Python Data Science

Personal Notes

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Contents

1 IPython

1.1 Help

- `help([symbol])` or `[symbol]?`: display the docstring of the symbol
  - Example: `help(map)` or `map?`
- `[symbol]??`: display the source code of the symbol (only if written in Python)
- `<Tab>`-completion: display matching `dir()` entries
- `*` (wildcard): matches any (also empty) string

1.2 Readline Commands

- `C` means `Ctrl`
- `M` means `Alt`

1.2.1 Navigation

- `C-a`: move to beginning of the line
- `C-e`: move to end of the line
- `C-f`: move one character forward
- `C-b`: move one character backward
- `A-f`: move one word forward
- `A-b`: move one word backward

1.2.2 Manipulation

- `C-d`: delete character under the cursor
- `A-d`: delete rest of the word under the cursor (right side)
- `C-k`: delete to the end of the line (right side)
- `C-u`: delete the beginning of the line (left side)
- `C-y`: yank (paste) text deleted before
- `C-t`: transpose; move character under the cursor one position to the left

1.2.3 History

- `C-p`: previous command (type multiple times to move back through the history)
- `C-n`: next command (type multiple times to move forth through the history)
- `C-r`: search backward in history

1.2.4 Miscellaneous

- `C-l`: clear screen
- `C-c`: cancel current command
- `C-d`: terminate session

1.3 Magic Commands

- `%paste`: paste code from the clipboard
- `%cpaste`: paste multiple code snippets interactively, end with `--`
- `%run`: run a script and keep the loaded symbols in the REPL
- `%history`: display the command history
  - `%history -n 1-4`: display from the first to the fourth command
- `%rerun`: run a part of the history again
- `%save`: store the history in a file
- `%lsmagic`: list magic functions
- `%xmode`: set exception reporting mode
  - `Plain`: most compact, least information
  - `Context`: more information
  - `Verbose`: most detailed output
- `%load_ext`: load the extension with the given name
- `%%file`/`%%writefile`: write the following code section to a file with the given file name `-a` for appending instead of overwriting

To get help on a magic command, use the question mark notation as with any other command. Example: `%rerun?` shows the documentation for the `%rerun` magic command.

- `%automagic`: toggle automagic setting

If `%automagic` is set, shell commands like `cat`, `cp`, `env`, `ls`, `man`, `mkdir`, `more`, `mv`, `pwd`, `rm`, `rmdir` can be used without prefixes. Otherwise, a `%` prefix is needed.

1.4 History

Lines of input and output are numbered so that single lines can be addressed:

- `In`: list of all inputs
  - `In[4]`: fourth input line
- `Out`: map of all outputs
  - `Out[2]`: second output line

- `_` (single underscore): last output
- `__` (double underscore): second to the last output
- `___` (triple underscore): third to the last output
- `_n` (single underscore with number): n to the last output `_4 = Out[4]`

1.5 Shell Interaction

- `!` at the beginning of a line: execute a shell command
- `files = !ls -l`: store output of a shell command as a list
  - `files.grep('foo')`: filter list by `'foo'`
  - `files.fields(1, 2)`: display columns 1 and 2 of the output
- `!mkdir {folder}`: create a directory with the variable `folder`'s value as a name
  - surround a Python variable with curly braces to make it available for the shell

1.6 Miscellaneous

- `;` at the end of a line: suppress output

1.7 Debugging

Python's standard debugger is `pdb`. IPython comes with an enhanced version `ipdb`.

- `%debug`: start a debugging session starting from the last exception
- `%pdb on`: start debugging session automatically when an exception occurs

Debugging sessions have special commands (usually, only the first letters needs to be typed):

- `l(ist)`: show the current location in the file
- `u(p)`/`d(own)`: move up and down in the call stack
- `n(ext)`: execute current line and move to next line (step over)
- `s(tep)`: enter the function (step in)
- `r(eturn)`: leave the function (step out)
- `q(uit)`: leave the debugging session and exit the program execution
- `c(ontinue)`: leave the debugging session, but keep the program running
- <Enter>: repeat previous command
- `p(rint)`: print variables
- `h(help)`: display a list of all available commands or help to the command argument supplied

1.8 Timing and Profiling

1.8.1 Timing

- `%time`: measure the execution time of a single statement/function call

    - The garbage collector will be deactivated so that the result is not biased.
- `%timeit`: measure the average execution time of a single statement/function call after repeated runs
    - The number of runs will be determined automatically.
- `%%timeit`: as above, but working on whole sections of code

1.8.2 Runtime Profiling

- `%prun`: runtime profile of a single statement/function call using Python's built-in profiler
- `%lprun`: line by line runtime profile of a single statement/function call
    - install with `pip install line_profiler` on the shell
    - load with `%load_ext line_profiler` in IPython

1.8.3 Memory Profiling

- install with `pip install memory_profiler` on the shell
- load with `%load_ext memory_profiler` in IPython
- `%memit`: memory profile of a single statement/function call
- `%mprun`: line by line memory profile of a single function call

`%mprun` requires the profiled code to be in it's own module. Example session:

```
%load_ext memory_profiler
%%file fibonacci.py
def fib(n):
    if n == 1 or n == 2:
        return 1
    return fib(n-1) + fib(n-2)

from fibonacci import fib
%mprun -f fib fib(35)
```

2 NumPy

Arrays of numbers are the fundamental data structure for data analysis. Python's primitive values have a large overhead. This information is redundant in lists, because the same type information is stored for every element. NumPy arrays are much more efficent than Python's lists—especially for big data sets. Python also offers an array type without redundant type information. However, this array type doesn't offer the fast and powerful operations of NumPy's `ndarray` type.

Conventionally, the NumPy library is imported as follows:

```
import numpy as np
```

2.1 Array Creation

2.1.1 Arrays of Python Lists

NumPy arrays can be created from Python lists:

```
>>> ints = np.array([2, 4, 6, 8]) # integer array
>>> floats = np.array([2, 4, 6, 8.1]) # upcast to float because of 8.1
>>> floats = np.array([2, 4, 6, 8], dtype='float') # with explicit type parameter
>>> ints = np.array([1.1, 2.2, 3.3], dtype='int') # with explicit type parameter
```

NumPy arrays can be multi-dimensional:

```
matrix = np.array([[1, 2, 3],
                   [4, 5, 6],
                   [7, 8, 9]])
```

2.1.2 Arrays from Scratch

Numpy offers various functions to generate arrays from scratch. Where a dimension is required (`size`), a single number (length), a tuple of two (rows, columns) or more (1st dimension, 2nd dimension, 3rd dimension, etc.) can be passed.

- `np.zeros(size, dtype)`: array of zeros
- `np.ones(size, dtype)`: array of ones
- `np.full(size, value)`: array filled with the given value
- `np.arange(start, end, step)`: array with values from start (inclusive) to end (exclusive) and given step width; `length=(end-start)/step`
- `np.linspace(from, to, n)`: array with evenly spaced values in interval `[from,to]` (both inclusive) of length n
- `np.random.random(size)`: uniformly distributed random values
- `np.random.normal(mean, sd, size)`: normally distributed array with the given mean and standard deviation
- `np.random.randint(from, to, size)`: random integers in the interval `[from,to)` (inclusive/exclusive)
- `np.random.choice(a, size, replace, p)`: random values from the array `a` or up to the upper bound value `a` with (`replace=True`) or without (`replace=False`) replacement and an optional array of probabilities `p`
- `np.eye(n)`: identity matrix with n rows and columns (values at indices with equal row/column index are 1)
- `np.empty(size)`: uninitialized array, values from current memory content (garbage)

2.1.3 Data Types

The `dtype` parameter can either be passed as a string literal or using a pre-defined constant:

1. literal: `dtype='int32'`

2. constant: `dtype=np.int32`

Common numeric types are:

- boolean: `Bool_`
- signed integers: `int8`, `int16`, `int32`, `int64`
  - `int_`: system's default `long`
  - `intc`: system's default `int`
- unsigned integers: `uint8`, `uint16`, `uint32`, `uint64`
- floating point: `float16`, `float32`, `float64`
  - `float_`: system default
- complex numbers: `complex64`, `complex128`
  - `complex_`: system default

2.2 Array Manipulation

NumPy arrays offer a rich set of attributes and operation for their manipulation. Since NumPy arrays are the foundation of many higher-level libraries, data manipulation in Python is often NumPy array manipulation.

2.2.1 Attributes

These read-only attributes can be used to retrieve information about an array:

- `ndim`: number of dimensions
- `shape`: size of each dimension
- `size`: total size of the array (the number of elements)
- `dtype`: data type of the array's elements
- `itemsize`: byte size of a single element
- `nbytes`: byte size of the entire array

In general, `nbytes` is equal to `itemsize` multiplied by `size`.

```
>>> np.random.seed(0) # for reproducable results
>>> arr = np.random.randint(10, 100, (3, 3))
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])
>>> arr.ndim
2
>>> arr.shape
(3, 3)
>>> arr.size
9
>>> arr.dtype
```

```
dtype('int64')
>>> arr.itemsize
8
>>> arr.nbytes
72
>>> arr.itemsize * arr.size
72
```

2.2.2 Indexing

Values of NumPy arrays can both be retrieved and modified by the means of indexing.

