{p_B}{\kappa}\phi\left(\frac{\delta\Delta t - \Delta p}{\kappa}\right)\right].\end{aligned}$$

As defined in the main text, $p_A(t_A,t_B), p_B(t_A,t_B)$ are the equilibrium prices of firm A and firm B, given any value of t_A and t_B, and $\Delta p(t_A,t_B) = p_A(t_A,t_B) - p_B(t_A,t_B)$. For simplicity, we slightly abuse the notations $p_A(t_A,t_B)$, $p_B(t_A,t_B)$, $\Delta p(t_A,t_B)$ as $\hat{p_A}$, $\hat{p_B}$, $\Delta\hat{p}$, respectively. Denote

$$x = \frac{\delta\Delta t - \Delta\hat{p}}{\kappa}. \tag{A.12}$$

Then, the equilibrium conditions $\frac{\partial \Pi_A}{\partial p_A}(\hat{p_A},\hat{p_B})=0$ and $\frac{\partial \Pi_B}{\partial p_B}(\hat{p_A},\hat{p_B})=0$ can be rewritten as follows:

$$\frac{\Phi(x)}{\phi(x)}=\frac{\hat{p_A}}{\kappa}, \text{ and } \frac{\Phi(-x)}{\phi(x)}=\frac{\hat{p_B}}{\kappa}. \tag{A.13}$$

Furthermore, the equation $\frac{\partial \Pi_A}{\partial p_A}(\hat{p_A},\hat{p_B})-\frac{\partial \Pi_B}{\partial p_B}(\hat{p_A},\hat{p_B})=0$ can be equivalently expressed as:

$$\frac{2\Phi(x)-1}{\phi(x)}=\frac{\Delta\hat{p}}{\kappa}. \tag{A.14}$$

Based on Equation (A.13), we can express the equilibrium profits as follows:

$$\begin{aligned}\Pi_A&=2\hat{p_A}\Phi(x)=2\kappa\frac{\Phi^2(x)}{\phi(x)},\\ \Pi_B&=2\hat{p_B}\Phi(-x)=2\kappa\frac{\Phi^2(-x)}{\phi(x)}.\end{aligned} \tag{A.15}$$

Here, we consider four sub-games, and we use the superscripts $\{NN,YY,NY,YN\}$ to denote each sub-game. Similarly, we use x^{ij}, $i,j\in\{Y,N\}$ to denote the x under each sub-game. Also, using Δp^{ij}, κ^{ij}, $i,j\in\{Y,N\}$ to denote the Δp and κ under each sub-game.

- **Sub-game : NN**

  In this case, both firms choose to be non-transparent, i.e., $t_A=t_B=0$, then Equation (A.12) reduces to $x^{NN}=-\frac{\Delta p^{NN}}{\sqrt{2}\sigma}$. Based on it, the formula for the first-order condition (A.14) is equivalent to

$$\frac{2\Phi(x^{NN})-1}{\phi(x^{NN})}=-x^{NN}.$$

  Define $g(x)=\frac{2\Phi(x)-1}{\phi(x)}+x$, and it is increasing in x with $g(0)=0$. Hence, we have $x^{NN}=0$, and correspondingly $\Delta p^{NN}=0$. According to (A.15), the profits of firm A and firm B are given by

$$\Pi(0,0):=\Pi_A(0,0)=\Pi_B(0,0)=2\sigma\sqrt{2}\frac{\Phi^2(0)}{\phi(0)}. \tag{A.16}$$

- **Sub-game : YY**

  In this case, both firms choose to be fully transparent, i.e., $t_A=t_B=\bar{t}$, then Equation (A.12) reduces to $x^{YY}=-\frac{\Delta p^{YY}}{\sqrt{2}(1-\bar{t})\sigma}$, and the first-order condition (A.14) is equivalent to

$$\frac{2\Phi(x^{YY})-1}{\phi(x^{YY})}=-x^{YY}.$$

  By the same token, we have $x^{YY}=0$ and $\Delta p^{YY}=0$. According to (A.15), the profit of firm A and firm B are given by

$$\Pi(\bar{t},\bar{t}):=\Pi_A(\bar{t},\bar{t})=\Pi_B(\bar{t},\bar{t})=2\sigma\sqrt{2}(1-\bar{t})\frac{\Phi^2(0)}{\phi(0)}. \tag{A.17}$$

Hence, the proof is complete. □

A.5. Proof of Lemma 6

- **Sub-game : NY**

  In this case, firm A chooses not to be transparent while firm B chooses to be transparent, i.e., $t_A = 0, t_B = \bar{t}$. Then Equation (A.12) turns to be $x^{NY} = \frac{-\delta\bar{t} - \Delta p^{NY}}{\sigma\sqrt{1+(1-\bar{t})^2}}$, and the first-order condition (A.14) is equivalent to

$$\frac{2\Phi(x^{NY})-1}{\phi(x^{NY})} = \frac{-\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}} - x^{NY}.$$

  As defined, $g(x) = \frac{2\Phi(x)-1}{\phi(x)} + x$, we can treat x^{NY} as the root of

$$g(x) = -\frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}. \tag{A.18}$$

  Since $g(x) = \frac{2\Phi(x)-1}{\phi(x)} + x$ is increasing in x, $g(0) = 0$, and $-\frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}$ is treated as a value and irrelevant to x, there is a unique x^{NY} satisfying (A.18). Inserting x^{NY} to (A.15), the equilibrium profits under sub-game NY are

$$\begin{aligned}\Pi_A(0,\bar{t}) &= 2\sigma\sqrt{1+(1-\bar{t})^2}\frac{\Phi^2(x^{NY})}{\phi(x^{NY})},\\ \Pi_B(0,\bar{t}) &= 2\sigma\sqrt{1+(1-\bar{t})^2}\frac{\Phi^2(-x^{NY})}{\phi(x^{NY})}.\end{aligned} \tag{A.19}$$

- **Sub-game : YN**

  In this case, firm A chooses to be transparent while firm B chooses not to be transparent, i.e., $t_A = \bar{t}, t_B = 0$. Then Equation (A.12) turns to be $x^{YN} = \frac{\delta\bar{t} - \Delta p^{YN}}{\sigma\sqrt{1+(1-\bar{t})^2}}$, and the first-order condition (A.14) is equivalent to

$$\frac{2\Phi(x^{YN})-1}{\phi\left(x^{YN}\right)} = \frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}} - x^{YN}.$$

