

Python Data Science

Personal Notes

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1 IPython

1.1 Help

- `help([symbol])` or `[symbol]?:` display the docstring of the symbol
 - Example: `help(map)` or `map?`
- `[symbol]??:` display the source code of the symbol (only if written in Python)
- `<Tab>`-completion: display matching `dir()` entries

- * (wildcard): matches any (also empty) string

1.2 Readline Commands

- C means Ctrl
- M means Alt

1.2.1 Navigation

- C-a: move to beginning of the line
- C-e: move to end of the line
- C-f: move one character forward
- C-b: move one character backward
- A-f: move one word forward
- A-b: move one word backward

1.2.2 Manipulation

- C-d: delete character under the cursor
- A-d: delete rest of the word under the cursor (right side)
- C-k: delete to the end of the line (right side)
- C-u: delete the beginning of the line (left side)
- C-y: yank (paste) text deleted before
- C-t: transpose; move character under the cursor one position to the left

1.2.3 History

- C-p: previous command (type multiple times to move back through the history)
- C-n: next command (type multiple times to move forth through the history)
- C-r: search backward in history

1.2.4 Miscellaneous

- C-l: clear screen
- C-c: cancel current command
- C-d: terminate session

1.3 Magic Commands

- %paste: paste code from the clipboard
- %cpaste: paste multiple code snippets interactively, end with --
- %run: run a script and keep the loaded symbols in the REPL
- %history: display the command history

- %history -n 1-4: display from the first to the fourth command
- %rerun: run a part of the history again
- %save: store the history in a file
- %lsmagic: list magic functions
- %xmode: set exception reporting mode
 - Plain: most compact, least information
 - Context: more information
 - Verbose: most detailed output
- %load_ext: load the extension with the given name
- %%file/%%writefile: write the following code section to a file with the given file name
 - a for appending instead of overwriting

To get help on a magic command, use the question mark notation as with any other command. Example: %rerun? shows the documentation for the %rerun magic command.

- %automagic: toggle automagic setting

If %automagic is set, shell commands like cat, cp, env, ls, man, mkdir, more, mv, pwd, rm, rmdir can be used without prefixes. Otherwise, a % prefix is needed.

1.4 History

Lines of input and output are numbered so that single lines can be addressed:

- In: list of all inputs
 - In[4]: fourth input line
- Out: map of all outputs
 - Out[2]: second output line
- _ (single underscore): last output
- __ (double underscore): second to the last output
- ___ (triple underscore): third to the last output
- _n (single underscore with number): n to the last output _4 = Out[4]

1.5 Shell Interaction

- ! at the beginning of a line: execute a shell command
- files = !ls -l: store output of a shell command as a list
 - files.grep('foo'): filter list by 'foo'
 - files.fields(1, 2): display columns 1 and 2 of the output
- !mkdir {folder}: create a directory with the variable folder's value as a name
 - surround a Python variable with curly braces to make it available for the shell

1.6 Miscellaneous

- ; at the end of a line: suppress output

1.7 Debugging

Python's standard debugger is `pdb`. IPython comes with an enhanced version `ipdb`.

- `%debug`: start a debugging session starting from the last exception
- `%pdb on`: start debugging session automatically when an exception occurs

Debugging sessions have special commands (usually, only the first letters needs to be typed):

- `l(ist)`: show the current location in the file
- `u(p)/d(own)`: move up and down in the call stack
- `n(ext)`: execute current line and move to next line (step over)
- `s(tep)`: enter the function (step in)
- `r(eturn)`: leave the function (step out)
- `q(uit)`: leave the debugging session and exit the program execution
- `c(ontinue)`: leave the debugging session, but keep the program running
- `<Enter>`: repeat previous command
- `p(rint)`: print variables
- `h(help)`: display a list of all available commands or help to the command argument supplied

1.8 Timing and Profiling

1.8.1 Timing

- `%time`: measure the execution time of a single statement/function call
 - The garbage collector will be deactivated so that the result is not biased.
- `%timeit`: measure the average execution time of a single statement/function call after repeated runs
 - The number of runs will be determined automatically.
- `%%timeit`: as above, but working on whole sections of code

1.8.2 Runtime Profiling

- `%prun`: runtime profile of a single statement/function call using Python's built-in profiler
- `%lprun`: line by line runtime profile of a single statement/function call
 - install with `pip install line_profiler` on the shell
 - load with `%load_ext line_profiler` in IPython

1.8.3 Memory Profiling

- install with `pip install memory_profiler` on the shell
- load with `%load_ext memory_profiler` in IPython
- `%memit`: memory profile of a single statement/function call

- `%mprun`: line by line memory profile of a single function call

`%mprun` requires the profiled code to be in it's own module. Example session:

```
%load_ext memory_profiler
%%file fibonacci.py
def fib(n):
    if n == 1 or n == 2:
        return 1
    return fib(n-1) + fib(n-2)

from fibonacci import fib
%mprun -f fib fib(35)
```

2 NumPy

Arrays of numbers are the fundamental data structure for data analysis. Python's primitive values have a large overhead. This information is redundant in lists, because the same type information is stored for every element. NumPy arrays are much more efficient than Python's lists—especially for big data sets. Python also offers an array type without redundant type information. However, this array type doesn't offer the fast and powerful operations of NumPy's `ndarray` type.

Conventionally, the NumPy library is imported as follows:

```
import numpy as np
```

2.1 Array Creation

2.1.1 Arrays of Python Lists

NumPy arrays can be created from Python lists:

```
>>> ints = np.array([2, 4, 6, 8]) # integer array
>>> floats = np.array([2, 4, 6, 8.1]) # upcast to float because of 8.1
>>> floats = np.array([2, 4, 6, 8], dtype='float') # with explicit type parameter
>>> ints = np.array([1.1, 2.2, 3.3], dtype='int') # with explicit type parameter
```

NumPy arrays can be multi-dimensional:

```
matrix = np.array([[1, 2, 3],
                   [4, 5, 6],
                   [7, 8, 9]])
```

2.1.2 Arrays from Scratch

Numpy offers various functions to generate arrays from scratch. Where a dimension is required (size), a single number (length), a tuple of two (rows, columns) or more (1st dimension, 2nd dimension, 3rd dimension, etc.) can be passed.

- `np.zeros(size, dtype)`: array of zeros
- `np.ones(size, dtype)`: array of ones
- `np.full(size, value)`: array filled with the given value
- `np.arange(start, end, step)`: array with values from start (inclusive) to end (exclusive) and given step width; `length=(end-start)/step`
- `np.linspace(from, to, n)`: array with evenly spaced values in interval [from,to] (both inclusive) of length n
- `np.random.random(size)`: uniformly distributed random values
- `np.random.normal(mean, sd, size)`: normally distributed array with the given mean and standard deviation
- `np.random.randint(from, to, size)`: random integers in the interval [from,to) (inclusive/exclusive)
- `np.random.choice(a, size, replace, p)`: random values from the array a or up to the upper bound value a with (`replace=True`) or without (`replace=False`) replacement and an optional array of probabilities p
- `np.eye(n)`: identity matrix with n rows and columns (values at indices with equal row/column index are 1)
- `np.empty(size)`: uninitialized array, values from current memory content (garbage)

2.1.3 Data Types

The dtype parameter can either be passed as a string literal or using a pre-defined constant:

1. literal: `dtype='int32'`
2. constant: `dtype=np.int32`

Common numeric types are:

- boolean: `Bool_`
- signed integers: `int8`, `int16`, `int32`, `int64`
 - `int_`: system's default long
 - `intc`: system's default int
- unsigned integers: `uint8`, `uint16`, `uint32`, `uint64`
- floating point: `float16`, `float32`, `float64`
 - `float_`: system default
- complex numbers: `complex64`, `complex128`
 - `complex_`: system default

2.2 Array Manipulation

NumPy arrays offer a rich set of attributes and operation for their manipulation. Since NumPy arrays are the foundation of many higher-level libraries, data manipulation in Python is often NumPy array manipulation.

2.2.1 Attributes

These read-only attributes can be used to retrieve information about an array:

- `ndim`: number of dimensions
- `shape`: size of each dimension
- `size`: total size of the array (the number of elements)
- `dtype`: data type of the array's elements
- `itemsize`: byte size of a single element
- `nbytes`: byte size of the entire array

In general, `nbytes` is equal to `itemsize` multiplied by `size`.

```
>>> np.random.seed(0) # for reproducible results
>>> arr = np.random.randint(10, 100, (3, 3))
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])
>>> arr.ndim
2
>>> arr.shape
(3, 3)
>>> arr.size
9
>>> arr.dtype
dtype('int64')
>>> arr.itemsize
8
>>> arr.nbytes
72
>>> arr.itemsize * arr.size
72
```

2.2.2 Indexing

Values of NumPy arrays can both be retrieved and modified by the means of indexing.

The indexing of single dimension arrays works with square brackets, just like indexing of Python lists:

- `arr[0]`: first element
- `arr[n]`: nth element
- `arr[-1]`: last element (first element counted from the end)
- `arr[-3]`: third last element (third element counted from the end)

For multi dimension arrays, a comma separated tuple has to be passed in square brackets:

- `arr[0, 0]`: first element of the first dimension
- `arr[3, 5]`: fifth element of the third dimension

```
>>> np.random.seed(0) # for reproducible results
>>> arr = np.random.randint(10, 100, (3, 3))
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])

>>> arr[0, 0]
54

>>> arr[1, 2]
19

>>> arr[-1, -1]
46

>>> arr[2, 2]
46
```

2.2.3 Slicing

The slicing syntax of Python lists also works for NumPy arrays:

- `[start:stop:step]`, with values omitted defaulting to:
 - `start=0`
 - `stop=[size of dimension]`
 - `step=1`
- For a negative step size, the defaults for start and stop are swapped.

```
>>> arr = np.arange(1, 10)
>>> arr
array([1, 2, 3, 4, 5, 6, 7, 8, 9])

>>> arr[2:5] # third (inclusive) to fifth (exclusive)
array([3, 4, 5])
```

```
>>> arr[::2] # every other (beginning with first)
array([1, 3, 5, 7])
```

```
>>> arr[::-1] # reversed
array([9, 8, 7, 6, 5, 4, 3, 2, 1])
```

If a step is indicated, two colons are required. Otherwise, step is interpreted as the stop.