The indexing of single dimension arrays works with square brackets, just like indexing of Python lists:

- `arr[0]`: first element
- `arr[n]`: nth element
- `arr[-1]`: last element (first element counted from the end)
- `arr[-3]`: third last element (third element counted from the end)

For multi dimension arrays, a comma separated tuple has to be passed in square brackets:

- `arr[0, 0]`: first element of the first dimension
- `arr[3, 5]`: fifth element of the third dimension

```
>>> np.random.seed(0) # for reproducable results
>>> arr = np.random.randint(10, 100, (3, 3))
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])

>>> arr[0, 0]
54

>>> arr[1, 2]
19

>>> arr[-1, -1]
46

>>> arr[2, 2]
46
```

2.2.3 Slicing

The slicing syntax of Python lists also works for NumPy arrays:

- `[start:stop:step]`, with values omitted defaulting to:
  - `start=0`
  - `stop=[size of dimension]`
  - `step=1`
- For a negative step size, the defaults for `start` and `stop` are swapped.

```
>>> arr = np.arange(1, 10)
>>> arr
array([1, 2, 3, 4, 5, 6, 7, 8, 9])

>>> arr[2:5] # third (inclusive) to fifth (exclusive)
array([3, 4, 5])

>>> arr[::2] # every other (beginning with first)
array([1, 3, 5, 7])

>>> arr[::-1] # reversed
array([9, 8, 7, 6, 5, 4, 3, 2, 1])
```

If a step is indicated, two colons are required. Otherwise, step is interpreted as the stop.

Multi-dimension arrays can be sliced by providing multiple, comma-separated slices:

- `[start1:stop1:step1, start2:stop2:step2]`, for slicing the first and second dimension.
- Indexing and slicing can be combined in order to access individual columns/rows:
  - `[:, 0]`: all rows, first column
  - `[0, :]`: first row, all columns
    * `[0]`: shorthand (`:` can be omitted)

```
>>> np.random.seed(0) # for reproducable results
>>> arr = np.random.randint(10, 100, (3, 3))
>>> arr
array([[54, 57, 74],
       [77, 77, 19],
       [93, 31, 46]])

>>> arr[::2, 0:2] # columns 0 and 1 of every other row
array([[54, 57],
       [93, 31]])

>>> arr[:, 0] # first column
array([54, 77, 93])
```

```
>>> arr[0, :] # first row
array([54, 57, 74])

>>> arr[0] # first row (shorthand)
array([54, 57, 74])
```

Unlike Python lists, slices of NumPy arrays are *views to* the original data, not *copies of* it. To get a copy of a slice that can be modified without affecting the underlying array, the `copy()` method can be used. Using the array from above:

```
>>> s = arr[::2, 0:2] # view on columns 0 and 1 of every other row
>>> s
array([[54, 57],
       [93, 31]])

>>> s[0,1] = 88
>>> s[1,0] = 99
>>> s
array([[54, 88],
       [99, 31]])

>>> t = arr[1, 0:2].copy() # copy of columns 0 and 1 of the second row
>>> t
array([77, 77])

>>> t[0] = 11
>>> t[1] = 22
>>> t
array([11, 22])

>>> arr
array([[54, 88, 74],  # 88 introduced through s
       [77, 77, 19],  # 11 and 22 missing (working on copy t)
       [99, 31, 46]]) # 99 introduced through s
```

2.2.4 Reshaping

There are two options to reshape an existing array:

1. The function `reshape(size)`, which reshapes the underlying array to the given size (dimension indications).
    - The new size must match the array's size.
    - Good: `arr.size=60`, `arr.reshape((6, 10))`, because `6*10=60`
    - Bad: `arr.size=16`, `arr.reshape((4, 6))`, because `4*6>16`

2. Using the slicing parameter `np.newaxis`, which converts a one-dimensional to a two-dimensional array.
    - `arr[np.newaxis, :]`: array elements as columns
    - `arr[:, np.newaxis]`: array elements as rows

```
>>> np.arange(1, 10).reshape((3, 3))
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])

>>> np.arange(1, 4)[np.newaxis, :]
array([[1, 2, 3]])

>>> np.arange(1, 4)[:, np.newaxis]
array([[1],
       [2],
       [3]])
```

2.2.5 Concatenation

The options to concatenate arrays of same and different dimensions are:

1. The function `np.concatenate(arrays, axis)`, which works on arrays of the same dimensions.
    - `arrays`: a list or tuple of arrays
    - `axis`: index of the axis, along which the concatenation takes place (0: rows, 1: columns, 2: third dimension)
2. Functions, which concatenate the given arrays of (possible) different dimensions:
    - `np.vstack(arrays)`: stack the arrays vertically
    - `np.hstack(arrays)`: stack the arrays horizontally
    - `np.dstack(arrays)`: stack the arrays along the third dimension

```
>>> a = np.arange(1, 5) # 1, 2, 3, 4
>>> b = np.arange(5, 9) # 5, 6, 7, 8
>>> np.concatenate((a, b))
array([1, 2, 3, 4, 5, 6, 7, 8])

>>> x = a.reshape((2, 2))
>>> x
array([[1, 2],
       [3, 4]])

>>> y = b.reshape((2, 2))
>>> y
array([[5, 6],
```

```
       [7, 8]])

>>> np.concatenate((x, y), axis=0) # along rows
array([[1, 2],
       [3, 4],
       [5, 6],
       [7, 8]])

>>> np.vstack((x, y)) # same, but shorter
array([[1, 2],
       [3, 4],
       [5, 6],
       [7, 8]])

>>> np.concatenate((x, y), axis=1) # along columns
array([[1, 2, 5, 6],
       [3, 4, 7, 8]])

>>> np.hstack((x, y)) # same, but shorter
array([[1, 2, 5, 6],
       [3, 4, 7, 8]])

>>> i = np.arange(1, 4).reshape((3, 1))
>>> i
array([[1],
       [2],
       [3]])

>>> j = np.arange(4, 10).reshape(3, 2)
>>> j
array([[4, 5],
       [6, 7],
       [8, 9]])

>>> np.hstack((i, j))
array([[1, 4, 5],
       [2, 6, 7],
       [3, 8, 9]])

>>> m = np.arange(1, 4)
>>> m
array([1, 2, 3])

>>> n = np.arange(4, 10).reshape((2, 3))
```

```
>>> n
array([[4, 5, 6],
       [7, 8, 9]])

>>> np.vstack((m, n))
array([[1, 2, 4],
       [4, 5, 6],
       [7, 8, 9]])
```

2.2.6 Splitting

An array split up at `N` split points will result in `N+1` arrays. As for reshaping and concatenation, there are two fundamental ways to split arrays:

1. The function `np.split(array, splitpoints)`.
   - `array`: an array of any dimension
   - `splitpoints`: a list of indices
     - a divider (positive integer value) can be used to split the array up into `n` equally sized chunks
2. Functions, which split an array along a specific dimension.
   - `np.hsplit(array, splitpoints)`: split the array along the horizontal axis
   - `np.vsplit(array, splitpoints)`: split the array along the vertically axis
   - `np.dsplit(array, splitpoints)`: split the array along a third dimension

```
>>> a = np.arange(1, 9)
>>> a
array([1, 2, 3, 4, 5, 6, 7, 8])

>>> np.split(a, [4]) # split at index 4 (beginning of second chunk)
[array([1, 2, 3, 4]), array([5, 6, 7, 8])]

>>> np.split(a, 2) # divide into 2 equally sized parts
[array([1, 2, 3, 4]), array([5, 6, 7, 8])]

>>> np.split(a, [2, 6]) # split at indices 2 and 6
[array([1, 2]), array([3, 4, 5, 6]), array([7, 8])]

>>> b = np.arange(1, 10).reshape((3, 3))
>>> b
array([[1, 2, 3],
       [4, 5, 6],
       [7, 8, 9]])

>>> i, j = np.hsplit(b, [2]) # split off first two columns
```

```
>>> i
array([[1, 2],
       [4, 5],
       [7, 8]])

>>> j
array([[3],
       [6],
       [9]])

>>> m, n = np.vsplit(b, [1]) # split off first row
>>> m
array([[1, 2, 3]])

>>> n
array([[4, 5, 6],
       [7, 8, 9]])
```

2.3 Universal Functions

- Loop-based operations on arrays resp. on their elements are slow, because Python performs type-checks and lookups for every function calll.
- NumPy's universal functions (UFuncs) are statically typed and compiled. They can be performed on an array as a whole—and will be applied to each element. This is much faster and more convenient.
  - Loops over arrays should be rewritten in terms of UFuncs. The bigger the array, the larger the gain.
- UFuncs can be applied:
  - to an array and a scalar value:
    * `np.arange(1, 4) * 2 # [2, 4, 6]`
  - to an array and another array:
    * `np.arange(1, 4) * np.arange(7, 10) # [8, 10, 12]`

2.3.1 Common UFuncs

Many of Python's native operators can be used as shorthands for UFuncs:

Shorthand	UFunc	Description
`+`	`np.add`	Addition
`-`	`np.subtract`	Subtraction
`-` (unary)	`np.negative`	Negative Prefix
`*`	`np.multiply`	Multiplication
`/`	`np.divide`	Division

Shorthand	UFunc	Description
`//`	`np.floor_divide`	Floor Division
`**`	`np.power`	Exponentiation
`%`	`np.mod`	Modulus (remainder)
`np.abs`	`np.absolute`	Absolute value

