Part IA — Numbers and Sets Example Sheet 1

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QUESTION 1

As $(n+1) \equiv (n+4) \pmod 3$, and exactly one of $n, n+1, n+2$ is divisible by 3, it follows that exactly of $n, n+2, n+4$ is divisible by 3 also.

So $3, 5, 7$ are three primes of this form, but this only occurs once.

QUESTION 2

Must have last digits 3,5,7,9. Consider the block of numbers $10k$ to $10k + 10$. We see that

$$10k \equiv 10 \pmod{30}$$

otherwise if $10k \equiv 0 \pmod{30}$, then the number with last digit 3 in our block would be divisible by 3. We follow a similar strategy with primes greater than 3, and obtain

$$k \equiv 1 \pmod{3}$$

QUESTION 3

QUESTION 4

QUESTION 5

QUESTION 6

QUESTION 7

QUESTION 8

QUESTION 9

QUESTION 10

QUESTION 11

QUESTION 12

QUESTION 13