Statistics 104 — Sample Midterm

May 2015

PROBLEM 1

The table below contains bus fares in 20 different cities

1.2	1.3	1.4	1.6	1.65	1.8	1.9	2.0	2.2	2.25
2.5	2.6	2.7	2.9	3.0	3.2	4.0	5.0	8	10

(a) Calculate sample mean $\bar{Y}$, standard deviation s and median $\hat{\theta}$.

(b) Carry out a t-test of the hypothesis $H_0 : \mu = 3$ vs $H_a : \mu < 3$. Find the p-value and state your conclusions at $\alpha = 0.05$.

(c) Calculate the 95% t-interval for μ.

(d) Test the hypothesis $H_0 : \theta = 3$ vs $H_a : \theta < 3$ using the binomial test at $\alpha = 0.05$. State your conclusion.

(e) Find a 95% confidence interval for θ using the normal approximation to the binomial distribution.

(f) When should you use a nonparametric test and does this apply to the data in this problem?

PROBLEM 2

The data in the following table compare reaction times (in seconds) of participants in front of a computer screen. Participants in sample 1 have been given decaffeinated coffee while sample 2 has been given regular coffee.

sample	1	2	3	4	5	6	7	8
S1	4	6	7	9	11	13	14	30
S2	5	8	10	12				

(a) Calculate sample means $\bar{X}$ and $\bar{Y}$ as well as sample medians $\hat{\theta}_X$ and $\hat{\theta}_y$ where $X_1, .., X_8$ refer to sample 1 data and $Y_1, .., Y_4$ to sample 2 data.

(b) Carry out a two-sample t-test for the hypothesis $H_0 : \mu_Y = \mu_X$ vs $H_a : \mu_Y > \mu_X$ at $\alpha = 0.05$. State your conclusions.

(c) Calculate a 95% CI for $\mu_Y - \mu_X$ based on the t-distribution. Assume unequal variances.

(d) Carry out the Wilcoxon test at $\alpha = 0.05$ for the hypothesis in (b).

(e) Find the Hodges-Lehmann estimate of $\theta_Y - \theta_X$ and a 90% CI.

(f) Calculate the exponential scores and find the p-value for the hypothesis in (b).

(f) Test the hypothesis $H_0 : \sigma_1 = \sigma_2$ vs $H_a : \sigma_1 > \sigma_2$ using the the Siegel-Tukey test at $\alpha = .05$.

(g) Use the Kolomogorov-Smirnov statistic to test $H_0 : F_X(x) = F_Y(x)$ vs $H_a : F_X(x) \neq F_Y(x)$ at $\alpha = 0.05$.

Problem 3

(a) Restate the hypothesis $H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 < \mu_2$ in terms of $F_1(x)$ and $F_2(x)$. Be precise in your statement.

(b) Suppose in part (a) the alternative hypothesis is two-sided. How can this alternative be expressed in terms of $F_1(x)$ and $F_2(x)$?

(c) Explain the difference between a skewed distribution and a heavy-tailed distribution.

(d) Calculate the power for a normal test with known variance when testing $H_0 : \mu = 0$ vs $H_a : \mu < 0$ when the true mean is $\mu = -5$ and the variance is $\sigma^2 = 9$ for a sample of size $n = 15$ from a normal distribution. Use $\alpha = .05$. Calculate the power of the binomial test for this problem.

(e) Calculate the power for the binomial test with known variance when testing $H_0 : \mu = 0$ vs $H_a : \mu < 0$ when the true mean is $\mu = -5$ and the variance is $\sigma^2 = 9$ for a sample of size $n = 15$ from a Laplace distribution. Use $\alpha = .05$.