  Recall that $g(x) = \frac{2\Phi(x)-1}{\phi(x)} + x$. We get $g(-x) = -g(x)$. Based on it, we conclude that $x^{YN} = -x^{NY}$. Then, the equilibrium profits are

$$\begin{aligned}\Pi_A(\bar{t},0) &= 2\sigma\sqrt{1+(1-\bar{t})^2}\frac{\Phi^2(x^{YN})}{\phi\left(x^{YN}\right)} = \Pi_B(0,\bar{t}),\\ \Pi_B(\bar{t},0) &= 2\sigma\sqrt{1+(1-\bar{t})^2}\frac{\Phi^2(-x^{YN})}{\phi\left(x^{YN}\right)} = \Pi_A(0,\bar{t}).\end{aligned} \tag{A.20}$$

Hence, the proof is complete. □

A.6. Proof of Lemma 7

We summarize the profits under different sub-games YY, YN, NY, and NN in Table 3. From Table 3, given firm A chooses N, whether firm B chooses Y or N is to compare $\Pi_B(0,\bar{t})$ and $\Pi(0,0)$. It is equivalent to observing the sign of

$$\frac{\Pi_B(0,\bar{t}) - \Pi(0,0)}{2\sigma} = \sqrt{1+(1-\bar{t})^2}\frac{\Phi^2\left(-x^{NY}\right)}{\phi\left(x^{NY}\right)} - \sqrt{2}\frac{\Phi^2(0)}{\phi(0)}. \tag{A.21}$$

		Firm B	
		Y	N
Firm A	Y	$\Pi(\bar{t},\bar{t})$, $\Pi(\bar{t},\bar{t})$	$\Pi_A(\bar{t},0)$, $\Pi_B(\bar{t},0)$
	N	$\Pi_A(0,\bar{t})$, $\Pi_B(0,\bar{t})$	$\Pi(0,0)$, $\Pi(0,0)$

Table 3 A bimatrix game representation of the Transparency Subgame

Recall that $g(x) = \frac{2\Phi(x)-1}{\phi(x)} + x$, and as defined in the above Sub-game: NY, x^{NY} is the unique root of

$$g(x) = -\frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}, \tag{A.22}$$

and $g(0) = 0$. Denote $n = \delta/\sigma$, when n increases, the right side $-\frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}$ decreases. Then, the unique root x^{NY} of (A.22) will also decrease. Hence, x^{NY} is decreasing in n. Combing the fact that $\frac{\Phi^2(-x)}{\phi(x)}$ decreases in x, we have

$$\frac{\partial \frac{\Phi^2(-x^{NY})}{\phi(x^{NY})}}{\partial n} = \frac{\partial \frac{\Phi^2(-x^{NY})}{\phi(x^{NY})}}{\partial x^{NY}} \cdot \frac{\partial x^{NY}}{\partial n} > 0.$$

Hence, formula (A.21) increases in n, we can conclude that there is a unique root of $\Pi_B(0,\bar{t}) - \Pi(0,0) = 0$. We denote it as n_1 and $\Pi_B(0,\bar{t}) \geq \Pi(0,0)$ iff $n \geq n_1$.

By the same token, given firm B chooses Y, whether firm A will choose N or Y is to compare $\Pi_A(0,\bar{t})$ and $\Pi(\bar{t},\bar{t})$. It is equivalent to observing the sign of

$$\frac{\Pi_A(0,\bar{t}) - \Pi(\bar{t},\bar{t})}{2\sigma} = \sqrt{1+(1-\bar{t})^2}\frac{\Phi^2(x^{NY})}{\phi(x^{NY})} - \sqrt{2}(1-\bar{t})\frac{\Phi^2(0)}{\phi(0)}. \tag{A.23}$$

Since $\frac{\Phi^2(x)}{\phi(x)}$ increases in x and x^{NY} decreases in n, we have

$$\frac{\partial \frac{\Phi^2(x^{NY})}{\phi(x^{NY})}}{\partial n} = \frac{\partial \frac{\Phi^2(x^{NY})}{\phi(x^{NY})}}{\partial x^{NY}} \cdot \frac{\partial x^{NY}}{\partial n} < 0.$$

Hence, (A.23) decreases in n. We can conclude that there is a unique root of $\Pi_A(0,\bar{t}) - \Pi(\bar{t},\bar{t}) = 0$. We denote it as n_2 and $\Pi(\bar{t},\bar{t}) \geq \Pi_A(0,\bar{t})$ iff $n \geq n_2$.

Specifically, according to (A.22), we conclude that

$$n_i = \left(\frac{1-2\Phi(\bar{x}_i)}{\phi(\bar{x}_i)} - \bar{x}_i\right)\frac{\sqrt{1+(1-\bar{t})^2}}{\bar{t}}, \quad i = \{1,2\} \tag{A.24}$$

and based on (A.21) and (A.23), $\bar{x}_1$ and $\bar{x}_2$ satisfy the following equations

$$\frac{\Phi^2(-\bar{x}_1)}{\phi(\bar{x}_1)} = \sqrt{\frac{2}{1+(1-\bar{t})^2}}\frac{\Phi^2(0)}{\phi(0)}, \quad \frac{\Phi^2(\bar{x}_2)}{\phi(\bar{x}_2)} = \sqrt{\frac{2}{1+(1-\bar{t})^2}}(1-\bar{t})\frac{\Phi^2(0)}{\phi(0)}, \tag{A.25}$$

respectively. Hence, the proof is complete. $\square$

A.7. Proof of Theorem 2

First, we need to compare n_1 and n_2 and prove that $n_1 < n_2$. Recall that

$$n_i = \left(\frac{1-2\Phi(\bar{x}_i)}{\phi(\bar{x}_i)} - \bar{x}_i\right)\frac{\sqrt{1+(1-\bar{t})^2}}{\bar{t}}, \quad i=\{1,2\}. \tag{A.26}$$

