Problem 1. Bickel and Doksum, problem 6.3.8

Problem 2. Consider the following multinomial model: Let X_i, $i = 1, \ldots, n$ be iid with

$$P(X_1 = j) = \theta_j, \quad j = 1, \ldots, k, \qquad 0 < \theta_j < 1 \quad \text{and} \quad \sum_{j=1}^{k} \theta_j = 1.$$

Let $\theta = (\theta_1, \ldots, \theta_{k-1})$. Show that the Wald test for testing

$$H_0 : \ \ \theta = \theta_0 \quad \text{versus} \quad H_1 : \ \ \theta \neq \theta_0$$

for specified $\theta_0 = (\theta_{0,1}, \ldots, \theta_{0,k-1})$ is equivalent to Pearson's χ^2-test, i.e. the test rejects if

$$\sum_{j=1}^{k} \frac{N_j - n\,\theta_{0,j})^2}{n\,\theta_{0,j}} \geq \chi^2_{k-1}(1-\alpha),$$

where $N_j = \sum_{i=1}^{n} \mathbf{1}(X_i = j)$, and $\chi^2_{k-1}(1-\alpha)$ denotes the $(1-\alpha)$-quantile of the χ^2_{k-1}-distribution.

HINT: First show that the Fisher information matrix $I(\theta)$ for $n = 1$ is given by

$$I_{ij}(\theta) = \begin{cases} \frac{1}{\theta_j} + \frac{1}{\theta_k} & \text{for } \ i = j \\ \frac{1}{\theta_k} & \text{else,} \end{cases}$$

where $i, j = 1, \ldots, k-1$ and $\theta_k = 1 - \sum_{j=1}^{k-1} \theta_j$.

Problem 3. Let X denote a random variable with $P(X = -1) = p_1$, $P(X = 1) = p_2$ with $p_1, p_2 > 0$, $p_1 + p_2 < 1$ and given that $X \neq \pm 1$ the random variable X follows a continuous distribution with pdf f. Calculate $\mathrm{E}(X)$.

Problem 4. Let X, Y be random variables (on the same probability space) with $X \sim F$ and $Y \sim G$. Show that $\mathrm{E}_F G(X) = 1 - \mathrm{E}_G F(Y)$ by

(a) using partial integration of Riemann-Stieltjes integrals,

(b) using a probabilistic interpretation of the quantities under consideration.

Here we assume that F and G do not have common jump points. This implies that the above Riemann-Stieltjes integrals exist and also that $P(X = Y) = 0$.