There are a lot of additional mathematical UFuncs:

- `np.sin/np.arcsin`: Sine and Arcsine
- `np.cos/np.arccos`: Cosine and Arcosine
- `np.tan/np.arctan`: Tangents and Cotangents
- `np.exp2`: `2^x`
- `np.exp`: `e^x`
- `np.log`: base-e logarithm
- `np.log2`: base-2 logarithm
- `np.log10`: base-10 logarithm

2.3.2 Advanced Features

Rather than creating a new array for the return value, the result of a UFunc can be stored in an existing array using the `out` parameter. This also works with slices:

```
>>> x = np.arange(1, 6)
>>> x
array([1, 2, 3, 4, 5])

>>> y = np.zeros(5, dtype=np.int)
>>> y
array([0, 0, 0, 0, 0])

>>> np.power(x, 2, out=y)
>>> y
array([1, 4, 9, 16, 25])

>>> z = np.zeros(10)
>>> z
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

>>> np.power(x, 2, out=z[::2]) # overwrite every other element
>>> z
array([1, 0, 4, 0, 9, 0, 16, 0, 25, 0])
```

Every UFunc comes with a `reduce` operation, which repeatedly applies an operation to the elements of an array until only a single result remains.

```
>>> x = np.arange(1, 5)
>>> x
array([1, 2, 3, 4, 5])

>>> np.add.reduce(x) # Sum: 1 + 2 + 3 + 4 + 5
15

>>> np.multiply.reduce(x) # Factorial: 1 * 2 * 3 * 4 * 5
120
```

Instead of just storing the end results, each intermediary step can be stored using the `accumulate` function:

```
>>> x = np.arange(1, 5)
>>> x
array([1, 2, 3, 4, 5])

>>> np.add.accumulate(x)
array([1, 3, 6, 10, 15)

>>> np.multiply.accumulate(x)
array([1, 2, 6, 24, 120])
```

The `outer` operation computes the output of all pairs of two inputs, which could be used to create a multiplication table, for example:

```
>>> a = np.arange(1, 6)
>>> a
array([1, 2, 3, 4, 5])

>>> b = np.arange(1, 9)[1::2]
>>> b
array([2, 4, 6, 8])

>>> np.multiply.outer(b, a) # column, row
array([[ 2,  4,  6,  8, 10],
       [ 4,  8, 12, 16, 20],
       [ 6, 12, 18, 24, 30],
       [ 8, 16, 24, 32, 40]])
```

*	1	2	3	4	5
2	2	4	6	8	10
4	4	8	12	16	20
6	6	12	18	24	30
8	8	16	24	32	40

2.4 Aggregations

Aggregations reduce an array or one of its dimensions to a single value. In contrast to Python's built-in aggregate functions (`sum`, `min`, `max`), NumPy's implementations can operate on multi-dimensional arrays—and are much faster.

- Aggregate functions take an optional `axis` parameter, which describes *the array dimension to be collapsed*:
  - `axis=0`: collapse columns
  - `axis=1`: collapse rows

```
>>> a = np.random.randint(1, 10, size=(3, 4))
>>> a
array([[7, 8, 5, 5],
       [6, 1, 7, 2],
       [7, 2, 8, 8]])

>>> a.sum()
66

>>> a.sum(axis=0)
array([20, 11, 20, 16])

>>> a.sum(axis=1)
array([25, 16, 25])
```

All aggregate functions can be called using the syntax `np.function(array, [parameters])`. Except for `np.median` and `np.percentile`, the following functions can be called directly on the array using the syntax `array.function([parameters])`.

Function	Returns
`np.sum`	sum
`np.prod`	product
`np.min`	minimum value
`np.max`	maximum value
`np.argmin`	index of minimum value
`np.argmax`	index of maximum value
`np.mean`	mean («average») value
`np.median`	median («middle») value
`np.var`	variance
`np.std`	standard deviation
`np.percentile(q=n)`	nth percentile, n in `[0, 100]`
`np.any`	is *any* value true?
`np.all`	are *all* values true?

Special NaN-aware functions exist for every function (execpt for the boolean functions `np.any` and `np.all`). They have the prefix `nan` and can only be called on `nd`, not directly on the array. Since NaN belongs to the IEEE-754 standard, arrays containing NaN must have the type float or double.

```
>>> a = np.array([1, 2, 3, np.NAN, 5])
>>> a
array([ 1.,  2.,  3., nan,  5.])

>>> np.sum(a)
nan

>>> np.nansum(a)
11.0
```

2.5 Broadcasting

Broadcasting is a set of rules for applying binary UFuncs (addition, multiplication, etc.) on arrays of different sizes and/or dimensions.

Rule 1: If the arrays have a different number of dimensions, the *shape* of the array with fewer dimensions is padded with ones on the left.

```
>>> a = np.ones(9).reshape(3, 3)
>>> a
array([[1., 1., 1.],
       [1., 1., 1.],
       [1., 1., 1.]])

>>> b = np.arange(1, 4)
>>> b
array([1, 2, 3])

>>> a.shape
(3, 3)

>>> b.shape
(3,)
```

Result: The shape of `b` is one-padded on the left: `(3,)` → `(1, 3)`. Thus, `array([1, 2, 3])` becomes `array([[1, 2, 3]])`.

Rule 2: If the shape of the arrays does not match in any dimension, the array with a shape of one is stretched in that dimension to match the other shape.

Result: The rows of `b` are stretched (i.e. repeated), the shape changes again: `(1, 3)` → `(3,` 3). Thus, `array([[1, 2, 3]])` becomes:

```
array([[1, 2, 3].
       [1, 2, 3],
       [1, 2, 3]])
```

This is only a *conceptual* transformation, no memory is wasted when stretching!

3. If the dimensions neither match nor are equal to one, an error is raised.

```
>>> x = np.ones(6).reshape(2, 3)
>>> x
array([[1., 1., 1.],
       [1., 1., 1.]])

>>> y = np.arange(1, 3)
>>> y
array([1, 2])

>>> x.shape
(2, 3)

>>> y.shape
(2,)
```

Result: Error.

In order to perform binary operations on incompatible arrays (according these broadcasting rules), the arrays can be re-shaped manually:

```
>>> x + y
ValueError: operands could not be broadcast together with shapes (2,3) (2,)

>>> x + y.reshape(2, 1)
array([[2., 2., 2.],
       [3., 3., 3.]])
```

2.6 Boolean Arrays

Python's comparison operators have NumPy equivalents. They are applied to each element and return a boolean array, indicating the result of every comparison:

Shorthand	UFunc	Description
`==`	`np.equal`	equal
`!=`	`np.not_equal`	not equal
`<`	`np.less`	less than
`>`	`np.great`	greater than

Shorthand	UFunc	Description
`<=`	`np.less_equal`	less than or equal
`>=`	`np.greater_equal`	greater than or equal

```
>>> a = np.random.randint(1, 10, size=(3, 3))
>>> a
array([[3, 4, 6],
       [7, 4, 2],
       [3, 6, 5]])

>>> a == 5
array([[False, False, False],
       [False, False, False],
       [False, False,  True]])

>>> np >= 5
array([[False, False,  True],
       [ True, False, False],
       [False,  True,  True]])


>>> np.less(a, 5)
array([[ True,  True, False],
       [False,  True,  True],
       [ True, False, False]])
```

The number of true values can be counted using the `np.count_nonzero` or the `np.sum` function, which counts `False` as 0 and `True` as 1. Using the array `a` from above:

```
>>> b = a >= 5
>>> b
array([[False, False,  True],
       [ True, False, False],
       [False,  True,  True]])

>>> np.count_nonzero(b)
4

>>> np.sum(b)
4

>>> np.count_nonzero(b, axis=0)
array([1, 1, 2])
```

```
>>> np.sum(b, axis=1)
array([1, 1, 2])
```

2.6.1 Bitmasks

Boolean arrays can be used for indexing, where every `True` item of the index array is returned:

```
>>> x = np.random.randint(1, 100, size=(4, 4))
>>> x
array([[58, 26, 64,  3],
       [91, 64, 44, 31],
       [14, 81, 77,  8],
       [64, 42, 56, 37]])

>>> above_mean = (x > x.mean())
>>> above_mean
array([[ True, False,  True, False],
       [ True,  True, False, False],
       [False,  True,  True, False],
       [ True, False,  True, False]])

>>> x[above_mean]
array([58, 64, 91, 64, 81, 77, 64, 56])
```

Selection criteria can be combined using the bitwise operands, which are shorthand for NumPy's element-wise logical UFuncs:

Shorthand	UFunc	Description
&	`np.bitwise_and`	and
|	`np.bitwise_or`	or
^	`np.bitwise_xor`	exclusive or
~	`np.bitwise_not`	not

Using the arrays `x` and `above_mean` from above:

```
>>> even = (x % 2 == 0)
>>> even
array([[ True,  True,  True, False],
       [False,  True,  True, False],
       [ True, False, False,  True],
       [ True,  True,  True, False]])

>>> x[even & above_mean]
```

```
array([58, 64, 64, 64, 56])

>>> x[np.bitwise_or(even, above_mean)]
array([58, 26, 64, 91, 64, 44, 14, 81, 77,  8, 64, 42, 56])

>>> odd = np.bitwise_not(even)
>>> odd
array([[False, False, False,  True],
       [ True, False, False,  True],
       [False,  True,  True, False],
       [False, False, False,  True]])
```