Define

$$n(x) = \frac{1-2\Phi(x)}{\phi(x)} - x.$$

We have $n(x)$ is decreasing in $x<0$ due to $n'(x) = \frac{x(1-2\Phi(x))}{\phi(x)} - 3 < 0$. Hence, proving $n_1 < n_2$ is equal to prove $\bar{x}_1 > \bar{x}_2$. As we know, $\bar{x}_1$ and $\bar{x}_2$ satisfy the following equations

$$\frac{\Phi^2(-\bar{x}_1)}{\phi(\bar{x}_1)} = \sqrt{\frac{2}{1+(1-\bar{t})^2}}\frac{\Phi^2(0)}{\phi(0)}, \quad \frac{\Phi^2(\bar{x}_2)}{\phi(\bar{x}_2)} = \sqrt{\frac{2}{1+(1-\bar{t})^2}}(1-\bar{t})\frac{\Phi^2(0)}{\phi(0)}. \tag{A.27}$$

Define function $f(x) = \frac{\Phi^2(x)}{\phi(x)}$. Taking the derivative of $f(x)$ with respect to x gives

$$f'(x) = \frac{\Phi(x)}{\phi(x)}\left(x\Phi(x) + 2\phi(x)\right).$$

Further, taking the derivative of $f'(x)$ with respect to x gives

$$f''(x) = 2\left(x\Phi(x)+\phi(x)\right) + \frac{\Phi^2(x)(1+x^2)}{\phi(x)}.$$

Denote $g(x) = x\Phi(x)+\phi(x)$. Taking the derivative of $g(x)$ gives

$$g'(x) = \Phi(x) > 0.$$

So $g(x)$ is increasing in x. According to L'Hôpital's rule, $\lim_{x\to-\infty} x\Phi(x) = \lim_{x\to-\infty}\frac{\Phi(x)}{1/x} = \lim_{x\to-\infty} -\frac{\phi(x)}{\frac{1}{x^2}} = \lim_{x\to-\infty} -x^2\phi(x) = 0$. Hence, we have $g(x)\to 0$ when $x\to-\infty$. So $g(x)>0$, which also means that

$$f'(x) = \frac{\Phi(x)}{\phi(x)}\left(g(x)+\phi(x)\right) > 0$$

and

$$f''(x) = 2g(x) + \frac{\Phi^2(x)(1+x^2)}{\phi(x)} > 0.$$

We conclude that $f(x)$ is an increasing convex function. For convexity, we have

$$f(x) + f(-x) > 2\cdot f(0)$$

Then, we get

$$\begin{aligned}
\frac{\Phi^2(\bar{x}_1)}{\phi(\bar{x}_1)} &> 2\frac{\Phi^2(0)}{\phi(0)} - \frac{\Phi^2(-\bar{x}_1)}{\phi(\bar{x}_1)} \\
&= \left(2 - \sqrt{\frac{2}{1+(1-\bar{t})^2}}\right)\frac{\Phi^2(0)}{\phi(0)} \\
&\geq \sqrt{\frac{2}{1+(1-\bar{t})^2}}(1-\bar{t})\frac{\Phi^2(0)}{\phi(0)} \\
&= \frac{\Phi^2(\bar{x}_2)}{\phi(\bar{x}_2)}
\end{aligned}$$

The inequality holds due to

$$
\begin{aligned}
& 2-\sqrt{\frac{2}{1+(1-\bar{t})^2}} \geq \sqrt{\frac{2}{1+(1-\bar{t})^2}}(1-\bar{t}) \\
\iff & \sqrt{\frac{2}{1+(1-\bar{t})^2}} \leq \frac{2}{2-\bar{t}} \\
\iff & \frac{1}{1+(1-\bar{t})^2} \leq \frac{2}{\left(1+(1-\bar{t})\right)^2} \\
\iff & 2+2(1-\bar{t})^2 \geq 1+(1-\bar{t})^2+2(1-\bar{t}) \\
\iff & (1-\bar{t})^2-2(1-\bar{t})+1 \geq 0.
\end{aligned}
$$

Hence, $\bar{x}_1 > \bar{x}_2$ for $f(x)=\frac{\Phi^2(x)}{\phi(x)}$ increases in x. Namely, $n_1 < n_2$ is proved.

Next, we consider n in the following three scenarios.

- When $n \leq n_1$, we have $\Pi(0,0) \geq \Pi_B(0,\bar{t})$ and $\Pi_A(0,\bar{t}) \geq \Pi(\bar{t},\bar{t})$. Irrespective of the choice made by the other firm, the firm will always choose N. So both firms choose N, and the equilibrium result will be NN.
- When $n_1 < n < n_2$, we have $\Pi_B(0,\bar{t}) > \Pi(0,0)$ and $\Pi_A(0,\bar{t}) > \Pi(\bar{t},\bar{t})$. If one of the firms chooses Y of N, the other firm will make the opposite choice. Therefore, the equilibrium result will be YN of NY.
- When $n \geq n_2$, we have $\Pi_B(0,\bar{t}) \geq \Pi(0,0)$ and $\Pi(\bar{t},\bar{t}) \geq \Pi_A(0,\bar{t})$. Irrespective of the choice made by the other firm, the firm will always choose Y. So both firms choose Y, and the equilibrium result will be YY. □

A.8. Proof of Lemma 8

When $\alpha > 0, \delta \neq 0$, the utility function is given by

$$
\begin{aligned}
U_{AA} &= q-p_A+\alpha+t_A\delta+(1-t_A)\epsilon, \\
U_{BA} &= q-p_B-\alpha+t_B\delta+(1-t_B)\epsilon, \\
U_{AB} &= q-p_A-\alpha+t_A\delta+(1-t_A)\epsilon, \\
U_{BB} &= q-p_B+\alpha+t_B\delta+(1-t_B)\epsilon.
\end{aligned}
\tag{A.28}
$$

