Part IB — Numerical Analysis Example Sheet 3

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QUESTION 1

First, $\mathbf{u}_1^T$ is just the first row of A, ie $(10, 6, -2, 1)$, and $\mathbf{l}_1$ is the first column of A scaled so that $L_{1,1} = 1$, ie. $(1, 1, -\frac{1}{5}, \frac{1}{10})$. Calculating

$$\mathbf{l}_1\mathbf{u}_1^T = \begin{pmatrix} 10 & 6 & -2 & 1 \\ 10 & 6 & -2 & 1 \\ -2 & -\frac{6}{5} & \frac{2}{5} & -\frac{1}{5} \\ 1 & \frac{6}{10} & -\frac{1}{5} & \frac{1}{10} \end{pmatrix}$$

Now

$$\begin{aligned} \mathbf{A}_1 &:= A - \mathbf{l}_1\mathbf{u}_1^T \\ &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & -3 & -1 \\ 0 & \frac{16}{5} & -\frac{14}{5} & \frac{6}{5} \\ 0 & \frac{24}{10} & -\frac{9}{10} & \frac{29}{10} \end{pmatrix} \end{aligned}$$

And so $\mathbf{u}_2^T = (0, 4, -3, -1)$, $\mathbf{l}_2 = (0, 1, \frac{4}{5}, \frac{6}{10})$.
Next,

QUESTION 2

QUESTION 3

The s-step Adams-Bashforth method has $\rho(w) = w^2(w-1)$. Letting $\xi = w - 1$ and expanding,

$$\begin{aligned}\frac{w^2(w-1)}{\log w} &= \\ &= \end{aligned}$$

QUESTION 4

QUESTION 5

QUESTION 6

QUESTION 7

QUESTION 8

QUESTION 9

Consider the two-step BDF method: $\mathbf{y}_{n+2} - \frac{4}{3}\mathbf{y}_{n+1} + \frac{1}{3}\mathbf{y}_n = \frac{2}{3}hf(t_{n+2}, \mathbf{y}_{n+2})$. Applied to $y' = \lambda y$ we get

$$y_{n+2} - \frac{4}{3}y_{n+1} + \frac{1}{3}y_n = \frac{2}{3}h\lambda y_{n+2}$$

$$(3 - 2h\lambda)y_{n+2} - 4y_{n+1} + y_n = 0$$

We try $y_n = k^n$ and obtain

$$(3 - 2h\lambda)k^2 - 4k + 1 = 0$$

So

$$\begin{aligned} k &= \frac{4 \pm \sqrt{16 - 4(3 - 2h\lambda)}}{(6 - 4h\lambda)} \\ &= \frac{2 \pm \sqrt{1 + 2h\lambda}}{3 - 2h\lambda} \end{aligned}$$

Hence

$$y_n = A\left(\frac{2 + \sqrt{1 + 2h\lambda}}{3 - 2h\lambda}\right)^n + B\left(\frac{2 - \sqrt{1 + 2h\lambda}}{3 - 2h\lambda}\right)^n$$

QUESTION 10

QUESTION 11

QUESTION 12