Multi-dimension arrays can be sliced by providing multiple, comma-separated slices:

- [start1:stop1:step1, start2:stop2:step2], for slicing the first and second dimension.
- Indexing and slicing can be combined in order to access individual columns/rows:
 - [:, 0]: all rows, first column
 - [0, :]: first row, all columns
 - * [0]: shorthand (: can be omitted)

```
>>> np.random.seed(0) # for reproducible results
```

```
>>> arr = np.random.randint(10, 100, (3, 3))
```

```
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])
```

```
>>> arr[::2, 0:2] # columns 0 and 1 of every other row
array([[54, 57],
       [93, 31]])
```

```
>>> arr[:, 0] # first column
array([54, 77, 93])
```

```
>>> arr[0, :] # first row
array([54, 57, 74])
```

```
>>> arr[0] # first row (shorthand)
array([54, 57, 74])
```

Unlike Python lists, slices of NumPy arrays are *views to* the original data, not *copies of* it. To get a copy of a slice that can be modified without affecting the underlying array, the `copy()` method can be used. Using the array from above:

```
>>> s = arr[::2, 0:2] # view on columns 0 and 1 of every other row
```

```
>>> s
array([[54, 57],
       [93, 31]])
```

```
>>> s[0,1] = 88
```

```

>>> s[1,0] = 99
>>> s
array([[54, 88],
       [99, 31]])

>>> t = arr[1, 0:2].copy() # copy of columns 0 and 1 of the second row
>>> t
array([77, 77])

>>> t[0] = 11
>>> t[1] = 22
>>> t
array([11, 22])

>>> arr
array([[54, 88, 74], # 88 introduced through s
       [77, 77, 19], # 11 and 22 missing (working on copy t)
       [99, 31, 46]]) # 99 introduced through s

```

2.2.4 Reshaping

There are two options to reshape an existing array:

1. The function `reshape(size)`, which reshapes the underlying array to the given size (dimension indications).
 - The new size must match the array's size.
 - Good: `arr.size=60, arr.reshape((6, 10))`, because $6*10=60$
 - Bad: `arr.size=16, arr.reshape((4, 6))`, because $4*6>16$
2. Using the slicing parameter `np.newaxis`, which converts a one-dimensional to a two-dimensional array.
 - `arr[np.newaxis, :]`: array elements as columns
 - `arr[:, np.newaxis]`: array elements as rows

```

>>> np.arange(1, 10).reshape((3, 3))
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])

```

```

>>> np.arange(1, 4)[np.newaxis, :]
array([[1, 2, 3]])

```

```

>>> np.arange(1, 4)[:, np.newaxis]
array([[1],
       [2],

```

```
[3]])
```

2.2.5 Concatenation

The options to concatenate arrays of same and different dimensions are:

1. The function `np.concatenate(arrays, axis)`, which works on arrays of the same dimensions.
 - `arrays`: a list or tuple of arrays
 - `axis`: index of the axis, along which the concatenation takes place (0: rows, 1: columns, 2: third dimension)
2. Functions, which concatenate the given arrays of (possible) different dimensions:
 - `np.vstack(arrays)`: stack the arrays vertically
 - `np.hstack(arrays)`: stack the arrays horizontally
 - `np.dstack(arrays)`: stack the arrays along the third dimension

```
>>> a = np.arange(1, 5) # 1, 2, 3, 4
>>> b = np.arange(5, 9) # 5, 6, 7, 8
>>> np.concatenate((a, b))
array([1, 2, 3, 4, 5, 6, 7, 8])
```

```
>>> x = a.reshape((2, 2))
>>> x
array([[1, 2],
       [3, 4]])
```

```
>>> y = b.reshape((2, 2))
>>> y
array([[5, 6],
       [7, 8]])
```

```
>>> np.concatenate((x, y), axis=0) # along rows
array([[1, 2],
       [3, 4],
       [5, 6],
       [7, 8]])
```

```
>>> np.vstack((x, y)) # same, but shorter
array([[1, 2],
       [3, 4],
       [5, 6],
       [7, 8]])
```

```
>>> np.concatenate((x, y), axis=1) # along columns
```

```

array([[1, 2, 5, 6],
       [3, 4, 7, 8]])

>>> np.hstack((x, y)) # same, but shorter
array([[1, 2, 5, 6],
       [3, 4, 7, 8]])

>>> i = np.arange(1, 4).reshape((3, 1))
>>> i
array([[1],
       [2],
       [3]])

>>> j = np.arange(4, 10).reshape(3, 2)
>>> j
array([[4, 5],
       [6, 7],
       [8, 9]])

>>> np.hstack((i, j))
array([[1, 4, 5],
       [2, 6, 7],
       [3, 8, 9]])

>>> m = np.arange(1, 4)
>>> m
array([1, 2, 3])

>>> n = np.arange(4, 10).reshape((2, 3))
>>> n
array([[4, 5, 6],
       [7, 8, 9]])

>>> np.vstack((m, n))
array([[1, 2, 4],
       [4, 5, 6],
       [7, 8, 9]])

```

2.2.6 Splitting

An array split up at N split points will result in $N+1$ arrays. As for reshaping and concatenation, there are two fundamental ways to split arrays:

1. The function `np.split(array, splitpoints)`.

- `array`: an array of any dimension
- `splitpoints`: a list of indices
 - a divider (positive integer value) can be used to split the array up into `n` equally sized chunks

2. Functions, which split an array along a specific dimension.

- `np.hsplit(array, splitpoints)`: split the array along the horizontal axis
- `np.vsplit(array, splitpoints)`: split the array along the vertically axis
- `np.dsplit(array, splitpoints)`: split the array along a third dimension

```
>>> a = np.arange(1, 9)
>>> a
array([1, 2, 3, 4, 5, 6, 7, 8])

>>> np.split(a, [4]) # split at index 4 (beginning of second chunk)
[array([1, 2, 3, 4]), array([5, 6, 7, 8])]

>>> np.split(a, 2) # divide into 2 equally sized parts
[array([1, 2, 3, 4]), array([5, 6, 7, 8])]

>>> np.split(a, [2, 6]) # split at indices 2 and 6
[array([1, 2]), array([3, 4, 5, 6]), array([7, 8])]

>>> b = np.arange(1, 10).reshape((3, 3))
>>> b
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])

>>> i, j = np.hsplit(b, [2]) # split off first two columns
>>> i
array([[1, 2],
       [4, 5],
       [7, 8]])

>>> j
array([[3],
       [6],
       [9]])

>>> m, n = np.vsplit(b, [1]) # split off first row
>>> m
array([[1, 2, 3]])

>>> n
```

```
array([[4, 5, 6],
       [7, 8, 9]])
```

2.3 Universal Functions

- Loop-based operations on arrays resp. on their elements are slow, because Python performs type-checks and lookups for every function call.
- NumPy's universal functions (UFuncs) are statically typed and compiled. They can be performed on an array as a whole—and will be applied to each element. This is much faster and more convenient.
 - Loops over arrays should be rewritten in terms of UFuncs. The bigger the array, the larger the gain.
- UFuncs can be applied:
 - to an array and a scalar value:
* `np.arange(1, 4) * 2` # [2, 4, 6]
 - to an array and another array:
* `np.arange(1, 4) * np.arange(7, 10)` # [8, 10, 12]

2.3.1 Common UFuncs

Many of Python's native operators can be used as shorthands for UFuncs:

Shorthand	UFunc	Description
+	<code>np.add</code>	Addition
-	<code>np.subtract</code>	Subtraction
- (unary)	<code>np.negative</code>	Negative Prefix
*	<code>np.multiply</code>	Multiplication
/	<code>np.divide</code>	Division
//	<code>np.floor_divide</code>	Floor Division
**	<code>np.power</code>	Exponentiation
%	<code>np.mod</code>	Modulus (remainder)
<code>np.abs</code>	<code>np.absolute</code>	Absolute value

There are a lot of additional mathematical UFuncs:

- `np.sin/np.arcsin`: Sine and Arcsine
- `np.cos/np.arccos`: Cosine and Arcosine
- `np.tan/np.arctan`: Tangents and Cotangents
- `np.exp2`: 2^x
- `np.exp`: e^x
- `np.log`: base-e logarithm
- `np.log2`: base-2 logarithm
- `np.log10`: base-10 logarithm

2.3.2 Advanced Features

Rather than creating a new array for the return value, the result of a UFunc can be stored in an existing array using the out parameter. This also works with slices:

```
>>> x = np.arange(1, 6)
>>> x
array([1, 2, 3, 4, 5])

>>> y = np.zeros(5, dtype=np.int)
>>> y
array([0, 0, 0, 0, 0])

>>> np.power(x, 2, out=y)
>>> y
array([1, 4, 9, 16, 25])

>>> z = np.zeros(10)
>>> z
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

>>> np.power(x, 2, out=z[::2]) # overwrite every other element
>>> z
array([1, 0, 4, 0, 9, 0, 16, 0, 25, 0])
```

Every UFunc comes with a reduce operation, which repeatedly applies an operation to the elements of an array until only a single result remains.

```
>>> x = np.arange(1, 5)
>>> x
array([1, 2, 3, 4, 5])

>>> np.add.reduce(x) # Sum: 1 + 2 + 3 + 4 + 5
15

>>> np.multiply.reduce(x) # Factorial: 1 * 2 * 3 * 4 * 5
120
```

Instead of just storing the end results, each intermediary step can be stored using the accumulate function:

```
>>> x = np.arange(1, 5)
>>> x
array([1, 2, 3, 4, 5])

>>> np.add.accumulate(x)
```



```
array([1, 3, 6, 10, 15])
```

```
>>> np.multiply.accumulate(x)
array([1, 2, 6, 24, 120])
```

The outer operation computes the output of all pairs of two inputs, which could be used to create a multiplication table, for example:

```
>>> a = np.arange(1, 6)
>>> a
array([1, 2, 3, 4, 5])
```

```
>>> b = np.arange(1, 9)[1::2]
>>> b
array([2, 4, 6, 8])
```

```
>>> np.multiply.outer(b, a) # column, row
array([[ 2,  4,  6,  8, 10],
       [ 4,  8, 12, 16, 20],
       [ 6, 12, 18, 24, 30],
       [ 8, 16, 24, 32, 40]])
```

*	1	2	3	4	5
2	2	4	6	8	10
4	4	8	12	16	20
6	6	12	18	24	30
8	8	16	24	32	40

2.4 Aggregations

Aggregations reduce an array or one of its dimensions to a single value. In contrast to Python's built-in aggregate functions (`sum`, `min`, `max`), NumPy's implementations can operate on multi-dimensional arrays—and are much faster.

- Aggregate functions take an optional `axis` parameter, which describes *the array dimension to be collapsed*:
 - `axis=0`: collapse columns
 - `axis=1`: collapse rows

```
>>> a = np.random.randint(1, 10, size=(3, 4))
>>> a
array([[7, 8, 5, 5],
       [6, 1, 7, 2],
       [7, 2, 8, 8]])
```

```
>>> a.sum()  
66
```

```
>>> a.sum(axis=0)  
array([20, 11, 20, 16])
```

```
>>> a.sum(axis=1)  
array([25, 16, 25])
```

All aggregate functions can be called using the syntax `np.function(array, [parameters])`. Except for `np.median` and `np.percentile`, the following functions can be called directly on the array using the syntax `array.function([parameters])`.

Function	Returns
<code>np.sum</code>	sum
<code>np.prod</code>	product
<code>np.min</code>	minimum value
<code>np.max</code>	maximum value
<code>np.argmin</code>	index of minimum value
<code>np.argmax</code>	index of maximum value
<code>np.mean</code>	mean («average») value
<code>np.median</code>	median («middle») value
<code>np.var</code>	variance
<code>np.std</code>	standard deviation
<code>np.percentile(q=n)</code>	nth percentile, n in [0, 100]
<code>np.any</code>	is <i>any</i> value true?
<code>np.all</code>	are <i>all</i> values true?

Special NaN-aware functions exist for every function (except for the boolean functions `np.any` and `np.all`). They have the prefix `nan` and can only be called on `nd`, not directly on the array. Since NaN belongs to the IEEE-754 standard, arrays containing NaN must have the type `float` or `double`.

```
>>> a = np.array([1, 2, 3, np.NaN, 5])  
>>> a  
array([ 1.,  2.,  3., nan,  5.])
```

```
>>> np.sum(a)  
nan
```

```
>>> np.nansum(a)  
11.0
```

2.5 Broadcasting

Broadcasting is a set of rules for applying binary UFuncs (addition, multiplication, etc.) on arrays of different sizes and/or dimensions.

Rule 1: If the arrays have a different number of dimensions, the *shape* of the array with fewer dimensions is padded with ones on the left.

```
>>> a = np.ones(9).reshape(3, 3)
>>> a
array([[1., 1., 1.],
       [1., 1., 1.],
       [1., 1., 1.]])
```

```
>>> b = np.arange(1, 4)
>>> b
array([1, 2, 3])
```

```
>>> a.shape
(3, 3)
```

```
>>> b.shape
(3,)
```

Result: The shape of `b` is one-padded on the left: $(3,) \rightarrow (1, 3)$. Thus, `array([1, 2, 3])` becomes `array([[1, 2, 3]])`.