The demand functions of firms in each segment are as follows:

$$
\begin{aligned}
D_{AA} &= \mathbb{P}\left(U_{AA} \geq U_{BA}\right) = \mathbb{P}\left(2\alpha-\Delta p+\delta\Delta t+(1-t_A)\epsilon > (1-t_B)\epsilon\right) = \Phi\left(\frac{2\alpha-\Delta p+\delta\Delta t}{\kappa}\right), \\
D_{AB} &= \mathbb{P}\left(U_{AB} \geq U_{BB}\right) = \mathbb{P}\left(-2\alpha-\Delta p+\delta\Delta t+(1-t_A)\epsilon > (1-t_B)\epsilon\right) = \Phi\left(\frac{-2\alpha-\Delta p+\delta\Delta t}{\kappa}\right), \\
D_{BA} &= 1-D_{AA} = 1-\Phi\left(\frac{2\alpha-\Delta p+\delta\Delta t}{\kappa}\right) = \Phi\left(\frac{-2\alpha+\Delta p-\delta\Delta t}{\kappa}\right), \\
D_{BB} &= 1-D_{AB} = 1-\Phi\left(\frac{-2\alpha-\Delta p+\delta\Delta t}{\kappa}\right) = \Phi\left(\frac{2\alpha+\Delta p-\delta\Delta t}{\kappa}\right),
\end{aligned}
\tag{A.29}
$$

where $\Delta t = t_A - t_B, \Delta p = p_A - p_B$. $\square$

A.9. Proof of Lemma 9

Based on Lemma 8, we examine the equilibrium obtained for each firm's best-response prices by applying the first-order conditions. Taking derivative of $\Pi_i = p_i D_i$ w.r.t p_i, we have

$$
\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A} &= p_A \left(\frac{\partial D_{AA}}{\partial p_A} + \frac{\partial D_{AB}}{\partial p_A} \right) + D_{AA} + D_{AB} \\
&= \frac{-p_A}{\kappa} \left[\phi\left(\frac{2\alpha - \Delta p + \delta \Delta t}{\kappa} \right) + \phi\left(\frac{2\alpha + \Delta p + \delta \Delta t}{\kappa} \right) \right] + 1 + \Phi\left(\frac{2\alpha - \Delta p + \delta \Delta t}{\kappa} \right) - \Phi\left(\frac{2\alpha + \Delta p - \delta \Delta t}{\kappa} \right), \\
\frac{\partial \Pi_B}{\partial p_B} &= p_B \left(\frac{\partial D_{BA}}{\partial p_B} + \frac{\partial D_{BB}}{\partial p_B} \right) + D_{BA} + D_{BB} \\
&= \frac{-p_B}{\kappa} \left[\phi\left(\frac{2\alpha - \Delta p + \delta \Delta t}{\kappa} \right) + \phi\left(\frac{2\alpha + \Delta p - \delta \Delta t}{\kappa} \right) \right] + 1 - \Phi\left(\frac{2\alpha - \Delta p + \delta \Delta t}{\kappa} \right) + \Phi\left(\frac{2\alpha + \Delta p - \delta \Delta t}{\kappa} \right).
\end{aligned}
\tag{A.30}
$$

Here, we abuse a little and use $\hat{p_A}, \hat{p_B}$ to represent $p_A(t_A, t_B), p_B(t_A, t_B)$, respectively. Then, $\frac{\partial \Pi_A}{\partial p_A}(\hat{p_A}, \hat{p_B}) = 0$ is equivalent to

$$\frac{1 - \Phi(x_1) + \Phi(x_2)}{\phi(x_1) + \phi(x_2)} = \frac{\hat{p_A}}{\kappa},$$

and $\frac{\partial \Pi_B}{\partial p_B}(\hat{p_A}, \hat{p_B}) = 0$ is equivalent to

$$\frac{1 + \Phi(x_1) - \Phi(x_2)}{\phi(x_1) + \phi(x_2)} = \frac{\hat{p_B}}{\kappa},$$

where

$$x_1 = \frac{2\alpha + \Delta\hat{p} - \delta\Delta t}{\kappa}, \quad x_2 = \frac{2\alpha - \Delta\hat{p} + \delta\Delta t}{\kappa}. \tag{A.31}$$

$\frac{\partial \Pi_A}{\partial p_A}(\hat{p_A}, \hat{p_B}) - \frac{\partial \Pi_B}{\partial p_B}(\hat{p_A}, \hat{p_B}) = 0$ is equivalent to

$$\frac{2\left[\Phi(x_2) - \Phi(x_1)\right]}{\phi(x_1) + \phi(x_2)} = \frac{\hat{p_A} - \hat{p_B}}{\kappa}, \tag{A.32}$$

The equilibrium profit functions are

$$
\begin{aligned}
\Pi_A(t_A, t_B) &= \hat{p_A} D_A(t_A, t_B) = \frac{\left[1 - \Phi(x_1) + \Phi(x_2)\right]^2}{\phi(x_1) + \phi(x_2)} \kappa, \\
\Pi_B(t_A, t_B) &= \hat{p_B} D_B(t_A, t_B) = \frac{\left[1 - \Phi(x_2) + \Phi(x_1)\right]^2}{\phi(x_1) + \phi(x_2)} \kappa.
\end{aligned}
\tag{A.33}
$$

There are totally four sub-games. Similarly, we use the superscripts $\{NN, YY, NY, YN\}$ to denote each sub-game. Using x_1^{ij} and x_2^{ij}, $i, j \in \{Y, N\}$ to denote the x_1 and x_2 under each sub-game, respectively. Also, using Δp^{ij}, κ^{ij}, $i, j \in \{Y, N\}$ to denote the Δp and κ under each sub-game.

Next, we analyze the two cases with the same operational transparency strategy, i.e., NN and YY.

