Part IA — Extra Countability Questions

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QUESTION 1

(i)

Proposition.

$$\aleph_0 + 2^{\aleph_0} = 2^{\aleph_0}$$

Proof. It is clear that $2^{\aleph_0} \le \aleph_0 + 2^{\aleph_0}$. Going the other direction,

$$2^{\aleph_0} + \aleph_0 \le \aleph_0^{\aleph_0} + \aleph_0 \le \aleph_0^{\aleph_0+1} \le (2^{\aleph_0})^{\aleph_0+1} \le 2^{\aleph_0}$$

as $(\aleph_0 + 1)\aleph_0 = \aleph_0$. Hence we infer the proof by Cantor-Bernstein on

$$\aleph_0 + 2^{\aleph_0} \le 2^{\aleph_0} \text{ and } 2^{\aleph_0} \le \aleph_0 + 2^{\aleph_0}$$

□

(ii) Bernstein's Lemma says

$$\delta + \gamma = \alpha \cdot \beta \Rightarrow \text{ surjection } \gamma \to \alpha \text{ or injection } \beta \to \delta$$

Want to show that if $\aleph_0 + \alpha = 2^{\aleph_0}$ then α must be equal to $2^{\aleph_0}$.

QUESTION 5

QUESTION 6

QUESTION 7

QUESTION 8

QUESTION 9

QUESTION 10

QUESTION 11

QUESTION 12

QUESTION 13