Part IA — Numbers and Sets Example Sheet 4.5

Supervised by Gareth Taylor Examples worked through by Christopher Turnbull

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Let $f: X \to Y$ be a function. For $A \subseteq X$ let $f(A) = \{f(x) : x \in A\}$. For $B \subseteq Y$ let $f^{-1}(B) = \{x \in X : f(x) \in B\}$. Give proofs or counterexamples of the following claims:

- (a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$
- (b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$
- (c) $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$
- (d) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$
- (e) $f^{-1}(f(A)) = A$
- (f) $f(f^{-1}(B)) = B$