- **Sub-game: NN** In this case, firm A and firm B both choose non-transparency, i.e., $t_A = t_B = 0$, and the first-order condition (A.32) is equivalent to

$$\frac{2\left[\Phi(x_2^{NN}) - \Phi(x_1^{NN})\right]}{\phi(x_1^{NN}) + \phi(x_2^{NN})} = \frac{\Delta p^{NN}}{\kappa^{NN}}. \tag{A.34}$$

Based on (A.31),

$$x_1^{NN} = \frac{2\alpha + \Delta p^{NN}}{\kappa^{NN}}, \quad x_2^{NN} = \frac{2\alpha - \Delta p^{NN}}{\kappa^{NN}},$$

Here, if $\Delta p^{NN} \geq 0$, we have $x_2^{NN} \leq x_1^{NN}$. Then, $\Phi(x_2^{NN}) - \Phi(x_1^{NN}) \leq 0$. And vice versa. So, we conclude that $(\Phi(x_2^{NN}) - \Phi(x_1^{NN})) \cdot \Delta p^{NN} \leq 0$. Hence, (A.34) holds only when $\Delta p^{NN} = 0$. Then, we get $x_1^{NN} = x_2^{NN} = \frac{\sqrt{2}\alpha}{\sigma}$, for $\kappa^{NN} = \sigma\sqrt{2}$. Finally,

$$p_A^{NN} = p_B^{NN} = \frac{\kappa^{NN}}{\phi(x_1^{NN}) + \phi(x_2^{NN})} = \frac{\sigma}{\sqrt{2}\phi(\frac{\sqrt{2}\alpha}{\sigma})},$$

and

$$D_A = D_B = 1.$$

Therefore,

$$\Pi(0,0) := \Pi_A(0,0) = \Pi_B(0,0) = \frac{\sigma}{\sqrt{2}\phi(\frac{\sqrt{2}\alpha}{\sigma})}.$$

- **Sub-game: YY** In this case, firm A and firm B both choose operational transparency, i.e., $t_A = t_B = \bar{t}$, then the first-order condition (A.32) is equivalent to

$$\frac{2\left[\Phi(x_2^{YY}) - \Phi(x_1^{YY})\right]}{\phi(x_1^{YY}) + \phi(x_2^{YY})} = \frac{\Delta p^{YY}}{\kappa^{YY}}.$$

Similarly, based on (A.31), we have

$$x_1^{YY} = \frac{2\alpha + \Delta p^{YY}}{\kappa^{YY}}, \quad x_2^{YY} = \frac{2\alpha - \Delta p^{YY}}{\kappa^{YY}},$$

By the same token, we have

$$p_A^{YY} = p_B^{YY} = \frac{\sigma(1-\bar{t})}{\sqrt{2}\phi(\frac{\sqrt{2}\alpha}{\sigma(1-\bar{t})})},$$

and

$$D_A = D_B = 1.$$

Hence,

$$\Pi(\bar{t},\bar{t}) := \Pi_A(\bar{t},\bar{t}) = \Pi_B(\bar{t},\bar{t}) = \frac{\sigma(1-\bar{t})}{\sqrt{2}\phi(\frac{\sqrt{2}\alpha}{\sigma(1-\bar{t})})}.$$

Hence, the proof is complete. $\square$

A.10. Proof of Lemma 10

- **Sub-game: NY** In this case, firm A chooses non-transparency and firm B chooses transparency, i.e., $t_A = 0, t_B = \bar{t}$. Then, the equilibrium profits are

$$\begin{aligned} \Pi_A(0,\bar{t}) &= \sigma\sqrt{1+(1-\bar{t})^2}\frac{\left[1-\Phi(x_1^{NY})+\Phi(x_2^{NY})\right]^2}{\phi(x_1^{NY})+\phi(x_2^{NY})}, \\ \Pi_B(0,\bar{t}) &= \sigma\sqrt{1+(1-\bar{t})^2}\frac{\left[1-\Phi(x_2^{NY})+\Phi(x_1^{NY})\right]^2}{\phi(x_1^{NY})+\phi(x_2^{NY})}. \end{aligned} \tag{A.35}$$

According to (A.31), we have

$$x_1^{NY} = \frac{2\alpha+\Delta p^{NY}+\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}, \quad x_2^{NY} = \frac{2\alpha-\Delta p^{NY}-\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}},$$

and the first-order condition (A.32) is equivalent to

$$\frac{2\left[\Phi(x_2^{NY})-\Phi(x_1^{NY})\right]}{\phi(x_1^{NY})+\phi(x_2^{NY})} = \frac{\Delta p^{NY}}{\sigma\sqrt{1+(1-\bar{t})^2}}. \tag{A.36}$$

We can establish that $\Delta p^{NY} < 0$ using a proof by contradiction. Assume that $\Delta p^{NY} \geq 0$, then the right-hand side of equation (A.36) would be positive. Since $\Phi(x)$ is an increasing function, it follows that $x_2^{NY} \geq x_1^{NY}$, which implies

$$x_1^{NY} - x_2^{NY} = \frac{2(\Delta p^{NY}+\delta\bar{t})}{\sigma\sqrt{1+(1-\bar{t})^2}} \leq 0.$$

Rearranging this inequality gives us $\Delta p^{NY} \leq -\delta\bar{t} < 0$, which is a contradiction to our initial assumption that $\Delta p^{NY} \geq 0$. As a result, we can conclude that $\Delta p^{NY} < 0$. It follows that $x_1^{NY} - x_2^{NY} = \frac{2(\Delta p^{NY}+\delta\bar{t})}{\sigma\sqrt{1+(1-\bar{t})^2}} > 0$, so we have

$$-\delta\bar{t} < \Delta p < 0.$$

Further, define

$$v = \frac{2\alpha}{\sigma\sqrt{1+(1-\bar{t})^2}} > 0, \quad x^{NY} = \frac{\Delta p^{NY}+\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}} > 0,$$

then

$$x_1^{NY} = v + x^{NY}, \quad x_2^{NY} = v - x^{NY}.$$

The first-order condition (A.36) turns to

$$\frac{2\left[\Phi(v-x^{NY})-\Phi(v+x^{NY})\right]}{\phi(v+x^{NY})+\phi(v-x^{NY})} = x^{NY} - \frac{\delta\bar{t}}{\sigma\sqrt{1+(1-\bar{t})^2}}.$$

To illustrate that x^{NY} is unique, we prove that $\frac{2[\Phi(v-x)-\Phi(v+x)]}{\phi(v+x)+\phi(v-x)} - x$ is decreasing. Define

$$f(x) = \frac{\left[\Phi(v-x)-\Phi(v+x)\right]}{\phi(v+x)+\phi(v-x)}.$$

Taking derivative of $f(x)$ gives

$$f'(x)=\frac{-\left[\phi(v+x)+\phi(v-x)\right]^2-\left[\Phi(v-x)-\Phi(v+x)\right]\left[(v-x)\phi(v-x)-(v+x)\phi(v+x)\right]}{\left[\phi(v+x)+\phi(v-x)\right]^2}$$

Since $v>0$ and $x>0$, we have $\Phi(v-x)-\Phi(v+x)<0$. We consider two cases:

— If $x>v>0$, then $(v-x)\phi(v-x)-(v+x)\phi(v+x)<0$. So we have $f'(x)<0$.