2.7 Fancy Indexing

Arrays can be indexed using arrays of indices to access multiple array elements at once.

```
>>> x = np.arange(5, 85, 5).reshape((4, 4))
>>> x
array([[ 5, 10, 15, 20],
       [25, 30, 35, 40],
       [45, 50, 55, 60],
       [65, 70, 75, 80]])

>>> x[[3, 1, 2], [2, 3, 1]] # select items (3,2), (1,3) and (2,1)
array([75, 40, 50])
```

2.7.1 Broadcasting

If array indices with different shapes are used, the index arrays are being broadcasted. The result of the index operation is shaped by the *broadcasted index array*, not by the array being indexed. Given the array x from above:

```
>>> rows = np.array([3, 1, 2])[:, np.newaxis)
>>> rows
array([[3],
       [1],
       [2]])

>>> cols = np.array([2, 3, 1]
>>> cols
array([2, 3, 1])

>>> x[rows, cols]
array([[75, 80, 70],
```

```
       [35, 40, 30],
       [55, 60, 50]])
```

The broadcasting of the index arrays is done like this:

	2	3	1
3	3,2	3,3	3,1
1	1,2	1,3	1,1
2	2,2	2,3	2,1

And the resulting array of the indexing operation looks like this:

	2	3	1
3	75	80	70
1	35	40	30
2	55	60	50

Array indices can be combined with scalar indices, slicing and masking:

```
>>> x[2, [1, 0, 3]] # scalar and array index
array([50, 45, 60])

>>> x[2:3, [1, 0, 3]] # slice and array index
array([50, 45, 60])

>>> rows = np.array([2, 3])[:, np.newaxis]
>>> cols = np.array([False, True, False, True])
>>> x[rows, cols] # array index and mask
array([[50, 60],
       [70, 80]])
```

2.7.2 Assignment

Fancy indexing can be used for assignments, too:

```
>>> x = np.arange(10)
>>> x
array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

>>> x[x % 2 == 0] = 0 # set all even values to zero
>>> x
array([0, 1, 0, 3, 0, 5, 0, 7, 0, 9])
```

However, the behaviour can be unexpected if index values are used multiple times:

```
>>> x = np.zeros(3)
>>> x
array([0, 0, 0])

>>> i = [0, 1, 1, 2, 2, 2]
>>> x[i] += 1
>>> x
array([1, 1, 1])
```

The values at indices 1 and 2 haven't been incremented three times, because the value of `x[i] + 1` is evaluated once at the beginning and then used multiple times. For repetitions, NumPy's functions have a `at` method, which performs unbuffered operations, i.e. results will be recalculated for every index element:

```
>>> x = np.zeros(3)
>>> x
array([0, 0, 0])

>>> i = [0, 1, 1, 2, 2, 2]
>>> np.add.at(x, i, 1)
>>> x
array([1, 2, 3])
```

2.8 Sorting

NumPy offers more efficient ways of sorting arrays than Python's native `sort()` function. An array can be sorted using the `np.sort()` function, which returns the sorted array:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> np.sort(x)
array([1, 2, 3, 4, 5])
```

By default, NumPy uses the quicksort algorithm. Other algorithms can be used by setting the `kind` parameter. Options are: `quicksort`, `mergesort`, `heapsort` and `stable`.

An array can also be sorted in-place, using the array's `sort()` method:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> x.sort()
>>> x
array([1, 2, 3, 4, 5])
```

The `np.argsort()` function sorts an array and returns an array of indices denoting the array's order. The returned array can be used for fancy indexing:

```
>>> x = np.array([5, 2, 4, 1, 3])
>>> i = np.argsort(x)
>>> i
array([3, 1, 4, 2, 0])

>>> x[i]
array([1, 2, 3, 4, 5])
```

Arrays can be sorted along rows and columns using the `axis` argument, which defines *along* (*not within!*) which axis the comparison and swapping is performed (0: along rows, 1: along columns):

```
>>> x = np.random.choice(10, (3, 3), replace=False)
>>> x
array([[7, 1, 9],
       [8, 0, 4],
       [2, 3, 6]])

>>> np.sort(x, axis=0) # along rows/within columns
array([[2, 0, 4],
       [7, 1, 6],
       [8, 3, 9]])

>>> np.sort(x, axis=1) # along columns/within rows
array([[1, 7, 9],
       [0, 4, 8],
       [2, 3, 6]])
```

Arrays can be sorted *partially*, i.e. the array is split into two sections, with the left partition containing all smaller values than the right partition. Arrays can be sorted partially using `np.partition()`, which requires the `kth` parameter denoting the size of the left partition (`K` elements):

```
>>> x = np.random.choice(10, 10, replace=False)
>>> x
array([9, 1, 6, 0, 8, 5, 3, 2, 7, 4])

>>> np.partition(x, 3)
array([1, 0, 2, 3, 4, 5, 6, 7, 8, 9])
```

Within the partitions, the elements are in arbitrary order. Partial sorting can also be done by row or column using the `axis` argument. To return the array of partially sorted indices, the function `np.argpartition()` can be used analogous to `np.argsort()`.

2.9 Structured Arrays

Storing heterogeneous data, say names and wages of employees, in different arrays of the same size is error prone: The relation of the data is not obvious, and sorting the arrays mixes up the entries. NumPy offers structured arrays, which can be defined with the `dtype` parameter using a compound data type specification in three ways:

1) using the dictionary method, indicating the field names and formats separately in two tuples:

```
dtype={'names': ('name', 'age', 'salary'),
       'formats': ('U20', 'u1', 'f4')}
```

2) using a list of tuples, defining the field name and its type together in one tuple per field:

```
dtype=np.dtype([('name', 'U20'),
                ('age', 'u1'),
                ('salary', 'f4')])
```

3) without specifying the field names, using automatic names from `f0` to `fn`, and defining the types as a comma-separated string:

```
dtype=np.dtype('U20,u1,f4')
```

A type indicator consists of three parts:

1. the endianness (optional): < for little endian, > for big endian
   - `<f4`: little endian float of four bytes
   - `>i8`: big endian integer of eight bytes
2. the data type (see the next table)
3. the size of the field *in bytes* (not in bits)

Indicator	Type	Example	Equivalent
`'b'`	byte	`'b'`	
`'i'`	signed integer	`'i4'`	`np.int32`
`'u'`	unsigned integer	`'u1'`	`np.uint8`
`'f'`	floating point	`'f8'`	`np.float64`
`'c'`	complex number	`'c16'`	`np.complex128`
`'S'` or `'a'`	string (ASCII)	`'S5'`	
`'U'`	unicode string	`'U10'`	`np.dtype(np.str_, 10)`
`'V'`	raw data (void)	`'V'`	`np.void`

The fields can be accessed by row, by column, by combining row and column, and also using bit masks:

```
>>> employees = np.zeros(3, dtype=np.dtype([('name', 'S10'), ('wage', 'f8')]))
```

```
>>> employees['name'] = ['Dilbert', 'Wally', 'Alice']
>>> employees['wage'] = [120000.00, 80000.00, 110000.00]
>>> employees
array([(b'Dilbert', 120000.),
       (b'Wally', 80000.),
       (b'Alice', 110000.)],
       dtype=[('name', 'S10'), ('wage', '<f8')])

>>> employees[employees['wage'] > 100000]]['name']
array([b'Dilbert', b'Alice'], dtype='|S10')
```

NumPy also allows storing arrays in fields of structured arrays, which can be achieved by providing an optional size indicator to every field definition:

```
>>> players = np.zeros(3, dtype=np.dtype([('name', 'U20'),
                                          ('pattern', 'S1', (3, 3))]))
>>> players[0]['name'] = 'John'
>>> glider = [[' ', 'O', ' '],
              [' ', ' ', 'O'],
              ['O', 'O', 'O']]
>>> players[0]['pattern'] = glider
```

NumPy offers the type `np.recarray`, which allows the individual fields to be accessed with dot notation instead of array indices:

```
>>> payroll = employees.view(np.recarray)
>>> payroll.name
array([b'Dilbert', b'Wally', b'Alice'], dtype='|S10')
```

The syntax is more convenient, but the performance of the access is lower.

NumPy's structured arrays are a very efficient way to store structured data. However, the Pandas library offers much more functionality for working with structured data.

2.10 Date and Time

Python's capabilities for handling date and time information, such as the modules `datetime`, `dateutil` and `pytz`, are convenient to use, but are too slow when it comes to big datasets.

NumPy defines its own type for that purpose: `datetime64`, which encodes date and time information as 64-bit integers, and can be used for vectorized operations:

```
>>> new_year = np.array('2019-01-01', dtype=np.datetime64)
>>> first_week = new_year + np.arange(7)
>>> first_week
array(['2019-01-01', '2019-01-02', '2019-01-03', '2019-01-04',
       '2019-01-05', '2019-01-06', '2019-01-07'], dtype='datetime64[D]')
```

The `timedelta64` data type is used to express the period between two points in time. Both `datetime64` and `timedelta64` are based on a *fundamental time unit* and can express a range of 2^{64} times that unit. There is a trade-off between resolution (precision) and range (time span): The smaller the fundamental time unit is chosen, the more precision and the less time span can be expressed. The fundamental time unit can be defined as follows:

```
>>> np.datetime64('2019-01-01', 'ns') # 'ns': nanoseconds
numpy.datetime64('2019-01-01T00:00:00.000000000')
```

The options available are:

- `Y`: year
- `M`: month
- `W`: week
- `D`: day
- `h`: hour
- `m`: minute
- `s`: second
- `ms`: millisecond
- `us`: microsecond
- `ns`: nanosecond
- `ps`: picosecond
- `fs`: femtosecond
- `as`: attosecond

Nanoseconds are a good compromise, for they are as precise as regular computers and and have a time span of about 500 years (now ± 250 years).