Rule 2: If the shape of the arrays does not match in any dimension, the array with a shape of one is stretched in that dimension to match the other shape.

Result: The rows of `b` are stretched (i.e. repeated), the shape changes again: $(1, 3) \rightarrow (3, 3)$. Thus, `array([[1, 2, 3]])` becomes:

```
array([[1, 2, 3],
       [1, 2, 3],
       [1, 2, 3]])
```

This is only a *conceptual* transformation, no memory is wasted when stretching!

3. If the dimensions neither match nor are equal to one, an error is raised.

```
>>> x = np.ones(6).reshape(2, 3)
>>> x
array([[1., 1., 1.],
       [1., 1., 1.]])

>>> y = np.arange(1, 3)
>>> y
```

```
array([1, 2])
```

```
>>> x.shape  
(2, 3)
```

```
>>> y.shape  
(2,)
```

Result: Error.

In order to perform binary operations on incompatible arrays (according these broadcasting rules), the arrays can be re-shaped manually:

```
>>> x + y  
ValueError: operands could not be broadcast together with shapes (2,3) (2,)
```

```
>>> x + y.reshape(2, 1)  
array([[2., 2., 2.],  
       [3., 3., 3.]])
```

2.6 Boolean Arrays

Python's comparison operators have NumPy equivalents. They are applied to each element and return a boolean array, indicating the result of every comparison:

Shorthand	UFunc	Description
<code>==</code>	<code>np.equal</code>	equal
<code>!=</code>	<code>np.not_equal</code>	not equal
<code><</code>	<code>np.less</code>	less than
<code>></code>	<code>np.great</code>	greater than
<code><=</code>	<code>np.less_equal</code>	less than or equal
<code>>=</code>	<code>np.greater_equal</code>	greater than or equal

```
>>> a = np.random.randint(1, 10, size=(3, 3))
```

```
>>> a  
array([[3, 4, 6],  
       [7, 4, 2],  
       [3, 6, 5]])
```

```
>>> a == 5  
array([[False, False, False],  
       [False, False, False],  
       [False, False, True]])
```

```
>>> np >= 5
array([[False, False,  True],
       [ True, False, False],
       [False,  True,  True]])
```

```
>>> np.less(a, 5)
array([[ True,  True, False],
       [False,  True,  True],
       [ True, False, False]])
```

The number of true values can be counted using the `np.count_nonzero` or the `np.sum` function, which counts False as 0 and True as 1. Using the array `a` from above:

```
>>> b = a >= 5
>>> b
array([[False, False,  True],
       [ True, False, False],
       [False,  True,  True]])
```

```
>>> np.count_nonzero(b)
4
```

```
>>> np.sum(b)
4
```

```
>>> np.count_nonzero(b, axis=0)
array([1, 1, 2])
```

```
>>> np.sum(b, axis=1)
array([1, 1, 2])
```

2.6.1 Bitmasks

Boolean arrays can be used for indexing, where every True item of the index array is returned:

```
>>> x = np.random.randint(1, 100, size=(4, 4))
>>> x
array([[58, 26, 64,  3],
       [91, 64, 44, 31],
       [14, 81, 77,  8],
       [64, 42, 56, 37]])

>>> above_mean = (x > x.mean())
```

```
>>> above_mean
array([[ True, False,  True, False],
       [ True,  True, False, False],
       [False,  True,  True, False],
       [ True, False,  True, False]])

>>> x[above_mean]
array([58, 64, 91, 64, 81, 77, 64, 56])
```

Selection criteria can be combined using the bitwise operands, which are shorthand for NumPy's element-wise logical UFuncs:

Shorthand	UFunc	Description
&	np.bitwise_and	and
\	np.bitwise_or	or
^	np.bitwise_xor	exclusive or
~	np.bitwise_not	not

Using the arrays `x` and `above_mean` from above:

```
>>> even = (x % 2 == 0)
>>> even
array([[ True,  True,  True, False],
       [False,  True,  True, False],
       [ True, False, False,  True],
       [ True,  True,  True, False]])

>>> x[even & above_mean]
array([58, 64, 64, 64, 56])

>>> x[np.bitwise_or(even, above_mean)]
array([58, 26, 64, 91, 64, 44, 14, 81, 77,  8, 64, 42, 56])

>>> odd = np.bitwise_not(even)
>>> odd
array([[False, False, False,  True],
       [ True, False, False,  True],
       [False,  True,  True, False],
       [False, False, False,  True]])
```

2.7 Fancy Indexing

Arrays can be indexed using arrays of indices to access multiple array elements at once.

```
>>> x = np.arange(5, 85, 5).reshape((4, 4))
>>> x
array([[ 5, 10, 15, 20],
       [25, 30, 35, 40],
       [45, 50, 55, 60],
       [65, 70, 75, 80]])

>>> x[[3, 1, 2], [2, 3, 1]] # select items (3,2), (1,3) and (2,1)
array([75, 40, 50])
```

2.7.1 Broadcasting

If array indices with different shapes are used, the index arrays are being broadcasted. The result of the index operation is shaped by the *broadcasted index array*, not by the array being indexed. Given the array `x` from above:

```
>>> rows = np.array([3, 1, 2])[:, np.newaxis]
>>> rows
array([[3],
       [1],
       [2]])

>>> cols = np.array([2, 3, 1])
>>> cols
array([2, 3, 1])

>>> x[rows, cols]
array([[75, 80, 70],
       [35, 40, 30],
       [55, 60, 50]])
```

The broadcasting of the index arrays is done like this:

	2	3	1
3	3,2	3,3	3,1
1	1,2	1,3	1,1
2	2,2	2,3	2,1

And the resulting array of the indexing operation looks like this:

	2	3	1
3	75	80	70
1	35	40	30

	2	3	1
2	55	60	50

Array indices can be combined with scalar indices, slicing and masking:

```
>>> x[2, [1, 0, 3]] # scalar and array index
array([50, 45, 60])

>>> x[2:3, [1, 0, 3]] # slice and array index
array([50, 45, 60])

>>> rows = np.array([2, 3])[:, np.newaxis]
>>> cols = np.array([False, True, False, True])
>>> x[rows, cols] # array index and mask
array([[50, 60],
       [70, 80]])
```

2.7.2 Assignment

Fancy indexing can be used for assignments, too:

```
>>> x = np.arange(10)
>>> x
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

>>> x[x % 2 == 0] = 0 # set all even values to zero
>>> x
array([0, 1, 0, 3, 0, 5, 0, 7, 0, 9])
```

However, the behaviour can be unexpected if index values are used multiple times:

```
>>> x = np.zeros(3)
>>> x
array([0, 0, 0])

>>> i = [0, 1, 1, 2, 2, 2]
>>> x[i] += 1
>>> x
array([1, 1, 1])
```

The values at indices 1 and 2 haven't been incremented three times, because the value of `x[i] + 1` is evaluated once at the beginning and then used multiple times. For repetitions, NumPy's functions have a `at` method, which performs unbuffered operations, i.e. results will be recalculated for every index element:


```
>>> x = np.zeros(3)
>>> x
array([0, 0, 0])

>>> i = [0, 1, 1, 2, 2, 2]
>>> np.add.at(x, i, 1)
>>> x
array([1, 2, 3])
```

2.8 Sorting

NumPy offers more efficient ways of sorting arrays than Python's native `sort()` function. An array can be sorted using the `np.sort()` function, which returns the sorted array:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> np.sort(x)
array([1, 2, 3, 4, 5])
```

By default, NumPy uses the quicksort algorithm. Other algorithms can be used by setting the `kind` parameter. Options are: `quicksort`, `mergesort`, `heapsort` and `stable`.

An array can also be sorted in-place, using the array's `sort()` method:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> x.sort()
>>> x
array([1, 2, 3, 4, 5])
```

The `np.argsort()` function sorts an array and returns an array of indices denoting the array's order. The returned array can be used for fancy indexing:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> i = np.argsort(x)
>>> i
array([3, 1, 4, 2, 0])

>>> x[i]
array([1, 2, 3, 4, 5])
```

Arrays can be sorted along rows and columns using the `axis` argument, which defines *along* (*not within!*) which axis the comparison and swapping is performed (0: along rows, 1: along columns):

```
>>> x = np.random.choice(10, (3, 3), replace=False)
>>> x
array([[7, 1, 9],
       [8, 0, 4],
```

```

[2, 3, 6]])

>>> np.sort(x, axis=0) # along rows/within columns
array([[2, 0, 4],
       [7, 1, 6],
       [8, 3, 9]])

>>> np.sort(x, axis=1) # along columns/within rows
array([[1, 7, 9],
       [0, 4, 8],
       [2, 3, 6]])

```

Arrays can be sorted *partially*, i.e. the array is split into two sections, with the left partition containing all smaller values than the right partition. Arrays can be sorted partially using `np.partition()`, which requires the `kth` parameter denoting the size of the left partition (`K` elements):

```

>>> x = np.random.choice(10, 10, replace=False)
>>> x
array([9, 1, 6, 0, 8, 5, 3, 2, 7, 4])

>>> np.partition(x, 3)
array([1, 0, 2, 3, 4, 5, 6, 7, 8, 9])

```

Within the partitions, the elements are in arbitrary order. Partial sorting can also be done by row or column using the `axis` argument. To return the array of partially sorted indices, the function `np.argsort()` can be used analogous to `np.argsort()`.

2.9 Structured Arrays

Storing heterogeneous data, say names and wages of employees, in different arrays of the same size is error prone: The relation of the data is not obvious, and sorting the arrays mixes up the entries. NumPy offers structured arrays, which can be defined with the `dtype` parameter using a compound data type specification in three ways:

- 1) using the dictionary method, indicating the field names and formats separately in two tuples:

```

dtype={'names': ('name', 'age', 'salary'),
      'formats': ('U20', 'u1', 'f4')}

```

- 2) using a list of tuples, defining the field name and its type together in one tuple per field:

```

dtype=np.dtype([('name', 'U20'),
                ('age', 'u1'),
                ('salary', 'f4')])

```

- 3) without specifying the field names, using automatic names from `f0` to `fn`, and defining the types as a comma-separated string:

```
dtype=np.dtype('U20,u1,f4')
```

A type indicator consists of three parts:

1. the endianness (optional): `<` for little endian, `>` for big endian
 - `<f4`: little endian float of four bytes
 - `>i8`: big endian integer of eight bytes
2. the data type (see the next table)
3. the size of the field *in bytes* (not in bits)

Indicator	Type	Example	Equivalent
'b'	byte	'b'	
'i'	signed integer	'i4'	<code>np.int32</code>
'u'	unsigned integer	'u1'	<code>np.uint8</code>
'f'	floating point	'f8'	<code>np.float64</code>
'c'	complex number	'c16'	<code>np.complex128</code>
'S' or 'a'	string (ASCII)	'S5'	
'U'	unicode string	'U10'	<code>np.dtype(np.str_, 10)</code>
'V'	raw data (void)	'V'	<code>np.void</code>

The fields can be accessed by row, by column, by combining row and column, and also using bit masks:

```
>>> employees = np.zeros(3, dtype=np.dtype([('name', 'S10'), ('wage', 'f8')]))
>>> employees['name'] = ['Dilbert', 'Wally', 'Alice']
>>> employees['wage'] = [120000.00, 80000.00, 110000.00]
>>> employees
array([(b'Dilbert', 120000.),
      (b'Wally', 80000.),
      (b'Alice', 110000.)],
      dtype=[('name', 'S10'), ('wage', '<f8')])
```