— If $v>x>0$, then $x_1>x_2>0$, where $x_1=v+x$, $x_2=v-x$. To prove $f'<0$ is equivalent to prove

$$\left[\Phi(x_2)-\Phi(x_1)\right]\left[x_1\phi(x_1)-x_2\phi(x_2)\right]-\left[\phi(x_1)+\phi(x_2)\right]^2<0.$$

Considering the given equation with x_1 taken as a parameter, we can define the given equation as a function of x_2 and denote it as

$$g(x_2)=\left[\Phi(x_2)-\Phi(x_1)\right]\left[x_1\phi(x_1)-x_2\phi(x_2)\right]-\left[\phi(x_1)+\phi(x_2)\right]^2,$$

where $x_2\in(0,x_1)$. In the following, we will show that $g(x_2)$ is an increasing function. Combined with the fact that $g(x_1)=-4\phi(x_1)^2<0$, we can conclude that $g(x_2)<0$, for $x_2\in(0,x_1)$, given any x_1. Hence, we proved that $f'(x)<0$. Next, we give rigorous proof.

The first derivative of $g(x_2)$ is given by

$$g'(x_2)=\phi(x_2)\left[x_1\phi(x_1)-x_2\phi(x_2)-(1-x_2^2)\left[\Phi(x_2)-\Phi(x_1)\right]+2x_2\left[\phi(x_1)+\phi(x_2)\right]\right].$$

To figure out the sign of $g'(x_2)$, we define

$$\begin{aligned}h(x_2)=\frac{g'(x_2)}{\phi(x_2)}&=x_1\phi(x_1)-x_2\phi(x_2)-(1-x_2^2)\left[\Phi(x_2)-\Phi(x_1)\right]+2x_2\left[\phi(x_1)+\phi(x_2)\right]\\&=-(1-x_2^2)\left[\Phi(x_2)-\Phi(x_1)\right]+(x_1+2x_2)\phi(x_1)+x_2\phi(x_2).\end{aligned}$$

The derivatives are given by

$$\begin{aligned}h'(x_2)&=2\left[\phi(x_1)+x_2\left[\Phi(x_2)-\Phi(x_1)\right]\right],\\h''(x_2)&=2\left[\Phi(x_2)-\Phi(x_1)+x_2\phi(x_2)\right],\\h'''(x_2)&=2\phi(x_2)(2-x_2^2).\end{aligned}$$

To prove $h(x_2)>0$, i.e., $g'(x_2)>0$, first we show that $h'(x_2)$ first decreases and then increases in $x_2\in(0,x_1)$.

* If $x_1\le\sqrt{2}$, then $x_2<x_1\le\sqrt{2}$, so $h''(x_2)$ increases in $x_2\in(0,x_1)$.
* If $x_1>\sqrt{2}$, then $h''(x_2)$ increases in $x_2\in(0,\sqrt{2}]$ and decreases in $x_2\in(\sqrt{2},x_1)$.

In both cases, there exists a x_2^* such that $h'(x_2)$ decreases in $(0, x_2^*]$ and increases in (x_2^*, x_1) since $h''(0) = 2[\Phi(0) - \Phi(x_1)] < 0$ and $h''(x_1) = 2x_1\phi(x_2) > 0$.

Then we turn to the monotonicity of $h(x_2)$. If $h'(x_2^*) > 0$, then $h(x_2)$ increases in $x_2 \in (0, x_1)$. If $h'(x_2^*) < 0$, combing with the fact that $h'(0) = h'(x_1) = 2\phi(x_1) > 0$, there exist $\bar{x}_2, \bar{\bar{x}}_2$ such that $h(x_2)$ increases in $(0, \bar{x}_2) \cup (\bar{\bar{x}}_2, x_1)$ and decreases in $(\bar{x}_2, \bar{\bar{x}}_2)$. According to the monotonicity, we have

$$\min h(x_2) = \min\{h(0), h(\bar{\bar{x}}_2)\}.$$

Since $h(0) = x_1\phi(x_1) + \Phi(x_1) > 0$, we only need to prove that $h(\bar{\bar{x}}_2) > 0$. We know that $\bar{\bar{x}}_2$ is the root of $h'(x_2) = 0$. That is, $h'(\bar{\bar{x}}_2) = 2[\phi(x_1) + \bar{\bar{x}}_2[\Phi(\bar{\bar{x}}_2) - \Phi(x_1)]] = 0$, which is equivalent to

$$\Phi(\bar{\bar{x}}_2) - \Phi(x_1) = -\frac{\phi(x_1)}{\bar{\bar{x}}_2}. \tag{A.37}$$

Inserting (A.37) to $h(x_2)$, we have

$$\begin{aligned} h(\bar{\bar{x}}_2) &= (1 - \bar{\bar{x}}_2^2)\frac{\phi(x_1)}{\bar{\bar{x}}_2} + (x_1 + 2\bar{\bar{x}}_2)\phi(x_1) + \bar{\bar{x}}_2\phi(\bar{\bar{x}}_2) \\ &= \frac{\phi(x_1)}{\bar{\bar{x}}_2} + (x_1 + \bar{\bar{x}}_2)\phi(x_1) + \bar{\bar{x}}_2\phi(\bar{\bar{x}}_2) \\ &> 0. \end{aligned}$$

Thus, it has been demonstrated that $h(x_2) > 0$. Namely, $g'(x_2) > 0$ when $x_2 \in (0, x_1)$, for any give x_1. Consequently, we can conclude that $f'(x) < 0$.