NumPy infers the time-zone automatically from the operating system.

3 Pandas

Pandas is a package built on top of NumPy, which offers powerful data operations familiar to those of data bases and spreadsheets. The fundamental data structures of Pandas are `Series`, `DataFrame` and `Index`. A `DataFrame` is a multidimensional array with labeled rows and columns, which supports heterogeneous and missing data—an issue often to be faced with in real-world data sets.

Pandas is idiomatically imported as `pd`:

```
>>> import pandas as pd
```

3.1 Series

What NumPy's `ndarray` is to Python's list, Pandas `Series` is to Python's dictionary: a fast and very powerful alternative. Whereas Python's dictionary maps a set of *arbitrary keys* to a set of *arbitrary values*, Pandas `Series` maps a set of *typed keys* to a set of *typed values*. A `Series` is made up of two sequences:

1. `values`: a NumPy array (`np.ndarray`)
2. `index`: a Pandas Index (`pd.Index`)

3.1.1 Creation

A Pandas `Series` can be created from scalars, lists and dictionaries.

If a `Series` is generated from list, the indices (first column) for the values (second column) are made up automatically, i.e. sequentially:

```
>>> pd.Series([1, 2, 3])
0    1
1    2
2    3
dtype: int64
```

An list of indices can be explicitly provided using the `index` parameter. The the lists of values and indices need to have the same length:

```
>>> pd.Series([1, 2, 3], index=['a', 'b', 'c'])
a    1
b    2
c    3
dtype: int64
```

However, indices can be noncontiguous and nonsequential:

```
>>> pd.Series([1, 2, 3], index=['Foo', 'Bar', 'Qux'])
Foo    1
Bar    2
Qux    3
dtype: int64
```

If a scalar value is used instead of list of values, the same value will be repeated for the length of the `index` list:

```
>>> pd.Series(42, index=[1, 2, 3])
1    42
2    42
3    42
dtype: int64
```

A `Series` can be created based on a dictionary with keys to be used as indices:

```
>>> pd.Series({'a': 1, 'b': 2, 'c': 3})
a    1
```

```
b    2
c    3
dtype: int64
```

An additional list of indices can be provided to further select values from the dictionary by their keys, and to specify the order of entries:

```
>>> pd.Series({'a': 1, 'b': 2, 'c': 3}, index=['c', 'a'])
c    3
a    1
dtype: int64
```

3.1.2 Access: Indexing and Selection

The elements of a `Series` can be accessed using indexing and slicing:

```
>>> s = pd.Series([1, 2, 3, 4, 5])
>>> s[0]
1

>>> s[4]
5

>>> s[1:4]
1    2
2    3
3    4
dtype: int64
```

If arbitrary (noncontiguous, nonsequential) indices are used, slicing is possible because of the fixed order of indices, but the upper bound is also included:

```
>>> payroll = pd.Series({'Dilbert': 120000, 'Wally': 80000, 'Alice': 110000})
>>> payroll['Dilbert':'Wally']
Dilbert  120000
Wally     80000
dtype: int64
```

Even though a non-numeric is used, a `Series` can also be sliced using a implicit index. Here, the upper bound is excluded:

```
>>> payroll[0:2]
Dilbert    120000
Wally       80000
dtype: int64
```

The elements of a `Series` can also be accessed through the means of masking and fancy indexing:

```
>>> payroll[(payroll >= 100000) & (payroll <= 150000)]
Dilbert    120000
Alice      110000
dtype: int64

>>> payroll[['Alice', 'Dilbert']]
Alice      110000
Dilbert    120000
dtype: int64
```

Python's native dictionary expressions are also supported:

```
>>> 'Dilbert' in payroll
True

>>> 'Asok' in payroll
False

>>> payroll.keys()
Index(['Dilbert', 'Wally', 'Alice'], dtype='object')

>>> list(payroll.items())
[('Dilbert', 120000), ('Wally', 80000), ('Alice', 110000)]

>>> payroll['Wally'] = 90000 # modify existing entry
>>> payroll['Asok']  = 12000 # add a new entry
```

3.1.3 Explicit and Implicit Indexing

When using a explicit integer index, indexing operations make use of the explicit indices (the actual index values provided), but slicing operations use the implicit indices (the items ordinal numbers). This can be confusing:

```
>>> ratings = pd.Series([2.3, 3.1, 3.9, 4.2, 4.8], index=[10, 20, 30, 40, 50])
>>> ratings[10] # explicit index
2.3

>>> ratings[1:3] # implicit index
20    3.1
30    3.9
dtype: float64
```

In order to reduce that confusion, a `Series` offers two attributes to access the indices:

- `loc`: the explicit index
- `iloc`: the implicit index

```
>>> ratings.loc[10]
2.3

>>> ratings.loc[10:30] # inclusive explicit indices from 10 to 30
10    2.3
20    3.1
30    3.9
dtype: float64


>>> ratings.iloc[0]
2.3

>>> ratings.iloc[0:3] # exclusive implicit indices from 0 to 3
10    2.3
20    3.1
30    3.9
dtype: float64
```

According to the Zen of Python («Explicit is better than implicit.»), slicing and indexing on `Series` using a integer index should be done using the `loc` and `iloc` attributes,

3.2 DataFrame

A Pandas `DataFrame` can be understood in terms of other data structures from two perspectives:

1. As a generalization of a NumPy array of two dimensions, with row indices and column names being flexible.
   - NumPy arrays are indexed as `arr[row, column]`: row first, column second.
   - Pandas `DataFrames` are indexed as `df[column][row]`: column first, row of the `Series` second.
2. As a specialization of a Python dictionary that maps a column name (key) to a `Series` of column data (value).

Generally speaking, a `DataFrame` is a sequence of `Series` sharing the index value. Important attributes are:

- `columns`: returns an `Index` object (column names)
- `index`: returns the index labels (row names)

3.2.1 Creation

A Pandas `DataFrame` can be created from `Series`, dictionaries and NumPy arrays.

If a single `Series` is provided, an optional column name for those values can be defined in a list:

```
>>> s = pd.Series([1, 2, 3])
>>> pd.DataFrme(s, columns=['values'])
   values
0       1
1       2
2       3
```

If a list of dictionaries is provided, each dictionary is mapped to a row. Missing entries of heterogeneous dictionaries are filled up with `NaN` in the resulting `DataFrame`:

```
>>> pd.DataFrame([{'a': 1, 'b': 2}, {'a': 5, 'c': 4}])
   a    b    c
0  1  2.0  NaN
1  5  NaN  4.0
```

If a dictionary of `Series` is provided, each `Series` becomes a column with its key mapped as the column name:

```
>>> s1 = pd.Series([2, 4, 6, 8])
>>> s2 = pd.Series[(3, 6, 9, 12])
>>> pd.DataFrame({'two': s1, 'three': s2})
   two  three
0    2      3
1    4      6
2    6      9
3    8     12
```

If a two-dimensional NumPy array is provided, the numeric column and row indices from the array are used, but can be set using the optional `columns` and `index` parameters:

```
>>> arr = np.arange(1, 10).reshape(3, 3)
>>> pd.DataFrame(arr)
   0  1  2
0  1  2  3
1  4  5  6
2  7  8  9

>>> pd.DataFrame(arr, columns=['A', 'B', 'C'], index=[1, 2, 3])
   A  B  C
1  1  2  3
2  4  5  6
```

```
3  7  8  9
```

If a structured NumPy array is provided, the field names serve as column names:

```
>>> employees = np.zeros(3, dtype=np.dtype([('name', 'S10'), ('wage', 'f8')]))
>>> employees['name'] = ['Dilbert', 'Wally', 'Alice']
>>> employees['wage'] = [120000.00, 80000.00, 110000.00]
>>> pd.DataFrame(employees)
         name      wage
0  b'Dilbert'  120000.0
1    b'Wally'   80000.0
2    b'Alice'  110000.0
```

3.2.2 Access: Indexing and Selection

The DataFrame for the following examples:

```
>>> population = {
... 'USA': 326625792,
... 'Russia': 142257520,
... 'Germany': 80594016,
... 'Switzerland': 8236303
... }

>>> area = {
... 'USA': 9147593,
... 'Russia': 16377742,
... 'Germany': 348672,
... 'Switzerland': 39997
... }

>>> data = pd.DataFrame({'pop': population, 'area': area})
>>> data
                  pop      area
Germany      80594016    348672
Russia      142257520  16377742
Switzerland   8236303     39997
USA         326625792   9147593
```

Individual columns can be accessed either dictionary-style or attribute-style, however the latter only works for columns with a string index that isn't used for any other DataFrame attribute:

```
>>> data['area']
Germany          348672
Russia         16377742
```

```
Switzerland       39997
USA             9147593
Name: area, dtype: int64

>>> data.area
Germany          348672
Russia         16377742
Switzerland       39997
USA             9147593
Name: area, dtype: int64

>>> data['area'] is data.area
True

>>> data['pop'] is data.pop
False # pop is a method of DataFrame!
```