```
>>> employees[employees['wage'] > 100000]['name']
array([b'Dilbert', b'Alice'], dtype='<S10')
```

NumPy also allows storing arrays in fields of structured arrays, which can be achieved by providing an optional size indicator to every field definition:

```
>>> players = np.zeros(3, dtype=np.dtype([('name', 'U20'),
                                          ('pattern', 'S1', (3, 3))]))
>>> players[0]['name'] = 'John'
>>> glider = [[' ', '0', ' '],
```

```

        [' ', ' ', '0'],
        ['0', '0', '0']]
>>> players[0]['pattern'] = glider

```

NumPy offers the type `np.recarray`, which allows the individual fields to be accessed with dot notation instead of array indices:

```

>>> payroll = employees.view(np.recarray)
>>> payroll.name
array([b'Dilbert', b'Wally', b'Alice'], dtype='<S10')

```

The syntax is more convenient, but the performance of the access is lower.

NumPy's structured arrays are a very efficient way to store structured data. However, the Pandas library offers much more functionality for working with structured data.

3 Pandas

Pandas is a package built on top of NumPy, which offers powerful data operations familiar to those of data bases and spreadsheets. The fundamental data structures of Pandas are Series, DataFrame and Index. A DataFrame is a multidimensional array with labeled rows and columns, which supports heterogeneous and missing data—an issue often to be faced with in real-world data sets.

Pandas is idiomatically imported as `pd`:

```

>>> import pandas as pd

```

3.1 Series

What NumPy's `ndarray` is to Python's list, Pandas Series is to Python's dictionary: a fast and very powerful alternative. Whereas Python's dictionary maps a set of *arbitrary keys* to a set of *arbitrary values*, Pandas Series maps a set of *typed keys* to a set of *typed values*. A Series is made up of two sequences:

1. values: a NumPy array (`np.ndarray`)
2. index: a Pandas Index (`pd.Index`)

3.1.1 Creation

A Pandas Series can be created from scalars, lists and dictionaries.

If a Series is generated from list, the indices (first column) for the values (second column) are made up automatically, i.e. sequentially:

```

>>> pd.Series([1, 2, 3])
0    1

```

```

1    2
2    3
dtype: int64

```

An list of indices can be explicitly provided using the `index` parameter. The the lists of values and indices need to have the same length:

```

>>> pd.Series([1, 2, 3], index=['a', 'b', 'c'])
a    1
b    2
c    3
dtype: int64

```

However, indices can be noncontiguous and nonsequential:

```

>>> pd.Series([1, 2, 3], index=['Foo', 'Bar', 'Qux'])
Foo    1
Bar    2
Qux    3
dtype: int64

```

If a scalar value is used instead of list of values, the same value will be repeated for the length of the index list:

```

>>> pd.Series(42, index=[1, 2, 3])
1    42
2    42
3    42
dtype: int64

```

A Series can be created based on a dictionary with keys to be used as indices:

```

>>> pd.Series({'a': 1, 'b': 2, 'c': 3})
a    1
b    2
c    3
dtype: int64

```

An additional list of indices can be provided to further select values from the dictionary by their keys, and to specify the order of entries:

```

>>> pd.Series({'a': 1, 'b': 2, 'c': 3}, index=['c', 'a'])
c    3
a    1
dtype: int64

```

3.1.2 Access: Indexing and Selection

The elements of a Series can be accessed using indexing and slicing:

```
>>> s = pd.Series([1, 2, 3, 4, 5])
>>> s[0]
1

>>> s[4]
5

>>> s[1:4]
1    2
2    3
3    4
dtype: int64
```

If arbitrary (noncontiguous, nonsequential) indices are used, slicing is possible because of the fixed order of indices, but the upper bound is also included:

```
>>> payroll = pd.Series({'Dilbert': 120000, 'Wally': 80000, 'Alice': 110000})
>>> payroll['Dilbert':'Wally']
Dilbert    120000
Wally       80000
dtype: int64
```

Even though a non-numeric is used, a Series can also be sliced using a implicit index. Here, the upper bound is excluded:

```
>>> payroll[0:2]
Dilbert    120000
Wally       80000
dtype: int64
```

The elements of a Series can also be accessed through the means of masking and fancy indexing:

```
>>> payroll[(payroll >= 100000) & (payroll <= 150000)]
Dilbert    120000
Alice      110000
dtype: int64

>>> payroll[['Alice', 'Dilbert']]
Alice      110000
Dilbert    120000
dtype: int64
```

Python's native dictionary expressions are also supported:

```

>>> 'Dilbert' in payroll
True

>>> 'Asok' in payroll
False

>>> payroll.keys()
Index(['Dilbert', 'Wally', 'Alice'], dtype='object')

>>> list(payroll.items())
[('Dilbert', 120000), ('Wally', 80000), ('Alice', 110000)]

>>> payroll['Wally'] = 90000 # modify existing entry
>>> payroll['Asok'] = 12000 # add a new entry

```

3.1.3 Explicit and Implicit Indexing

When using an explicit integer index, indexing operations make use of the explicit indices (the actual index values provided), but slicing operations use the implicit indices (the items ordinal numbers). This can be confusing:

```

>>> ratings = pd.Series([2.3, 3.1, 3.9, 4.2, 4.8], index=[10, 20, 30, 40, 50])
>>> ratings[10] # explicit index
2.3

>>> ratings[1:3] # implicit index
20    3.1
30    3.9
dtype: float64

```

In order to reduce that confusion, a Series offers two attributes to access the indices:

- loc: the explicit index
- iloc: the implicit index

```

>>> ratings.loc[10]
2.3

>>> ratings.loc[10:30] # inclusive explicit indices from 10 to 30
10    2.3
20    3.1
30    3.9
dtype: float64

```

```
>>> ratings.iloc[0]
2.3

>>> ratings.iloc[0:3] # exclusive implicit indices from 0 to 3
10    2.3
20    3.1
30    3.9
dtype: float64
```

According to the [Zen of Python](#) («Explicit is better than implicit.»), slicing and indexing on Series using a integer index should be done using the `loc` and `iloc` attributes,

3.2 DataFrame

A Pandas DataFrame can be understood in terms of other data structures from two perspectives:

1. As a generalization of a NumPy array of two dimensions, with row indices and column names being flexible.
 - NumPy arrays are indexed as `arr[row, column]`: row first, column second.
 - Pandas DataFrames are indexed as `df[column][row]`: column first, row of the Series second.
2. As a specialization of a Python dictionary that maps a column name (key) to a Series of column data (value).

Generally speaking, a DataFrame is a sequence of Series sharing the index value. Important attributes are:

- `columns`: returns an Index object (column names)
- `index`: returns the index labels (row names)

3.2.1 Creation

A Pandas DataFrame can be created from Series, dictionaries and NumPy arrays.

If a single Series is provided, an optional column name for those values can be defined in a list:

```
>>> s = pd.Series([1, 2, 3])
>>> pd.DataFrame(s, columns=['values'])
  values
0      1
1      2
2      3
```

If a list of dictionaries is provided, each dictionary is mapped to a row. Missing entries of heterogeneous dictionaries are filled up with NaN in the resulting DataFrame:


```
>>> pd.DataFrame([{'a': 1, 'b': 2}, {'a': 5, 'c': 4}])
   a    b    c
0  1  2.0 NaN
1  5  NaN  4.0
```

If a dictionary of Series is provided, each Series becomes a column with its key mapped as the column name:

```
>>> s1 = pd.Series([2, 4, 6, 8])
>>> s2 = pd.Series([3, 6, 9, 12])
>>> pd.DataFrame({'two': s1, 'three': s2})
   two  three
0    2     3
1    4     6
2    6     9
3    8    12
```

If a two-dimensional NumPy array is provided, the numeric column and row indices from the array are used, but can be set using the optional columns and index parameters:

```
>>> arr = np.arange(1, 10).reshape(3, 3)
>>> pd.DataFrame(arr)
   0  1  2
0  1  2  3
1  4  5  6
2  7  8  9

>>> pd.DataFrame(arr, columns=['A', 'B', 'C'], index=[1, 2, 3])
   A  B  C
1  1  2  3
2  4  5  6
3  7  8  9
```

If a structured NumPy array is provided, the field names serve as column names:

```
>>> employees = np.zeros(3, dtype=np.dtype([('name', 'S10'), ('wage', 'f8')]))
>>> employees['name'] = ['Dilbert', 'Wally', 'Alice']
>>> employees['wage'] = [120000.00, 80000.00, 110000.00]
>>> pd.DataFrame(employees)
   name      wage
0 b'Dilbert' 120000.0
1 b'Wally'   80000.0
2 b'Alice'   110000.0
```

3.2.2 Access: Indexing and Selection

The DataFrame for the following examples:

```
>>> population = {
...   'USA': 326625792,
...   'Russia': 142257520,
...   'Germany': 80594016,
...   'Switzerland': 8236303
... }

>>> area = {
...   'USA': 9147593,
...   'Russia': 16377742,
...   'Germany': 348672,
...   'Switzerland': 39997
... }

>>> data = pd.DataFrame({'pop': population, 'area': area})
>>> data
```

	pop	area
Germany	80594016	348672
Russia	142257520	16377742
Switzerland	8236303	39997
USA	326625792	9147593

Individual columns can be accessed either dictionary-style or attribute-style, however the latter only works for columns with a string index that isn't used for any other DataFrame attribute:

```
>>> data['area']
Germany      348672
Russia      16377742
Switzerland    39997
USA         9147593
Name: area, dtype: int64

>>> data.area
Germany      348672
Russia      16377742
Switzerland    39997
USA         9147593
Name: area, dtype: int64

>>> data['area'] is data.area
```

True

```
>>> data['pop'] is data.pop
False # pop is a method of DataFrame!
```

For assignments, only dictionary-style access works (on the left side):

```
>>> data['density'] = data['pop'] / data.area
>>> data
```

	pop	area	density
Germany	80594016	348672	231.145650
Russia	142257520	16377742	8.686028
Switzerland	8236303	39997	205.923019
USA	326625792	9147593	35.706201

The raw, underlying multi-dimensional array of data of a DataFrame can be accessed using the `values` attribute, which supports array-style indexing:

```
>>> data.values
array([[8.05940160e+07, 3.48672000e+05, 2.31145650e+02],
       [1.42257520e+08, 1.63777420e+07, 8.68602766e+00],
       [8.23630300e+06, 3.99970000e+04, 2.05923019e+02],
       [3.26625792e+08, 9.14759300e+06, 3.57062007e+01]])
```

```
>>> data.values[0, 0]
80594016.0
```

A transposed version of the DataFrame (which rows and columns swapped) can be accessed using the `T` attribute:

```
>>> data.T
```

	Germany	Russia	Switzerland	USA
pop	8.059402e+07	1.422575e+08	8.236303e+06	3.266258e+08
area	3.486720e+05	1.637774e+07	3.999700e+04	9.147593e+06
density	2.311456e+02	8.686028e+00	2.059230e+02	3.570620e+01

A DataFrame offers different index attributes:

- `loc`: explicit index to access values by column and row *names*
 - inclusive upper bound
 - supports name based slicing, masking, fancy indexing
- `iloc`: implicit index to access values by column and row *numbers*
 - zero-based, exclusive upper bound
 - supports row and column access by ordinal numbers

```
>>> data.loc['Germany':'Russia', 'pop':'area']
```

	pop	area
Germany	80594016	348672

```
Russia    142257520  16377742
```

```
>>> data.loc[data.density > 100, ['pop', 'density']]
```

```
      pop    density
Germany  80594016  231.145650
Switzerland  8236303  205.923019
```

```
>>> data.iloc[0:2, 0:2]
```

```
      pop    area
Germany  80594016  348672
Russia   142257520  16377742
```

3.3 Index

The Pandas Index is an immutable array/a ordered (multi)set that is used both for the indexing of Series and DataFrame.

An Index can be created from a list:

```
>>> pd.Index([1, 2, 3, 4, 5])
Int64Index([1, 2, 3, 4, 5], dtype='int64')
```

The elements of the Index can be accessed like list entries, i.e. by a single index and using slicing:

```
>>> idx = pd.Index([1, 2, 3, 4, 5])
>>> idx[2]
3
```

```
>>> idx[0:2]
Int64Index([1, 2], dtype='int64')
```

```
>>> idx[::2]
Int64Index([1, 3, 5], dtype='int64')
```

An Index is immutable, which is important when they are shared between different DataFrames and Series:

```
>>> idx[2] = 6
TypeError: Index does not support mutable operations
```

Like Python's native set, Index supports set operations like intersection, union and difference:

```
>>> idxA.intersection(idxB)
Int64Index([1, 3, 5], dtype='int64')
```

```

>>> idxA.union(idxB)
Int64Index([1, 2, 3, 4, 5, 7, 9], dtype='int64')

>>> idxA.difference(idxB)
Int64Index([7, 9], dtype='int64')

>>> idxB.difference(idxA)
Int64Index([2, 4], dtype='int64')

>>> idxA.symmetric_difference(idxB)
Int64Index([2, 4, 7, 9], dtype='int64')

Union, intersection and symmetric difference can be expressed by the means of operators:

>>> idxA & idxB # intersection
Int64Index([1, 3, 5], dtype='int64')

>>> idxA | idxB # union
Int64Index([1, 2, 3, 4, 5, 7, 9], dtype='int64')

>>> idxA ^ idxB # symmetric difference
Int64Index([2, 4, 7, 9], dtype='int64')

```

3.4 Operations

Pandas offers a lot of functions like NumPy's UFuncs that can be applied on a Series or DataFrame either using a method (with another Series or DataFrame as a argument) or using a Python operator:

Operator	Method	Description
+	add()	Addition
-	sub(), subtract()	Subtraction
*	mul(), multiply()	Multiplication
/	truediv(), div(), divide()	Division
//	floordiv()	Floor Division
%	mod()	Modulus (remainder)
**	pow()	Exponentiation

The index of the operands is preserved in the result. If the operands are heterogeneous, the result contains the union of the two indices, with NaN filled in for missing values:

```

>>> hours = pd.Series([25, 40, 32], index=['Alice', 'Bob', 'Malory'])
>>> rates = pd.Series([45, 50, 30], index=['Alice', 'Bob', 'Thomas'])
>>> hours * rates

```

```

Alice      1125.0
Bob        2000.0
Malory      NaN
Thomas      NaN
dtype: float64

```

An operation that mixes a Series and a DataFrame works like an operation on a one-dimensional and a multi-dimensional array; broadcasting rules (similar as those for NumPy) apply:

```

>>> wages = pd.DataFrame({'January': {'Alice': 4500, 'Bob': 4800},
...                        'February': {'Alice': 4200, 'Bob': 4500}})

```

```

>>> wages
      January  February
Alice    4500    4200
Bob      4800    4500

```

```

>>> increase = pd.Series({'Alice': 1.2, 'Bob': 1.1})

```

```

>>> increase

```

```

Alice    1.2
Bob      1.1
dtype: float64

```

```

>>> wages.T * increase # with transposition

```

```

      Alice  Bob
January  5400.0  5280.0
February  5040.0  4950.0

```

```

>>> wages.multiply(increase, axis=0) # with optional axis (increase as rows)

```

```

      January  February
Alice  5400.0  5040.0
Bob    5280.0  4950.0

```

Pandas always preserves indices and column names, so that the data context is maintained.

3.5 Handling Missing Data

Real-world data sets are rarely clean and homogeneous. Oftentimes, values are missing, and the lack of a value is indicated in different ways. Pandas marks the absence of a value in two different ways:

1. `None`: a Python singleton object, which is used in object collections (rather slow due to the overhead).
2. `NaN`: a special floating point value (not a number), which is defined in the IEEE-754 standard and used for numeric collections. NumPy's NaN reference is used: `np.nan`.

A Series and DataFrame containing a None or NaN «value» is upcast according to the types of the other elements: integer types are upcast to float64; booleans are upcast to object.

```
>>> pd.Series([1, 2, None]) # None replaced by NaN
0    1.0
1    2.0
2    NaN
dtype: float64
```

```
>>> pd.Series([1, 2, np.nan])
0    1.0
1    2.0
2    NaN
dtype: float64
```

```
>>> pd.Series([True, False, None]) # None preserved
0     True
1    False
2     None
dtype: object
```

```
>>> pd.Series([True, False, np.nan])
0     True
1    False
2     NaN
dtype: object
```

Any operation involving NaN yields NaN:

```
>>> 3 + np.nan
nan
```

```
>>> (3 + 7) * np.nan
nan
```

```
>>> pd.Series([1, 2, np.nan]) + pd.Series([1, np.nan, 3])
0    2.0
1    NaN
2    NaN
dtype: float64
```

Whereas NumPy supports special NaN-aware functions (`np.nansum()`, `np.nanmax()`), Pandas offers special functions to deal with absent values:

`isnull()` and `notnull()` return a boolean mask indicating if there is no value (`isnull`) or a value (`notnull`) at the respective index. These masks can be used for indexing:

```
>>> s = pd.Series([1, np.nan, 3])
>>> s.isnull()
0    False
1     True
2    False
dtype: bool
```

```
>>> s.notnull()
0     True
1    False
2     True
dtype: bool
```

```
>>> s[s.notnull()]
0    1.0
2    3.0
dtype: float64
```

`dropna()` removes None and NaN entries in a Series. In a DataFrame, the full row or column missing a value is removed, which can be defined using the optional axis parameter:

```
>>> farmers = ['Miller', 'Shaw', 'Watson']
>>> dogs = pd.Series([1, 2, 1], index=farmers)
>>> cats = pd.Series([3, 1, np.nan], index=farmers)
>>> cows = pd.Series([7, np.nan, 2], index=farmers)
>>> pigs = pd.Series([0, 2, np.nan], index=farmers)
>>> livestock = pd.DataFrame( {'dogs': dogs, 'cats': cats, 'cows': cows, 'pigs': pigs})
>>> livestock
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0
Shaw	2	1.0	NaN	2.0
Watson	1	NaN	2.0	NaN

```
>>> livestock.dropna() # default: axis='rows'
      dogs  cats  cows  pigs
Miller    1   3.0   7.0   0.0
```

```
>>> livestock.dropna(axis='columns')
      dogs
Miller    1
Shaw      2
Watson    1
```

By default, every row/column with at least one missing entry is dropped. If the optional `how` parameter is set to `all`, only rows/columns with missing values only are dropped:


```
>>> livestock.dropna() # default: how='any'
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0

```
>>> livestock.dropna(how='all')
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0
Shaw	2	1.0	NaN	2.0
Watson	1	NaN	2.0	NaN

The optional parameter `thresh` allows to define a threshold: only drop rows/columns with fewer values given:

```
>>> livestock.dropna(thresh=3) # drop rows with fewer than three values
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0
Shaw	2	1.0	NaN	2.0

```
>>> livestock.dropna(thresh=3, axis='columns')
```

	dogs
Miller	1
Shaw	2
Watson	1

`fillna()` fills in a value where one is missing. Either a scalar value can be passed, or the value from a neighbouring cell can be propagated using a combination of the method (`ffill/bfill`: forward and backward fill) and `axis` (rows/columns) parameters:

```
>>> livestock.fillna(0) # replace NaN with 0, which is useful for sums
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0
Shaw	2	1.0	0.0	2.0
Watson	1	0.0	2.0	0.0

```
>>> livestock.fillna(method='ffill', axis='rows') # propagate value to next row
```

	dogs	cats	cows	pigs
Miller	1	3.0	7.0	0.0
Shaw	2	1.0	7.0	2.0
Watson	1	1.0	2.0	2.0

```
>>> livestock.fillna(method='bfill', axis='columns') # ... from previous column
```

	dogs	cats	cows	pigs
Miller	1.0	3.0	7.0	0.0
Shaw	2.0	1.0	2.0	2.0
Watson	1.0	2.0	2.0	NaN

If there is no next or previous row or column, NaN entries could still remain after the `fillna()` operation.

3.6 Hierarchical Indexing

Pandas Series and DataFrame represent one- and two-dimensional data. But some data must be indexed by more than two indices, and values can only be accessed by a combination of all those indices. This concept is called *hierarchical indexing* or *multi-indexing*.

A index with multiple levels could be represented by a tuple (using Formula 1 teams and seasons as indices):

```
>>> index = [
    ('Mercedes', 2018), ('Mercedes', 2017),
    ('Ferrari', 2018), ('Ferrari', 2017),
    ('McLaren', 2018), ('McLaren', 2017)]
>>> points = pd.Series([655, 688, 571, 522, 62, 30], index=index)
>>> points
(Mercedes, 2018)    655
(Mercedes, 2017)    688
(Ferrari, 2018)     571
(Ferrari, 2017)     522
(McLaren, 2018)      62
(McLaren, 2017)     30
dtype: int64
```

However, storing a tuple as the index is inconvenient and inefficient for data access. Therefore Pandas offers MultiIndex, an efficient wrapper for tuple indices:

```
>>> multi_index = pd.MultiIndex.from_tuples(index)
>>> multi_index
MultiIndex(levels=[['Ferrari', 'McLaren', 'Mercedes'], [2017, 2018]],
            labels=[[2, 2, 0, 0, 1, 1], [1, 0, 1, 0, 1, 0]])
```

The MultiIndex has two levels (the team names and seasons), and they are combined with labels like this:

Team	labels[0]	labels[1]	Season
Mercedes	2	1	2018
Mercedes	2	0	2017
Ferrari	0	1	2018
Ferrari	0	0	2017
McLaren	1	1	2018
McLaren	1	0	2017

A Series created with a tuple index can be reindexed using a MultiIndex:

```
>>> points = points.reindex(multi_index)
>>> points
Mercedes 2018    655
          2017    688
Ferrari   2018    571
          2017    522
McLaren   2018     62
          2017     30
dtype: int64
```

The blank space below the team index means that the value from above is used.

A DataFrame with additional columns can be created based on the existing DataFrame:

```
>>> f1 = pd.DataFrame({
    'points': points,
    'races': [21, 20, 21, 20, 21, 20],
    'wins': [11, 12, 6, 5, 0, 0]})
>>> f1
```

		points	races	wins
Mercedes	2018	655	21	11
	2017	688	20	12
Ferrari	2018	571	21	6
	2017	522	20	5
McLaren	2018	62	21	0
	2017	30	20	0

The operations mentioned earlier can also be applied:

```
>>> win_ratio = f1['wins'] / f1['races']
>>> win_ratio
Mercedes 2018    0.523810
          2017    0.600000
Ferrari   2018    0.285714
          2017    0.250000
McLaren   2018    0.000000
          2017    0.000000
dtype: float64
```

3.6.1 Creation of Hierarchical Indices

A hierarchical index can be created implicitly, i.e. together with the Series or the DataFrame.

The index can be passed as an additional argument to the constructor as a list of index arrays:

```
>>> points = [655, 688, 571, 522]
>>> index = [['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari'],
             [2018, 2017, 2018, 2017]]
>>> pd.Series(points, index=index)
Mercedes 2018    655
          2017    688
Ferrari   2018    571
          2017    522
dtype: int64
```

Or a dictionary can be passed to the constructor, with appropriate index tuples as keys:

```
>>> points = {('Mercedes', 2018): 655, ('Mercedes', 2017): 688,
              ('Ferrari', 2018): 571, ('Ferrari', 2017): 522}
>>> pd.Series(points)
Mercedes 2018    655
          2017    688
Ferrari   2018    571
          2017    522
dtype: int64
```

Using one of MultiIndex class methods, a hierarchical index can be created explicitly. The resulting object can be passed to the constructor of a Series or a DataFrame as the index attribute.