For $f(x)$ is decreasing in x, we have x^{NY} is the unique root of the following equation.

$$2f(x) - x = -\frac{\delta\bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}. \tag{A.38}$$

- **Sub-game: YN** In this case, firm A chooses transparency and firm B chooses non-transparency, i.e., $t_A = \bar{t}, t_B = 0$, (A.31) reduces to

$$x_1^{YN} = \frac{2\alpha + \Delta p^{YN} - \delta\bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}, \quad x_2^{YN} = \frac{2\alpha - \Delta p^{YN} + \delta\bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}},$$

The first order condition (A.32) is equivalent to

$$\frac{2[\Phi(x_2^{YN}) - \Phi(x_1^{YN})]}{\phi(x_1^{YN}) + \phi(x_2^{YN})} = \frac{\Delta p^{YN}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}. \tag{A.39}$$

By the same token as Δp^{NY}, it can be concluded that $\Delta p^{YN} \in (0,\ \delta\bar{t})$. Similarly, we define

$$x^{YN} = \frac{\Delta p^{YN} - \delta\bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}} < 0,$$

then we have

$$x_1^{YN} = v + x^{YN}, \quad x_2^{YN} = v - x^{YN}.$$

The first-order condition (A.39) turns to

$$\frac{2\left[\Phi(v - x^{YN}) - \Phi(v + x^{YN})\right]}{\phi(v + x^{YN}) + \phi(v - x^{YN})} = x^{YN} + \frac{\delta \bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}.$$

Since $f(-x) = -f(x)$, $f(x)$ is also decreasing in $x < 0$. We can conclude that x^{YN} is the unique root of the following equation:

$$2f(x) - x = \frac{\delta \bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}. \tag{A.40}$$

Combined with (A.38), (A.40), and the property that $f(-x) = -f(x)$, we can conclude that

$$x^{YN} = -x^{NY}.$$

It also means

$$x_1^{YN} = x_2^{NY}, \quad x_2^{YN} = x_1^{NY}.$$

Then, combined with (A.35), the equilibrium profits have the following relationships.

$$\begin{aligned} \Pi_A(\bar{t}, 0) &= \sigma\sqrt{1 + (1 - \bar{t})^2}\frac{\left(1 - \Phi(x_1^{YN}) + \Phi(x_2^{YN})\right)^2}{\phi(x_1^{YN}) + \phi(x_2^{YN})} = \Pi_B(0, \bar{t}), \\ \Pi_B(\bar{t}, 0) &= \sigma\sqrt{1 + (1 - \bar{t})^2}\frac{\left(1 - \Phi(x_2^{YN}) + \Phi(x_1^{YN})\right)^2}{\phi(x_1^{YN}) + \phi(x_2^{YN})} = \Pi_A(0, \bar{t}). \end{aligned} \tag{A.41}$$

Hence, the proof is complete. $\square$

A.11. Proof of Lemma 11

First, we want to prove that $\lim_{\sigma \to 0} \frac{\Pi_A(\bar{t}, \bar{t})}{\Pi_A(0, \bar{t})} = +\infty$. Specifically,

$$\Pi_A(\bar{t}, \bar{t}) = \frac{\left[1 - \Phi(x_1^{YY}) + \Phi(x_2^{YY})\right]^2}{\phi(x_1^{YY}) + \phi(x_2^{YY})}\kappa^{YY}, \quad \Pi_A(0, \bar{t}) = \frac{\left[1 - \Phi(x_1^{NY}) + \Phi(x_2^{NY})\right]^2}{\phi(x_1^{NY}) + \phi(x_2^{NY})}\kappa^{NY}.$$

where

$$x_1^{NY} = \frac{2\alpha + \Delta p^{NY} + \delta \bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}}, \quad x_2^{NY} = \frac{2\alpha - \Delta p^{NY} - \delta \bar{t}}{\sigma\sqrt{1 + (1 - \bar{t})^2}},$$

The ratio between $\Pi_A(\bar{t}, \bar{t})$ and $\Pi_A(0, \bar{t})$ can be expressed as:

$$\frac{\Pi_A(\bar{t}, \bar{t})}{\Pi_A(0, \bar{t})} = \frac{\left[1 - \Phi(x_1^{YY}) + \Phi(x_2^{YY})\right]^2}{\left[1 - \Phi(x_1^{NY}) + \Phi(x_2^{NY})\right]^2} \cdot \frac{\phi(x_1^{NY}) + \phi(x_2^{NY})}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \cdot \frac{\kappa^{YY}}{\kappa^{NY}}, \tag{A.42}$$

As $\Delta p^{NY} \in [-\delta \bar{t}, \ 0]$, we get $x_1^{NY} \geq 0$, while x_2^{NY} can be positive or negative. Hence, we consider two cases, i.e., $x_2^{NY} \geq 0$ and $x_2^{NY} < 0$.

- when $x_2^{NY} \geq 0$, and σ tends to zero, we have

$$\lim_{\sigma\to 0} x_1^{NY} = +\infty, \quad \lim_{\sigma\to 0} x_2^{NY} = +\infty.$$

Then, (A.42) turns to

$$\lim_{\sigma\to 0} \frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})} = \lim_{\sigma\to 0} \frac{\phi(x_1^{NY}) + \phi(x_2^{NY})}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \cdot \sqrt{\frac{2(1-\bar{t})^2}{1+(1-\bar{t})^2}}$$

For $x_1^{NY} \geq 0$ and $x_2^{NY} \geq 0$, when σ goes to zero, we have

$$\phi(x_1^{NY}) + \phi(x_2^{NY}) \geq 2\cdot\phi(\frac{x_1^{NY} + x_2^{NY}}{2}) = 2\cdot\phi(\frac{2\alpha}{\kappa^{NY}}) \tag{A.43}$$

due to the concavity within $[x_2^{NY}, x_1^{NY}]$. At the same time,

$$\phi(x_1^{YY}) + \phi(x_2^{YY}) = 2\cdot\phi(\frac{2\alpha}{\kappa^{YY}}). \tag{A.44}$$

Hence

$$\frac{\phi(x_1^{NY}) + \phi(x_2^{NY})}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \geq \frac{\phi(\frac{2\alpha}{\kappa^{NY}})}{\phi(\frac{2\alpha}{\kappa^{YY}})} = e^{\frac{2\alpha^2}{\sigma^2}[\frac{1}{2(1-\bar{t})^2} - \frac{1}{1+(1-\bar{t})^2}]}. \tag{A.45}$$

For

$$\lim_{\sigma\to 0} e^{\frac{2\alpha^2}{\sigma^2}[\frac{1}{2(1-\bar{t})^2} - \frac{1}{1+(1-\bar{t})^2}]} = +\infty,$$

we conclude that when σ goes to zero,

$$\lim_{\sigma\to 0} \frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})} = +\infty.$$