For assignments, only dictionary-style access works (on the left side):

```
>>> data['density'] = data['pop'] / data.area
>>> data
                   pop      area     density
Germany       80594016    348672  231.145650
Russia       142257520  16377742    8.686028
Switzerland    8236303     39997  205.923019
USA          326625792   9147593   35.706201
```

The raw, underlying multi-dimensional array of data of a DataFrame can be accessed using the value attribute, which supports array-style indexing:

```
>>> data.values
array([[8.05940160e+07, 3.48672000e+05, 2.31145650e+02],
       [1.42257520e+08, 1.63777420e+07, 8.68602766e+00],
       [8.23630300e+06, 3.99970000e+04, 2.05923019e+02],
       [3.26625792e+08, 9.14759300e+06, 3.57062007e+01]])

>>> data.values[0, 0]
80594016.0
```

A transposed version of the DataFrame (which rows and columns swapped) can be accessed using the T attribute:

```
>>> data.T
              Germany        Russia   Switzerland           USA
pop      8.059402e+07  1.422575e+08  8.236303e+06  3.266258e+08
area     3.486720e+05  1.637774e+07  3.999700e+04  9.147593e+06
density  2.311456e+02  8.686028e+00  2.059230e+02  3.570620e+01
```

A `DataFrame` offers different index attributes:

- `loc`: explicit index to access values by column and row *names*
  - inclusive upper bound
  - supports name based slicing, masking, fancy indexing
- `iloc`: implicit index to access values by column and row *numbers*
  - zero-based, exclusive upper bound
  - supports row and column access by ordinal numbers

```
>>> data.loc['Germany':'Russia', 'pop':'area']
              pop      area
Germany  80594016    348672
Russia  142257520  16377742

>>> data.loc[data.density > 100, ['pop', 'density']]
                  pop     density
Germany      80594016  231.145650
Switzerland   8236303  205.923019

>>> data.iloc[0:2, 0:2]
              pop      area
Germany  80594016    348672
Russia  142257520  16377742
```

3.3 Index

The Pandas `Index` is an immutable array/a ordered (multi)set that is used both for the indexing of `Series` and `DataFrame`.

An `Index` can be created from a list:

```
>>> pd.Index([1, 2, 3, 4, 5])
Int64Index([1, 2, 3, 4, 5], dtype='int64')
```

The elements of the `Index` can be accessed like list entries, i.e. by a single index and using slicing:

```
>>> idx = pd.Index([1, 2, 3, 4, 5])
>>> idx[2]
3

>>> idx[0:2]
Int64Index([1, 2], dtype='int64')

>>> idx[::2]
Int64Index([1, 3, 5], dtype='int64')
```

An `Index` is immutable, which is important when they are shared between different `DataFrames` and `Series`:

```
>>> idx[2] = 6
TypeError: Index does not support mutable operations
```

Like Python's native set, `Index` supports set operations like intersection, union and difference:

```
>>> idxA.intersection(idxB)
Int64Index([1, 3, 5], dtype='int64')

>>> idxA.union(idxB)
Int64Index([1, 2, 3, 4, 5, 7, 9], dtype='int64')

>>> idxA.difference(idxB)
Int64Index([7, 9], dtype='int64')

>>> idxB.difference(idxA)
Int64Index([2, 4], dtype='int64')

>>> idxA.symmetric_difference(idxB)
Int64Index([2, 4, 7, 9], dtype='int64')
```

Union, intersection and symmetric difference can be expressed by the means of operators:

```
>>> idxA & idxB # intersection
Int64Index([1, 3, 5], dtype='int64')

>>> idxA | idxB # union
Int64Index([1, 2, 3, 4, 5, 7, 9], dtype='int64')

>>> idxA ^ idxB # symmetric difference
Int64Index([2, 4, 7, 9], dtype='int64')
```

3.4 Operations

Pandas offers a lot of functions like NumPy's UFuncs that can be applied on a `Series` or `DataFrame` either using a method (with another `Seris` or `DataFrame` as a argument) or using a Python operator:

Operator	Method	Description
`+`	`add()`	Addition
`-`	`sub()`, `subtract()`	Subtraction
`*`	`mul()`, `multiply()`	Multiplication
`/`	`truediv()`, `div()`, `divide()`	Division

Operator	Method	Description
`//`	`floordiv()`	Floor Division
`%`	`mod()`	Modulus (remainder)
`**`	`pow()`	Exponentiation

The index of the operands is preserved in the result. If the operands are heterogeneous, the result contains the union of the two indices, with `NaN` filled in for missing values:

```
>>> hours = pd.Series([25, 40, 32], index=['Alice', 'Bob', 'Malory'])
>>> rates = pd.Series([45, 50, 30], index=['Alice', 'Bob', 'Thomas'])
>>> hours * wages
Alice      1125.0
Bob        2000.0
Malory        NaN
Thomas        NaN
dtype: float64
```

An operation that mixes a `Series` and a `DataFrame` works like an operation on a one-dimensional and a multi-dimensional array; broadcasting rules (similar as those for NumPy) apply:

```
>>> wages = pd.DataFrame({'January': {'Alice': 4500, 'Bob': 4800},
...                       'February': {'Alice': 4200, 'Bob': 4500}})
>>> wages
       January  February
Alice     4500      4200
Bob       4800      4500

>>> increase = pd.Series({'Alice': 1.2, 'Bob': 1.1})
>>> increase
Alice    1.2
Bob      1.1
dtype: float64

>>> wages.T * increase # with transposition
           Alice     Bob
January   5400.0  5280.0
February  5040.0  4950.0

>>> wages.multiply(increase, axis=0) # with optional axis (increase as rows)
       January  February
Alice   5400.0    5040.0
Bob     5280.0    4950.0
```

Pandas always preserves indices and column names, so that the data context is maintained.

3.5 Handling Missing Data

Real-world data sets are rarely clean and homogeneous. Oftentimes, values are missing, and the lack of a value is indicated in different ways. Pandas marks the absence of a value in two different ways:

1. `None`: a Python singleton object, which is used in `object` collections (rather slow due to the overhead).
2. `NaN`: a special floating point value (not a number), which is defined in the IEEE-754 standard and used for numeric collections. NumPy's `NaN` reference is used: `np.nan`.

A `Series` and `DataFrame` containing a `None` or `NaN` «value» is upcast according to the types of the other elements: integer types are upcast to `float64`; booleans are upcast to `object`.

```
>>> pd.Series([1, 2, None]) # None replaced by NaN
0    1.0
1    2.0
2    NaN
dtype: float64

>>> pd.Series([1, 2, np.nan])
0    1.0
1    2.0
2    NaN
dtype: float64

>>> pd.Series([True, False, None]) # None preserved
0     True
1    False
2     None
dtype: object

>>> pd.Series([True, False, np.nan])
0     True
1    False
2      NaN
dtype: object
```

Any operation involving `NaN` yields `NaN`:

```
>>> 3 + np.nan
nan

>>> (3 + 7) * np.nan
```

```
nan

>>> pd.Series([1, 2, np.nan]) + pd.Series([1, np.nan, 3])
0    2.0
1    NaN
2    NaN
dtype: float64
```

Whereas NumPy supports special `NaN`-aware functions (`np.nansum()`, `np.nanmax()`), Pandas offers special functions to deal with absent values:

`isnull()` and `notnull()` return a boolean mask indicating if there is no value (`isnull`) or a value (`notnull`) at the respective index. These masks can be used for indexing:

```
>>> s = pd.Series([1, np.nan, 3])
>>> s.isnull()
0    False
1     True
2    False
dtype: bool

>>> s.notnull()
0     True
1    False
2     True
dtype: bool

>>> s[s.notnull()]
0    1.0
2    3.0
dtype: float64
```

`dropna()` removes `None` and `NaN` entries in a `Series`. In a `DataFrame`, the full row or column missing a value is removed, which can be defined using the optional `axis` parameter:

```
>>> farmers = ['Miller', 'Shaw', 'Watson']
>>> dogs = pd.Series([1, 2, 1], index=farmers)
>>> cats = pd.Series([3, 1, np.nan], index=farmers)
>>> cows = pd.Series([7, np.nan, 2], index=farmers)
>>> pigs = pd.Series([0, 2, np.nan], index=farmers)
>>> livestock = pd.DataFrame( {'dogs': dogs, 'cats': cats, 'cows': cows, 'pigs': pigs})
>>> livestock
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0
Shaw       2   1.0   NaN   2.0
Watson     1   NaN   2.0   NaN
```

```
>>> livestock.dropna() # default: axis='rows'
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0

>>> livestock.dropna(axis='columns')
        dogs
Miller     1
Shaw       2
Watson     1
```

By default, every row/column with at least one missing entry is dropped. If the optional `how` parameter is set to `all`, only rows/columns with missing values only are dropped:

```
>>> livestock.dropna() # default: how='any'
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0

>>> livestock.dropna(how='all')
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0
Shaw       2   1.0   NaN   2.0
Watson     1   NaN   2.0   NaN
```

The optional parameter `thresh` allows to define a threshold: only drop rows/columns with fewer values given:

```
>>> livestock.dropna(thresh=3) # drop rows with fewer than three values
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0
Shaw       2   1.0   NaN   2.0

>>> livestock.dropna(thresh=3, axis='columns')
        dogs
Miller     1
Shaw       2
Watson     1
```

`fillna()` fills in a value where one is missing. Either a scalar value can be passed, or the value from a neighbouring cell can be propagated using a combination of the `method` (`ffill`/`bfill`: forward and backward fill) and `axis` (`rows`/`columns`) parameters:

```
>>> livestock.fillna(0) # replace NaN with 0, which is useful for sums
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0
Shaw       2   1.0   0.0   2.0
Watson     1   0.0   2.0   0.0
```

```
>>> livestock.fillna(method='ffill', axis='rows') # propagate value to next row
        dogs  cats  cows  pigs
Miller     1   3.0   7.0   0.0
Shaw       2   1.0   7.0   2.0
Watson     1   1.0   2.0   2.0

>>> livestock.fillna(method='bfill', axis='columns') # ... from previous column
        dogs  cats  cows  pigs
Miller   1.0   3.0   7.0   0.0
Shaw     2.0   1.0   2.0   2.0
Watson   1.0   2.0   2.0   NaN
```

If there is no next or previous row or column, `NaN` entries could still remain after the `fillna()` operation.