The method `from_arrays` accepts a list of index arrays:

```
>>> pd.MultiIndex.from_arrays(['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari'],
                             [2018, 2017, 2018, 2017])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
            labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

The method `from_tuples` accepts a list of index tuples:

```
>>> pd.MultiIndex.from_tuples([('Mercedes', 2018), ('Mercedes', 2017),
                              ('Ferrari', 2018), ('Ferrari', 2017)])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
            labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

In the above examples, every item from the first index (['Mercedes', 'Ferrari']) has been combined with every item from the second index ([2018, 2017]) *manually*. This Cartesian product can also be created automatically using the `from_product` method:

```
>>> pd.MultiIndex.from_product(['Mercedes', 'Ferrari'], [2018, 2017])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
            labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

The index levels can also be combined manually using a nested list of labels passed to the constructor of MultiIndex. This is especially helpful, if only certain combinations of index

entries need to be created:

```
>>> index = pd.MultiIndex(levels=[['Manor', 'Haas'], [2015, 2016, 2017]],
                           labels=[[0,0,1,1], [0,1,1,2]])
>>> pd.Series([0, 1, 29, 47], index=index)
Manor  2015    0
       2016    1
Haas   2016   29
       2017   47
dtype: int64
```

Using a DataFrame, both rows and columns can have multiple indices:

```
>>> row_index = pd.MultiIndex.from_product(['Mercedes', 'Ferrari'],
                                           [2018, 2017])
>>> col_index = pd.MultiIndex.from_product(['Australia', 'Bahrain'],
                                           ['Driver 1', 'Driver 2'])
>>> pos = np.array([2, 8, 3, 2, 2, 3, 2, 3, 1, 3, 1, np.nan, 1, 4, 1, 4])
>>> f1 = pd.DataFrame(pos.reshape((4, 4)), index=row_index, columns=col_index)
>>> f1
```

		Australia		Bahrain	
		Driver 1	Driver 2	Driver 1	Driver 2
Mercedes	2018	2.0	8.0	3.0	2.0
	2017	2.0	3.0	2.0	3.0
Ferrari	2018	1.0	3.0	1.0	NaN
	2017	1.0	4.0	1.0	4.0

This allows for four-dimensional indices.

Both row and column index can be named by setting a list of row/column names with the appropriate length to the names attribute of the index:

```
>>> f1.index.names = ['Team', 'Season']
>>> f1.columns.names = ['GP', 'Driver']
>>> f1
```

		Australia		Bahrain	
		Driver 1	Driver 2	Driver 1	Driver 2
Team	Season				
Mercedes	2018	2.0	8.0	3.0	2.0
	2017	2.0	3.0	2.0	3.0
Ferrari	2018	1.0	3.0	1.0	NaN
	2017	1.0	4.0	1.0	4.0

3.6.2 Indexing and Slicing

Indexing and Slicing on Series is row based. This Series index has a species as the first (higher level) index, and the year as the second (lower level) index:

```
>>> idx = pd.MultiIndex.from_product(['cats', 'cows', 'dogs', 'pigs'],
                                     [2000, 2005, 2010])
>>> livestock = pd.Series([32, 16, 25, 60, 75, 52, 1, 1, 2, 4, 3, 7], index=idx)
>>> livestock
cows  2000    32
      2005    16
      2010    25
pigs  2000    60
      2005    75
      2010    52
dogs  2000     1
      2005     1
      2010     2
cats  2000     4
      2005     3
      2010     7
dtype: int64
```

Individual values can be accessed using full indexing by first indicating the higher level index and second the lower level index:

```
>>> livestock['cats', 2000]
4
>>> livestock['cows', 2010]
25
>>> livestock['pigs', 2005] - livestock['pigs', 2010]
23
```

If the lower level index is left unspecified, a Series with the lower level index retained is returned:

```
>>> livestock['cows']
2000    32
2005    16
2010    25
dtype: int64
```

Passing an empty slice for the higher level index allows indexing on the lower level index:

```
>>> livestock[:, 2010]
cows    25
pigs    52
```

```
dogs      2
cats      7
dtype: int64
```

Slicing on the explicit index is only available on a dataset with a sorted MultiIndex. Either the dataset is created using a sorted MultiIndex:

```
>>> idx = idx.sort_values()
>>> livestock = pd.Series([4, 3, 7, 32, 16, 25, 1, 1, 2, 60, 75, 52], index=idx)
```

Or the MultiIndex on the existing dataset is sorted, returning a new dataset:

```
>>> livestock = livestock.sort_index()
```

The indices are sorted lexicographically. Then the slicing operations can be performed (on the explicit index):

```
>>> livestock.loc['cats':'cows', 2000:2005]
cats  2000      4
      2005      3
cows   2000     32
      2005     16
dtype: int64
```

Selections can be made based on boolean masks:

```
>>> livestock[livestock > 10]
cows   2000     32
      2005     16
      2010     25
pigs   2000     60
      2005     75
      2010     52
dtype: int64
```

Values can be selected using fancy indexing:

```
>>> livestock[['cows', 'pigs']]
cows   2000     32
      2005     16
      2010     25
pigs   2000     60
      2005     75
      2010     52
dtype: int64
```

The indexing hierarchy on a DataFrame behaves like the one of a Series, expect that a DataFrame is indexed by columns first:

```
>>> row_idx = pd.MultiIndex.from_product([[2017, 2018],
                                           ['Jan', 'Jul']])
>>> col_idx = col_idx = pd.MultiIndex.from_product(['Tom', 'Jim'],
                                                    ['height', 'weight'])
>>> val = [[122, 35, 129, 37],
           [128, 37, 131, 39],
           [134, 39, 135, 41],
           [137, 40, 138, 43]]
>>> kids = pd.DataFrame(val, columns=col_idx, index=row_idx)
>>> kids
```

		Tom		Jim	
		height	weight	height	weight
2017	Jan	122	35	129	37
	Jul	128	37	131	39
2018	Jan	134	39	135	41
	Jul	137	40	138	43

```
>>> kids['Tom', 'height']
2017 Jan    122
      Jul    128
2018 Jan    134
      Jul    137
Name: (Tom, height), dtype: int64
```

For row-oriented selection on a DataFrame, the implicit index can be used:

```
>>> kids.iloc[0:2]
```

		Tom		Jim	
		height	weight	height	weight
2017	Jan	122	35	129	37
	Jul	128	37	131	39

The column index hierarchy can be expressed using the explicit index and tuples:

```
>>> kids.loc[:, ('Tom', 'weight')]
2017 Jan    35
      Jul    37
2018 Jan    39
      Jul    40
Name: (Tom, weight), dtype: int64
```

Because tuples do not support slices, Pandas offers the IndexSlice object:

```
>>> jan = pd.IndexSlice[:, 'Jan']
>>> weight = pd.IndexSlice[:, 'weight']
>>> kids.loc[jan, weight]
      Tom    Jim
```


		weight	weight
2017	Jan	35	37
2018	Jan	39	41

3.6.3 Rearranging Multi-Indices

Conceptually, a Series with two indices is a lot like a DataFrame, which maps the first index to the rows and the second index to the columns. A multi-index Series can be converted to a DataFrame using the Series `unstack()` method:

```
>>> idx = pd.MultiIndex.from_product([[2017, 2018],
                                      ['Bezos', 'Gates', 'Buffet']])
>>> billions = [72.8, 75.6, 86.0, 112, 84, 90]
>>> richest = pd.Series(billions, index=idx.sort_values())
>>> richest
2017 Bezos      72.8
     Buffet    75.6
     Gates     86.0
2018 Bezos    112.0
     Buffet    84.0
     Gates     90.0
dtype: float64
```

```
>>> richest.unstack()
      Bezos  Buffet  Gates
2017  72.8   75.6   86.0
2018 112.0   84.0   90.0
```

An optional level can be defined to indicate which index level is to be transformed into a column level:

```
>>> richest.unstack(level=0)
      2017  2018
Bezos  72.8 112.0
Buffet  75.6  84.0
Gates  86.0  90.0
```

```
>>> richest.unstack(level=1)
      Bezos  Buffet  Gates
2017  72.8   75.6   86.0
2018 112.0   84.0   90.0
```

The DataFrame can be converted back to a multi-index Series using the `stack()` method. The column index will become the lower level index of the row MultiIndex:

```
>>> richest.unstack(level=0).stack()
```

Bezos	2017	72.8
	2018	112.0
Buffet	2017	75.6
	2018	84.0
Gates	2017	86.0
	2018	90.0

dtype: float64

The indices of a dataset can be turned into regular columns using the `reset_index()` method, which allows to name the existing data column using an optional argument:

```
>>> richest.index.names = ['year', 'person']
>>> table = richest.reset_index(name='billions')
>>> table
```

	year	person	billions
0	2017	Bezos	72.8
1	2017	Buffet	75.6
2	2017	Gates	86.0
3	2018	Bezos	112.0
4	2018	Buffet	84.0
5	2018	Gates	90.0

Data columns can also be turned (back) into a MultiIndex using the `set_index()` method, which expects a list of columns to be used as indices:

```
>>> table.set_index(['year', 'person'])
```

		billions
year	person	
2017	Bezos	72.8
	Buffet	75.6
	Gates	86.0
2018	Bezos	112.0
	Buffet	84.0
	Gates	90.0

Aggregation methods have optional `level` and `axis` parameters, which allow for partial aggregations:

```
>>> richest.mean(level='year')
year
2017    78.133333
2018    95.333333
dtype: float64
```

```
>>> richest.mean(level='person')
person
Bezos    92.4
```

```
Buffet    79.8
Gates     88.0
dtype: float64
```

```
>>> richest.unstack(level=0).mean(axis=0)
year
2017    78.133333
2018    95.333333
dtype: float64
```

```
>>> richest.unstack(level=0).mean(axis=1)
person
Bezos     92.4
Buffet    79.8
Gates     88.0
dtype: float64
```

level and axis can also be combined, which is useful if both row and column use a Multi-Index.

3.6.4 Multi-Indices vs. Panels

Datasets using a MultiIndex are *sparse representations* of data: only the existing values are represented. Panels (classes Panel and Panel4D), in contrast, are *dense representations* of data. A value is stored for every combination of all indices. Since real-world data sets are often sparse, MultiIndex datasets are often more efficient than panels.

3.7 Combining Datasets

Conducting interesting studies of data often requires combining datasets from different sources. Pandas offers different facilities to perform this task: concatenations and database-style joins.

3.7.1 Concat and Append

To demonstrate the concatenation of datasets, this function is used to create a DataFrame quickly with values made up of column names and row indices:

```
def create_df(cols, index):
    data = {c: [str(c) + str(i) for i in index] for c in cols}
    return pd.DataFrame(data, index)
```

The function can be used thus:

```
>>> create_df('ABC', range(3))
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
```

Multiple Series or DataFrames can be combined using Pandas concat function, which expects a list of datasets:

```
>>> a = create_df('ABC', [1, 2, 3])
>>> b = create_df('ABC', [4, 5, 6])
>>> pd.concat([a, b])
   A  B  C
1  A1 B1 C1
2  A2 B2 C2
3  A3 B3 C3
4  A4 B4 C4
5  A5 B5 C5
6  A6 B6 C6
```

By default, the concatenation is performed row-wise (default parameter axis=0). The concatenation can be performed column-wise by setting the axis parameter either to 1:

```
>>> a = create_df('ABC', [1, 2, 3])
>>> b = create_df('DEF', [1, 2, 3])
>>> pd.concat([a, b], axis=1)
   A  B  C  D  E  F
1  A1 B1 C1 D1 E1 F1
2  A2 B2 C2 D2 E2 F2
3  A3 B3 C3 D3 E3 F3
```

By default, indices are preserved, even if the resulting index contains duplicates:

```
>>> a = create_df('ABC', [0, 1, 2])
>>> b = create_df('ABC', [2, 3, 4])
>>> pd.concat([a, b])
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
2  A2 B2 C2
3  A3 B3 C3
4  A4 B4 C4
```

The index 2 occurs twice in the resulting dataset above. There are different ways to deal with duplicate indices. The first is to raise an error in case of conflict by setting the verify_integrity flag to True:

```
>>> pd.concat([a, b], verify_integrity=True)
ValueError: Indexes have overlapping values: Int64Index([2], dtype='int64')
```

An other option is to ignore the existing indices and let Pandas create a new one by setting the `ignore_index` flag to `True`:

```
>>> pd.concat([a, b], ignore_index=True)
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
3  A2 B2 C2
4  A3 B3 C3
5  A4 B4 C4
```

The existing indices can be converted to a `MultiIndex` by introducing a higher-level index key describing the source of the entries in the resulting dataset using the `keys` parameter:

```
>>> pd.concat([a, b], keys=['a', 'b'])
   A  B  C
a 0  A0 B0 C0
  1  A1 B1 C1
  2  A2 B2 C2
b 2  A2 B2 C2
  3  A3 B3 C3
  4  A4 B4 C4
```

If datasets with columns in common are concatenated, the resulting dataset is a union of the source datasets (default parameter `join='outer'`). Missing values (in uncommon columns) are filled up as `NaN`:

```
>>> a = create_df('ABC', range(3))
>>> b = create_df('BCD', range(3))
>>> pd.concat([a, b])
   A  B  C  D
0  A0 B0 C0 NaN
1  A1 B1 C1 NaN
2  A2 B2 C2 NaN
0  NaN B0 C0 D0
1  NaN B1 C1 D1
2  NaN B2 C2 D2
```

If the resulting dataset should only consist of the columns in common of the source datasets, setting the parameter `join='inner'` will create a dataset as an intersection of the source columns:

```
>>> pd.concat([a, b], join='inner')
   B  C
```

```

0  B0  C0
1  B1  C1
2  B2  C2
0  B0  C0
1  B1  C1
2  B2  C2

```

For fine-grained control of the resulting columns, the parameter `join_axes` can be set to a Index object representing the output columns:

```

>>> pd.concat([a, b], join_axes=[pd.Index(['A', 'B', 'C'])])
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
0  NaN B0 C0
1  NaN B1 C1
2  NaN B2 C2

```

An existing Index object of the source datasets can also be used:

```

>>> pd.concat([a, b], join_axes=[a.columns])
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
0  NaN B0 C0
1  NaN B1 C1
2  NaN B2 C2

```

The `append()` method of a DataFrame is a shorthand for the `pd.concat()` function:

```

>>> a = create_df('ABC', range(3))
>>> b = create_df('ABC', [3, 4, 5])
>>> a.append(b)
   A  B  C
0  A0 B0 C0
1  A1 B1 C1
2  A2 B2 C2
3  A3 B3 C3
4  A4 B4 C4
5  A5 B5 C5

```

It should not be used when combining more than two datasets, because new indices and data buffers are created for every intermediary step.

3.7.2 Merge and Join

Pandas offers high-performance, in-memory join and merge operations. The `pd.merge()` function is the main interface, but `DataFrame` and `Series` also offer a `join()` method for higher convenience.

There are three types of joins:

1. one-to-one (1:1)
2. one-to-many (1:n)
3. many-to-many (n:m)

The type of join to be performed depends solely on the input data.

A one-to-one join is similar to column-wise concatenation. The datasets are automatically joined using a column common to both datasets:

```
>>> employees = pd.DataFrame(
    {'employee': ['Dilbert', 'Catbert', 'Pointy Haired Boss'],
     'department': ['Engineering', 'HR', 'Management']})
>>> employees
   employee  department
0    Dilbert  Engineering
1    Catbert           HR
2 Pointy Haired Boss  Management

>>> departments = pd.DataFrame(
    {'department': ['Management', 'HR', 'Engineering'],
     'location': ['upper floor', 'middle floor', 'basement']})
>>> departments
   department  location
0  Management  upper floor
1           HR  middle floor
2  Engineering  basement

>>> pd.merge(employees, departments)
   employee  department  location
0    Dilbert  Engineering  basement
1    Catbert           HR  middle floor
2 Pointy Haired Boss  Management  upper floor
```

The index of the input datasets is discarded; a new index is generated for the resulting dataset. The order of entries in the output may be different from the input.

If one of the key columns contains duplicates, a one-to-many join is performed. Using the same departments, but a extended employees dataset:

```
>>> employees = pd.DataFrame(
```

```

        {'employee': ['Dilbert', 'Wally', 'Catbert', 'Pointy Haired Boss'],
         'department': ['Engineering', 'Engineering', 'HR', 'Management']})
>>> employees
      employee  department
0      Dilbert  Engineering
1        Wally  Engineering
2      Catbert           HR
3 Pointy Haired Boss  Management

>>> pd.merge(employees, departments)
      employee  department  location
0      Dilbert  Engineering  basement
1        Wally  Engineering  basement
2      Catbert           HR  middle floor
3 Pointy Haired Boss  Management  upper floor

```

If the key columns on both sides contain duplicates, a many-to-many join is performed:

```

>>> employees = pd.DataFrame(
    {'name': ['Dilbert', 'Wally', 'Catbert'],
     'department': ['Engineering', 'Engineering', 'HR']})
>>> employees
      name  department
0 Dilbert  Engineering
1   Wally  Engineering
2 Catbert           HR

>>> skills = pd.DataFrame(
    {'skill': ['programming', 'thinking', 'thinking', 'manipulating'],
     'department': ['Engineering', 'Engineering', 'HR', 'HR']})
>>> skills
      skill  department
0 programming  Engineering
1   thinking  Engineering
2   thinking           HR
3 manipulating           HR

>>> pd.merge(employees, skills)
      name  department  skill
0 Dilbert  Engineering  programming
1 Dilbert  Engineering   thinking
2   Wally  Engineering  programming
3   Wally  Engineering   thinking
4 Catbert           HR   thinking
5 Catbert           HR  manipulating

```


These examples all assume *one column common to both datasets*, which is often not given in real-world datasets. The behaviour of `merge()` can be further specified to overcome this constraint.

If there are multiple common columns in both datasets, the column to be joined on can be defined using the `on` parameter:

```
>>> employees = pd.DataFrame(
    {'id': [1, 2, 3],
     'name': ['Dilbert', 'Wally', 'Catbert'],
     'department': ['Engineering', 'Engineering', 'HR']})
>>> employees
   id  name  department
0   1  Dilbert  Engineering
1   2   Wally  Engineering
2   3  Catbert           HR

>>> departments = pd.DataFrame(
    {'id': [1, 2],
     'department': ['Engineering', 'HR'],
     'location': ['basement', 'middle floor']})
>>> departments
   id  department  location
0   1  Engineering  basement
1   2           HR  middle floor

>>> pd.merge(employees, departments, on='department')
   id_x  name  department  id_y  location
0     1  Dilbert  Engineering     1  basement
1     2   Wally  Engineering     1  basement
2     3  Catbert           HR     2  middle floor
```

If the columns to be joined have a different name, the join can be defined using the `left_on` and `right_on` parameters:

```
>>> employees = pd.DataFrame(
    {'id': [1, 2, 3],
     'name': ['Dilbert', 'Wally', 'Catbert'],
     'department_id': [1, 1, 2]})
>>> employees
   id  name  department_id
0   1  Dilbert             1
1   2   Wally             1
2   3  Catbert             2

>>> departments = pd.DataFrame(
```

```

        {'id': [1, 2, 3],
         'department': ['Engineering', 'HR', 'Management']})
>>> departments
   id  department
0   1  Engineering
1   2             HR
2   3  Management

```

```

>>> pd.merge(employees, departments,
              left_on='department_id', right_on='id')
   id_x  name  department_id  id_y  department
0     1  Dilbert             1     1  Engineering
1     2   Wally             1     1  Engineering
2     3  Catbert             2     2             HR

```

Redundant columns can be removed from the output using the `drop()` method by providing the name of the column to be discarded, and the argument `axis=1` to specify that the column has to be dropped (as opposed to the row with `axis=0`):

```

>>> pd.merge(employees, departments,
              left_on='department_id', right_on='id').drop('id_x', axis=1)
   name  department_id  id_y  department
0  Dilbert             1     1  Engineering
1   Wally             1     1  Engineering
2  Catbert             2     2             HR

```

Joins can also be performed based on the index instead of on columns. Using the datasets `employees` and `department` from above with appropriate indices, the join can be performed by setting the `left_index` and `right_index` flags to `True`:

```

>>> employees = employees.set_index('id')
>>> employees
      name  department_id
id
1  Dilbert             1
2   Wally             1
3  Catbert             2

>>> departments = departments.set_index('id')
>>> departments
      department
id
1  Engineering
2           HR
3  Management

```

```
>>> pd.merge(employees, departments, left_index=True, right_index=True)
      name  department_id  department
id
1  Dilbert              1  Engineering
2    Wally              1           HR
3  Catbert              2   Management
```

Merging on the index is the default behaviour of the `join()` method:

```
>>> employees.join(departments)
      name  department_id  department
id
1  Dilbert              1  Engineering
2    Wally              1           HR
3  Catbert              2   Management
```

Merging on indices and columns can also be mixed, specifying either the `left_on/right_index` or the `left_index/right_on` parameter pairs:

```
>>> employees = pd.DataFrame({
    'id': [1, 2, 3],
    'name': ['Dilbert', 'Wally', 'Catbert'],
    'department_id': [1, 1, 2]})
>>> employees
   id  name  department_id
0   1  Dilbert           1
1   2   Wally           1
2   3  Catbert           2

>>> departments = pd.DataFrame({
    'id': [1, 2, 3],
    'department': ['Engineering', 'HR', 'Management']})
>>> departments = departments.set_index('id')
>>> departments
      department
id
1  Engineering
2           HR
3  Management

>>> pd.merge(employees, departments, left_on='department_id', right_index=True)
   id  name  department_id  department
0   1  Dilbert           1  Engineering
1   2   Wally           1  Engineering
2   3  Catbert           2           HR
```

The type of the join to be performed in terms of set arithmetic can be defined using the `how` keyword. The default option is `inner`; only entries common to both input datasets are contained in the result:

```
>>> employees = pd.DataFrame({
    'employee': ['Dilbert', 'Pointy Haired Boss', 'Dogbert'],
    'department': ['Engineering', 'Management', 'Evil Operations']})
>>> employees
```

	employee	department
0	Dilbert	Engineering
1	Pointy Haired Boss	Management
2	Dogbert	Evil Operations

```
>>> departments = pd.DataFrame({
    'department': ['Engineering', 'Management', 'Marketing'],
    'location': ['basement', 'upper floor', 'middle floor']})
>>> departments
```

	department	location
0	Engineering	basement
1	Management	upper floor
2	Marketing	middle floor

```
>>> pd.merge(employees, departments, how='inner')
```

	employee	department	location
0	Dilbert	Engineering	basement
1	Pointy Haired Boss	Management	upper floor

The option `outer` fills up missing entries (i.e. entries not common to both input datasets) with `NaN` in the result:

```
>>> pd.merge(employees, departments, how='outer')
```

	employee	department	location
0	Dilbert	Engineering	basement
1	Pointy Haired Boss	Management	upper floor
2	Dogbert	Evil Operations	NaN
3	NaN	Marketing	middle floor

The options `left` and `right` preserve all values from the left resp. right side, and fill up all the missing entries on the other side with `NaN`:

```
>>> pd.merge(employees, departments, how='left')
```

	employee	department	location
0	Dilbert	Engineering	basement
1	Pointy Haired Boss	Management	upper floor
2	Dogbert	Evil Operations	NaN

```
>>> pd.merge(employees, departments, how='right')
      employee  department  location
0      Dilbert  Engineering  basement
1  Pointy Haired Boss  Management  upper floor
2           NaN    Marketing  middle floor
```

If the two input datasets have columns with the same name that are not used to perform the join operation, a suffix (`_x` and `_y`) is added to both columns to prevent conflicts:

```
>>> employees.index.names = ['id']
>>> employees = employees.reset_index()
>>> employees
   id      employee  department
0  0      Dilbert  Engineering
1  1  Pointy Haired Boss  Management
2  2      Dogbert  Evil Operations
```

```
>>> departments.index.names = ['id']
>>> departments = departments.reset_index()
>>> departments
   id  department  location
0  0  Engineering  basement
1  1  Management  upper floor
2  2   Marketing  middle floor
```

```
>>> pd.merge(employees, departments, on='department')
   id_x      employee  department  id_y  location
0     0      Dilbert  Engineering     0  basement
1     1  Pointy Haired Boss  Management     1  upper floor
```

A list of custom suffixes can be set using the `suffixes` parameter:

```
>>> pd.merge(employees, departments, on='department', suffixes=['_emp', '_dep'])
   id_emp      employee  department  id_dep  location
0       0      Dilbert  Engineering     0  basement
1       1  Pointy Haired Boss  Management     1  upper floor
```

3.8 Aggregation

Computing aggregations is an essential technique for efficient summarization of data sets. The `planets` dataset of the `seaborn` package is useful for practicing aggregations:

```
>>> import seaborn as sns
>>> planets = sns.load_dataset('planets')
```

A good starting point is to get an overview over the dataset using the `describe()` function,

which is a convenience method that performs a couple of aggregations for the purpose of understanding rather than further processing the data:

```
>>> planets.describe()
      number  orbital_period      mass      distance      year
count  1035.000000      992.000000  513.000000   808.000000  1035.000000
mean     1.785507     2002.917596    2.638161   264.069282  2009.070531
std     1.240976     26014.728304    3.818617   733.116493    3.972567
min     1.000000      0.090706    0.003600    1.350000  1989.000000
25%     1.000000      5.442540    0.229000    32.560000  2007.000000
50%     1.000000     39.979500    1.260000    55.250000  2010.000000
75%     2.000000     526.005000    3.040000   178.500000  2012.000000
max     7.000000   730000.000000   25.000000  8500.000000  2014.000000
```

Important aggregation functions are:

Function	Returns
<code>count()</code>	number of entries (NaN not counted)
<code>min()</code>	minimum value
<code>max()</code>	maximum value
<code>sum()</code>	sum (addition)
<code>prod()</code>	product (multiplication)
<code>mean()</code>	mean (arithmetic average)
<code>median()</code>	median (middle value)
<code>std()</code>	standard deviation
<code>var()</code>	variance
<code>mad()</code>	mean absolute deviation

Aggregations on a DataFrame result in summarized columns. To aggregate rows instead of columns, the `axis` parameter can be set accordingly:

```
>>> planets.mean(axis='columns')
```

The `axis` parameters describe what is to be aggregated (the *columns* of each row), not what the result should be!

3.9 Grouping

Grouping allows to split a dataset up based on its values or index, perform computations within the groups and combine the group results together to overall results. Grouping is a three-step process:

1. `split`: breaking up and grouping a DataFrame (based on the values of a specified key or other property)
2. `apply`: perform computations within each group:

1. filter: remove or retain values for further processing
2. transform: map the input values to output values
3. aggregate: reduce the multitude of values to a single value (or a smaller amount of values)
4. apply: perform computations on the aggregation result(s)
3. combine: merge the results to a single resulting dataset

The `groupby()` method allows to perform those three steps together in an efficient way. When called on a `DataFrame`, it returns a `DataFrameGroupBy` object, which is a special (grouped) view onto the underlying `DataFrame`:

```
>>> import seaborn as sns
>>> planets = sns.load_dataset('planets')
>>> planets.groupby('year')
<pandas.core.groupby.groupby.DataFrameGroupBy object at 0x7f9db32f2eb8>
```

A `DataFrameGroupBy` is a collection of `DataFrames` that allows for the operations filter, transform, aggregate and apply. No computation is performed until an aggregation is applied (lazy evaluation), which returns a new `DataFrame`:

```
>>> planets.groupby('year').sum()
      number  orbital_period      mass  distance
year
1989        1      83.888000   11.68000    40.57
1992        6      91.803900    0.00000     0.00
1994        3      98.211400    0.00000     0.00
...
```

Selecting a column on a `DataFrameGroupBy` object returns a `SeriesGroupBy` object, which can be also used for aggregations and the like:

```
>>> planets.groupby('year')['distance']
<pandas.core.groupby.groupby.SeriesGroupBy object at 0x7f9db3224f60>
```

A `GroupBy` object allows to iterate over the individual groups, yielding the group key and the `DataFrame`:

```
>>> for (key, df) in planets.groupby('year'):
    print(key, ', '.join(df.columns))
1989 method, number, orbital_period, mass, distance, year
1992 method, number, orbital_period, mass, distance, year
1994 method, number, orbital_period, mass, distance, year
...
```

However, the `apply()` method is usually faster and more convenient than an explicit iteration.

When a method of a `DataFrame` is called on a `GroupBy` object, it is dispatched to each of the underlying `DataFrame` objects:

```
>>> planets.groupby('year').first()
              method  number  orbital_period    mass  distance
year
1989  Radial Velocity      1    83.888000  11.6800    40.57
1992    Pulsar Timing      3    25.262000     NaN     NaN
1994    Pulsar Timing      3    98.211400     NaN     NaN
...
```

As mentioned earlier, after grouping and before combining the data, different operations can be performed on the grouped data.

The `filter()` method executes a predicate function (or lambda expression) on every entry, retains it in the dataset (matching condition) or discards it from the dataset (not matching condition). The predicate function/lambda expression expects a DataFrame and returns a boolean:

```
>>> teams = ['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari']
>>> drivers = ['Hamilton', 'Bottas', 'Vettel', 'Raikkonen']
>>> points = [408, 247, 320, 251]
>>> championship = df.DataFrame(
    {'team': teams, 'driver': drivers, 'points': points})
>>> championship
   team    driver  points
0 Mercedes  Hamilton   408
1 Mercedes   Bottas   247
2  Ferrari   Vettel   320
3  Ferrari Raikkonen   251

>>> championship.groupby('team').filter(lambda x: x['points'].mean() > 300)
   team    driver  points
0 Mercedes  Hamilton   408
1 Mercedes   Bottas   247
```

The DataFrame is grouped by team. For every team the mean of points scored is calculated, and only entries with a team's point mean above 300 are retained. This filtering uses a predicate function:

```
>>> def below_600(x):
    return x['points'].sum() < 600
>>> championship.groupby('team').filter(below_600)
   team    driver  points
2  Ferrari   Vettel   320
3  Ferrari Raikkonen   251
```

The `transform()` method allows to map the input data record by record to output data of the same shape:


```
>>> championship.groupby('team')['points'].transform(lambda x: x / x.mean())
0    1.245802
1    0.754198
2    1.120841
3    0.879159
```

Each driver's ratio of points scored to the team is computed in terms of mean points per team. Notice that the points column was selected, so x refers to a Series, not to a DataFrame.

The aggregate() method allows to reduce a group in two fundamental ways:

First, by applying one or more aggregation functions that are passed either as a function or as a function name (string):

```
>>> championship.groupby('team').aggregate([min, 'max'])
      driver      points
      min      max   min  max
team
Ferrari  Raikkoenen  Vettel  251  320
Mercedes    Bottas  Hamilton  247  408
```

Second, by applying different aggregation functions for each column, by providing a dictionary that maps a function to every column:

```
>>> championship['position'] = [1, 5, 2, 3]
>>> championship.groupby('team').aggregate({'points': max, 'position': min})
      points  position
team
Ferrari    320         2
Mercedes   408         1
```

The apply() method allows to execute a function on every group result. It takes a DataFrame/Series and returns either a DataFrame/Series object, or the function reduces the group results further to a single scalar:

```
>>> championship.groupby('team')['points'].apply(sum)
team
Ferrari    571
Mercedes   655
Name: points, dtype: int64
```

The grouping of the data is not limited to a single column name. Different alternatives are available.

First, provide a list/array/series/index of group keys, telling every entry in which group to go:

```
>>> names = ['Harry Potter', 'Draco Malfoy', 'Hermine Granger', 'Ron Weasley']
```

```
>>> students = pd.Series(names)
>>> houses = ['Griffindor', 'Slytherin', 'Griffindor', 'Griffindor']
>>> students.groupby(houses).apply(lambda s: ', '.join(s))
Griffindor    Harry Potter, Hermine Granger, Ron Weasley
Slytherin                                Draco Malfoy
dtype: object
```

Second, provide a dictionary that maps the index keys to groups:

```
>>> courses = ['Math', 'English', 'History', 'Geography', 'Music', 'Biology']
>>> results = ['A', 'C', 'E', 'B', 'D', 'F']
>>> grouping = {'A': 'good', 'B': 'good', 'C': 'ok', 'D': 'ok', 'E': 'bad', 'F': 'bad'}
>>> marks = pd.DataFrame({'course': courses, 'result': results})
>>> marks = marks.set_index('result')
>>> marks
```

	course
result	
A	Math
C	English
E	History
B	Geography
D	Music
F	Biology

```
>>> marks.groupby(grouping).aggregate(lambda c: ', '.join(c))
course
bad    History, Biology
good   Math, Geography
ok     English, Music
```

Third, provide any function that maps a input (index) to a output (group):

```
>>> lectures = ['Math: Calculus', 'Math: Statistics',
                'Computer Science: Algorithms', 'Computer Science: Data Structures']
>>> professors = ['Smith', 'Myers', 'Dijkstra', 'Kernighan']
>>> plan = pd.DataFrame({'lecture': lectures, 'professor': professors})
>>> plan = plan.set_index('lecture')
>>> plan
```

	professor
lecture	
Math: Calculus	Smith
Math: Statistics	Myers
Computer Science: Algorithms	Dijkstra
Computer Science: Data Structures	Kernighan

```
>>> plan.groupby(lambda l: l.split(':')[0]).aggregate(lambda p: ', '.join(p))
```

	professor
Computer Science	Dijkstra, Kernighan
Math	Smith, Myers

And fourth, use a combination thereof, which results in a MultiIndex:

```
>>> marks.groupby([str.lower, grouping]).aggregate(lambda m: ' '.join(m))
           course
a good      Math
b good  Geography
c ok        English
d ok         Music
e bad      History
f bad      Biology
```

3.10 Miscellaneous

Pandas allows to read CSV files into a DataFrame. Given the CSV file `countries.csv`, it can be read as follows:

```
Country, Population, Area
USA, 326625792, 9147593
Russia, 142257520, 16377742
Germany, 80594016, 348672
Switzerland, 8236303, 39997

>>> countries = pd.read_csv('countries.csv')
>>> countries
```

	Country	Population	Area
0	USA	326625792	9147593
1	Russia	142257520	16377742
2	Germany	80594016	348672
3	Switzerland	8236303	39997