- when $x_2^{NY} < 0$, we have

$$\lim_{\sigma\to 0} x_1^{NY} = +\infty, \quad \lim_{\sigma\to 0} x_2^{NY} = -\infty.$$

When $x \to -\infty$, we have $\phi(x) > \Phi(x)$ due to $\lim_{x\to-\infty} \frac{\phi(x)}{\Phi(x)} = \lim_{x\to-\infty} \frac{\phi(x)\cdot(-x)}{\phi(x)} = +\infty$. So when $\sigma \to 0$, we have

$$\Phi(-x_1^{NY}) + \Phi(x_2^{NY}) < \phi(-x_1^{NY}) + \phi(x_2^{NY}) = \phi(x_1^{NY}) + \phi(x_2^{NY}).$$

Based on it, when $\sigma \to 0$, (A.42) turns to

$$\begin{aligned}
&\frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})} \\
&= \frac{1}{[\Phi(-x_1^{NY}) + \Phi(x_2^{NY})]^2} \cdot \frac{\phi(x_1^{NY}) + \phi(x_2^{NY})}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \cdot \sqrt{\frac{2(1-\bar{t})^2}{1+(1-\bar{t})^2}} \\
&> \frac{1}{[\phi(x_1^{NY}) + \phi(x_2^{NY})]^2} \frac{\phi(x_1^{NY}) + \phi(x_2^{NY})}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \cdot \sqrt{\frac{2(1-\bar{t})^2}{1+(1-\bar{t})^2}} \\
&= \frac{1}{\phi(x_1^{NY}) + \phi(x_2^{NY})} \frac{1}{\phi(x_1^{YY}) + \phi(x_2^{YY})} \cdot \sqrt{\frac{2(1-\bar{t})^2}{1+(1-\bar{t})^2}} \\
&\to +\infty.
\end{aligned}$$

Hence, we complete the proof.

Next, we want to prove that $\lim_{\sigma\to+\infty}\frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)}>1$. According to the profit functions (A.33) under equilibrium, we have

$$\Pi_A(0,0)=\frac{\left[1-\Phi(x_1^{NN})+\Phi(x_2^{NN})\right]^2}{\phi(x_1^{NN})+\phi(x_2^{NN})}\kappa^{NN},\quad \Pi_A(\bar{t},0)=\frac{\left[1-\Phi(x_1^{YN})+\Phi(x_2^{YN})\right]^2}{\phi(x_1^{YN})+\phi(x_2^{YN})}\kappa^{YN}.$$

The ratio between $\Pi_A(0,0)$ and $\Pi_A(\bar{t},0)$ can be expressed as:

$$\begin{aligned}\frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)}&=\frac{\left[1-\Phi(x_1^{NN})+\Phi(x_2^{NN})\right]^2}{\left[1-\Phi(x_1^{YN})+\Phi(x_2^{YN})\right]^2}\cdot\frac{\phi(x_1^{YN})+\phi(x_2^{YN})}{\phi(x_1^{NN})+\phi(x_2^{NN})}\cdot\frac{\kappa^{NN}}{\kappa^{NY}}\\&=\frac{1}{\left[1-\Phi(x_1^{YN})+\Phi(x_2^{YN})\right]^2}\cdot\frac{\phi(x_1^{YN})+\phi(x_2^{YN})}{2\phi(\frac{\sqrt{2}\alpha}{\sigma})}\cdot\sqrt{\frac{2}{1+(1-\bar{t}^2)}}.\end{aligned}$$

According to (A.31), we get

$$x_1^{YN}=\frac{2\alpha+\Delta p^{YN}-\delta\bar{t}}{\kappa^{YN}},\quad x_2^{YN}=\frac{2\alpha-\Delta p^{YN}+\delta\bar{t}}{\kappa^{YN}}.$$

when σ tends to $+\infty$, $x_1^{YN},x_2^{YN},x_1^{NN}$ and x_2^{NN} all tend to zero (Δp^{YN} is bounded), hence we have

$$\lim_{\sigma\to+\infty}\frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)}=\sqrt{\frac{2}{1+(1-\bar{t}^2)}}>1. \tag{A.46}$$

The proof is completed. □

A.12. Proof of Theorem 3

Based on the results of Lemma 9, Lemma 10 and Lemma 11, we can conclude the equilibrium results.

First, according to Lemma 11, we have

$$\lim_{\sigma\to 0}\frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})}=+\infty.$$

That is, when σ tends to zero, it is optimal for firm A to choose strategy Y instead of N, given firm B chooses Y. Next, according to the Lemma 9 and Lemma 10, we have $\Pi_A(\bar{t},\bar{t})=\Pi_B(\bar{t},\bar{t})$ and $\Pi_A(0,\bar{t})=\Pi_B(\bar{t},0)$. Then we get

$$\lim_{\sigma\to 0}\frac{\Pi_B(\bar{t},\bar{t})}{\Pi_B(\bar{t},0)}=\frac{\Pi_A(\bar{t},\bar{t})}{\Pi_A(0,\bar{t})}=+\infty.$$

It shows that it is optimal for firm B to choose strategy Y instead of N, given firm A chooses Y. Hence, when σ tends to zero, the equilibrium result is YY. That is, both firms will choose operational transparency.

Similarly, according to Lemma 11, we have

$$\lim_{\sigma\to+\infty}\frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)}>1.$$

That is, when σ tends to $+\infty$, it is optimal for firm A to choose strategy N instead of Y, given firm B chooses N. Next, according to the Lemma 9 and Lemma 10, we have $\Pi_A(0,0) = \Pi_B(0,0)$ and $\Pi_A(\bar{t},0) = \Pi_B(0,\bar{t})$. Then we get

$$\lim_{\sigma \to 0} \frac{\Pi_B(0,0)}{\Pi_B(0,\bar{t})} = \frac{\Pi_A(0,0)}{\Pi_A(\bar{t},0)} > 1.$$

It shows that it is optimal for firm B to choose strategy N instead of Y, given firm A chooses N. Hence, when σ tends to $+\infty$, the equilibrium result is NN. That is, both firms will not choose operational transparency. Hence, the proof is complete. $\square$