3.6 Hierarchical Indexing

Pandas `Series` and `DataFrame` represent one- and two-dimensional data. But some data must be indexed by more than two indices, and values can only be accessed by a combination of all those indices. This concept is called *hierarchical indexing* or *multi-indexing*.

A index with multiple levels could be represented by a tuple (using Formula 1 teams and seasons as indices):

```
>>> index = [
    ('Mercedes', 2018), ('Mercedes', 2017),
    ('Ferrari', 2018), ('Ferrari', 2017),
    ('McLaren', 2018), ('McLaren', 2017)]
>>> points = pd.Series([655, 688, 571, 522, 62, 30], index=index)
>>> points
(Mercedes, 2018)    655
(Mercedes, 2017)    688
(Ferrari, 2018)     571
(Ferrari, 2017)     522
(McLaren, 2018)      62
(McLaren, 2017)      30
dtype: int64
```

However, storing a tuple as the index is inconvenient and inefficient for data access. Therefore Pandas offers `MultiIndex`, an efficient wrapper for tuple indices:

```
>>> multi_index = pd.MultiIndex.from_tuples(index)
>>> multi_index
MultiIndex(levels=[['Ferrari', 'McLaren', 'Mercedes'], [2017, 2018]],
           labels=[[2, 2, 0, 0, 1, 1], [1, 0, 1, 0, 1, 0]])
```

The `MultiIndex` has two `levels` (the team names and seasons), and they are combined with `labels` like this:

Team	`labels[0]`	`labels[1]`	Season
Mercedes	2	1	2018
Mercedes	2	0	2017
Ferrari	0	1	2018
Ferrari	0	0	2017
McLaren	1	1	2018
McLaren	1	0	2017

A `Series` created with a tuple index can be reindexed using a `MultiIndex`:

```
>>> points = points.reindex(multi_index)
>>> points
Mercedes  2018    655
          2017    688
Ferrari   2018    571
          2017    522
McLaren   2018     62
          2017     30
dtype: int64
```

The blank space below the team index means that the value from above is used.

A `DataFrame` with additional columns can be created based on the existing `DataFrame`:

```
>>> f1 = pd.DataFrame({
    'points': points,
    'races': [21, 20, 21, 20, 21, 20],
    'wins': [11, 12, 6, 5, 0, 0]})
>>> f1
               points  races  wins
Mercedes 2018     655     21    11
         2017     688     20    12
Ferrari  2018     571     21     6
         2017     522     20     5
McLaren  2018      62     21     0
         2017      30     20     0
```

The operations mentioned earlier can also be applied:

```
>>> win_ratio = f1['wins'] / f1['races']
>>> win_ratio
Mercedes  2018    0.523810
          2017    0.600000
```

```
Ferrari   2018    0.285714
          2017    0.250000
McLaren   2018    0.000000
          2017    0.000000
dtype: float64
```

3.6.1 Creation of Hierarchical Indices

A hierarchical index can be created implicitly, i.e. together with the `Series` or the `DataFrame`.

The index can be passed as an additional argument to the constructor as a list of index arrays:

```
>>> points = [655, 688, 571, 522]
>>> index = [['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari'],
             [2018, 2017, 2018, 2017]]
>>> pd.Series(points, index=index)
Mercedes  2018    655
          2017    688
Ferrari   2018    571
          2017    522
dtype: int64
```

Or a dictionary can be passed to the constructor, with appropriate index tuples as keys:

```
>>> points = {('Mercedes', 2018): 655, ('Mercedes', 2017): 688,
              ('Ferrari', 2018): 571, ('Ferrari', 2017): 522}
>>> pd.Series(points)
Mercedes  2018    655
          2017    688
Ferrari   2018    571
          2017    522
dtype: int64
```

Using one of `MultiIndex` class methods, a hierarchical index can be created explicitly. The resulting object can be passed to the constructor of a `Series` or a `DataFrame` as the `index` attribute.

The method `from_arrays` accepts a list of index arrays:

```
>>> pd.MultiIndex.from_arrays([['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari'],
                               [2018, 2017, 2018, 2017]])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
           labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

The method `from_tuples` accepts a list of index tuples:

```
>>> pd.MultiIndex.from_tuples([('Mercedes', 2018), ('Mercedes', 2017),
                               ('Ferrari', 2018), ('Ferrari', 2017)])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
           labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

In the above examples, every item from the first index (`['Mercedes', 'Ferrari']`) has been combined with every item from the second index (`[2018, 2017]`) *manually*. This Cartesian product can also be created automatically using the `from_product` method:

```
>>> pd.MultiIndex.from_product([['Mercedes', 'Ferrari'], [2018, 2017]])
MultiIndex(levels=[['Ferrari', 'Mercedes'], [2017, 2018]],
           labels=[[1, 1, 0, 0], [1, 0, 1, 0]])
```

The index levels can also be combined manually using a nested list of labels passed to the constructor of `MultiIndex`. This is especially helpful, if only certain combinations of index entries need to be created:

```
>>> index = pd.MultiIndex(levels=[['Manor', 'Haas'], [2015, 2016, 2017]],
                          labels=[[0,0,1,1], [0,1,1,2]])
>>> pd.Series([0, 1, 29, 47], index=index)
Manor  2015     0
       2016     1
Haas   2016    29
       2017    47
dtype: int64
```

Using a `DataFrame`, both rows and columns can have multiple indices:

```
>>> row_index = pd.MultiIndex.from_product([['Mercedes', 'Ferrari'],
                                            [2018, 2017]])
>>> col_index = pd.MultiIndex.from_product([['Australia', 'Bahrain'],
                                            ['Driver 1', 'Driver 2']])
>>> pos = np.array([2, 8, 3, 2, 2, 3, 2, 3, 1, 3, 1, np.nan, 1, 4, 1, 4])
>>> f1 = pd.DataFrame(pos.reshape((4, 4)), index=row_index, columns=col_index)
>>> f1
              Australia          Bahrain
               Driver 1 Driver 2 Driver 1 Driver 2
Mercedes 2018       2.0      8.0      3.0      2.0
         2017       2.0      3.0      2.0      3.0
Ferrari  2018       1.0      3.0      1.0      NaN
         2017       1.0      4.0      1.0      4.0
```

This allows for four-dimensional indices.

Both row and column index can be named by setting a list of row/column names with the appropriate length to the names attribute of the index:

```
>>> f1.index.names = ['Team', 'Season']
```

```
>>> f1.columns.names = ['GP', 'Driver']
>>> f1
GP               Australia          Bahrain
Driver            Driver 1 Driver 2 Driver 1 Driver 2
Team     Season
Mercedes 2018          2.0      8.0      3.0      2.0
         2017          2.0      3.0      2.0      3.0
Ferrari  2018          1.0      3.0      1.0      NaN
         2017          1.0      4.0      1.0      4.0
```

3.6.2 Indexing and Slicing

Indexing and Slicing on `Series` is row based. This `Series` index has a species as the first (higher level) index, and the year as the second (lower level) index:

```
>>> idx = pd.MultiIndex.from_product([['cats', 'cows', 'dogs', 'pigs'],
                                        [2000, 2005, 2010]])
>>> livestock = pd.Series([32, 16, 25, 60, 75, 52, 1, 1, 2, 4, 3, 7], index=idx)
>>> livestock
cows  2000    32
      2005    16
      2010    25
pigs  2000    60
      2005    75
      2010    52
dogs  2000     1
      2005     1
      2010     2
cats  2000     4
      2005     3
      2010     7
dtype: int64
```

Individual values can be accessed using full indexing by first indicating the higher level index and second the lower level index:

```
>>> livestock['cats', 2000]
4
>>> livestock['cows', 2010]
25
>>> livestock['pigs', 2005] - livestock['pigs', 2010]
23
```

If the lower level index is left unspecified, a `Series` with the lower level index retained is returned:

```
>>> livestock['cows']
2000    32
2005    16
2010    25
dtype: int64
```

Passing an empty slice for the higher level index allows indexing on the lower level index:

```
>>> livestock[:, 2010]
cows    25
pigs    52
dogs     2
cats     7
dtype: int64
```

Slicing on the explicit index is only available on a dataset with a sorted `MultiIndex`. Either the dataset is created using a sorted `MultiIndex`:

```
>>> idx = idx.sort_values()
>>> livestock = pd.Series([4, 3, 7, 32, 16, 25, 1, 1, 2, 60, 75, 52], index=idx)
```

Or the `MultiIndex` on the existing dataset is sorted, returning a new dataset:

```
>>> livestock = livestock.sort_index()
```

The indices are sorted lexicographically. Then the slicing operations can be performed (on the explicit index):

```
>>> livestock.loc['cats':'cows', 2000:2005]
cats  2000     4
      2005     3
cows  2000    32
      2005    16
dtype: int64
```

Selections can be made based on boolean masks:

```
>>> livestock[livestock > 10]
cows  2000    32
      2005    16
      2010    25
pigs  2000    60
      2005    75
      2010    52
dtype: int64
```

Values can be selected using fancy indexing:

```
>>> livestock[['cows', 'pigs']]
cows  2000    32
```

```
      2005    16
      2010    25
pigs  2000    60
      2005    75
      2010    52
dtype: int64
```

The indexing hierarchy on a `DataFrame` behaves like the one of a `Series`, expect that a `DataFrame` is indexed by columns first:

```
>>> row_idx = pd.MultiIndex.from_product([[2017, 2018],
                                           ['Jan', 'Jul']])
>>> col_idx = col_idx = pd.MultiIndex.from_product([['Tom', 'Jim'],
                                                     ['height', 'weight']])
>>> val = [[122, 35, 129, 37],
           [128, 37, 131, 39],
           [134, 39, 135, 41],
           [137, 40, 138, 43]]
>>> kids = pd.DataFrame(val, columns=col_idx, index=row_idx)
>>> kids
            Tom           Jim
         height weight height weight
2017 Jan    122     35    129     37
     Jul    128     37    131     39
2018 Jan    134     39    135     41
     Jul    137     40    138     43

>>> kids['Tom', 'height']
2017  Jan    122
      Jul    128
2018  Jan    134
      Jul    137
Name: (Tom, height), dtype: int64
```

For row-oriented selection on a `DataFrame`, the implicit index can be used:

```
>>> kids.iloc[0:2]
            Tom           Jim
         height weight height weight
2017 Jan    122     35    129     37
     Jul    128     37    131     39
```

The column index hierarchy can be expressed using the explicit index and tuples:

```
>>> kids.loc[:, ('Tom', 'weight')]
2017  Jan    35
      Jul    37
```

```
2018  Jan    39
      Jul    40
Name: (Tom, weight), dtype: int64
```

Because tuples do not support slices, Pandas offers the `IndexSlice` object:

```
>>> jan = pd.IndexSlice[:, 'Jan']
>>> weight = pd.IndexSlice[:, 'weight']
>>> kids.loc[jan, weight]
            Tom    Jim
         weight weight
2017 Jan     35     37
2018 Jan     39     41
```

3.6.3 Rearranging Multi-Indices

Conceptually, a `Series` with two indices is a lot like a `DataFrame`, which maps the first index to the rows and the second index to the columns. A multi-index `Series` can be converted to a `DataFrame` using the `Series` `unstack()` method:

```
>>> idx = pd.MultiIndex.from_product([[2017, 2018],
                                      ['Bezos', 'Gates', 'Buffet']])
>>> billions = [72.8, 75.6, 86.0, 112, 84, 90]
>>> richest = pd.Series(billions, index=idx.sort_values())
>>> richest
2017  Bezos      72.8
      Buffet     75.6
      Gates      86.0
2018  Bezos     112.0
      Buffet     84.0
      Gates      90.0
dtype: float64

>>> richest.unstack()
      Bezos  Buffet  Gates
2017   72.8    75.6   86.0
2018  112.0    84.0   90.0
```

An optional level can be defined to indicate which index level is to be transformed into a column level:

```
>>> richest.unstack(level=0)
        2017   2018
Bezos   72.8  112.0
Buffet  75.6   84.0
Gates   86.0   90.0
```

```
>>> richest.unstack(level=1)
      Bezos  Buffet  Gates
2017   72.8    75.6   86.0
2018  112.0    84.0   90.0
```

The `DataFrame` can be converted back to a multi-index `Series` using the `stack()` method. The column index will become the lower level index of the row `MultiIndex`:

```
>>> richest.unstack(level=0).stack()
Bezos   2017     72.8
        2018    112.0
Buffet  2017     75.6
        2018     84.0
Gates   2017     86.0
        2018     90.0
dtype: float64
```

The indices of a dataset can be turned into regular columns using the `reset_index()` method, which allows to name the existing data column using an optional argument:

```
>>> richest.index.names = ['year', 'person']
>>> table = richest.reset_index(name='billions')
>>> table
   year  person  billions
0  2017   Bezos      72.8
1  2017  Buffet      75.6
2  2017   Gates      86.0
3  2018   Bezos     112.0
4  2018  Buffet      84.0
5  2018   Gates      90.0
```

Data columns can also be turned (back) into a `MultiIndex` using the `set_index()` method, which expects a list of columns to be used as indices:

```
>>> table.set_index(['year', 'person'])
             billions
year person
2017 Bezos       72.8
     Buffet      75.6
     Gates       86.0
2018 Bezos      112.0
     Buffet      84.0
     Gates       90.0
```

Aggregation methods have optional `level` and `axis` parameters, which allow for partial aggregations:

```
>>> richest.mean(level='year')
year
2017    78.133333
2018    95.333333
dtype: float64

>>> richest.mean(level='person')
person
Bezos     92.4
Buffet    79.8
Gates     88.0
dtype: float64

>>> richest.unstack(level=0).mean(axis=0)
year
2017    78.133333
2018    95.333333
dtype: float64

>>> richest.unstack(level=0).mean(axis=1)
person
Bezos     92.4
Buffet    79.8
Gates     88.0
dtype: float64
```

`level` and `axis` can also be combined, which is useful if both row and column use a `MultiIndex`.

3.6.4 Multi-Indices vs. Panels

Datasets using a `MultiIndex` are *sparse representations* of data: only the existing values are represented. Panels (classes `Panel` and `Panel4D`), in contrast, are *dense representations* of data. A value is stored for every combination of all indices. Since real-world data sets are often sparse, `MultiIndex` datasets are often more efficient than panels.

3.7 Combining Datasets

Conducting interesting studies of data often requires combining datasets from different sources. Pandas offers different facilities to perform this task: concatenations and database-style joins.

3.7.1 Concat and Append

To demonstrate the concatenation of datasets, this function is used to create a `DataFrame` quickly with values made up of column names and row indices:

```
def create_df(cols, index):
    data = {c: [str(c) + str(i) for i in index] for c in cols}
    return pd.DataFrame(data, index)
```

The function can be used thus:

```
>>> create_df('ABC', range(3))
    A   B   C
0  A0  B0  C0
1  A1  B1  C1
2  A2  B2  C2
```

Multiple `Series` or `DataFrames` can be combined using Pandas `concat` function, which expects a list of datasets:

```
>>> a = create_df('ABC', [1, 2, 3])
>>> b = create_df('ABC', [4, 5, 6])
>>> pd.concat([a, b])
    A   B   C
1  A1  B1  C1
2  A2  B2  C2
3  A3  B3  C3
4  A4  B4  C4
5  A5  B5  C5
6  A6  B6  C6
```

By default, the concatenation is performed row-wise (default parameter `axis=0`). The concatenation can be performed column-wise by setting the `axis` parameter either to `1`:

```
>>> a = create_df('ABC', [1, 2, 3])
>>> b = create_df('DEF', [1, 2, 3])
>>> pd.concat([a, b], axis=1)
    A   B   C   D   E   F
1  A1  B1  C1  D1  E1  F1
2  A2  B2  C2  D2  E2  F2
3  A3  B3  C3  D3  E3  F3
```

By default, indices are preserved, even if the resulting index contains duplicates:

```
>>> a = create_df('ABC', [0, 1, 2])
>>> b = create_df('ABC', [2, 3, 4])
>>> pd.concat([a, b])
    A   B   C
```

```
0  A0  B0  C0
1  A1  B1  C1
2  A2  B2  C2
2  A2  B2  C2
3  A3  B3  C3
4  A4  B4  C4
```

The index 2 occurs twice in the resulting dataset above. There are different ways to deal with duplicate indices. The first is to raise an error in case of conflict by setting the `verify_integrity` flag to `True`:

```
>>> pd.concat([a, b], verify_integrity=True)
ValueError: Indexes have overlapping values: Int64Index([2], dtype='int64')
```

An other option is to ignore the existing indices and let Pandas create a new one by setting the `ignore_index` flag to `True`:

```
>>> pd.concat([a, b], ignore_index=True)
    A   B   C
0  A0  B0  C0
1  A1  B1  C1
2  A2  B2  C2
3  A2  B2  C2
4  A3  B3  C3
5  A4  B4  C4
```

The existing indices can be converted to a `MultiIndex` by introducing a higher-level index key describing the source of the entries in the resulting dataset using the `keys` parameter:

```
>>> pd.concat([a, b], keys=['a', 'b'])
      A   B   C
a 0  A0  B0  C0
  1  A1  B1  C1
  2  A2  B2  C2
b 2  A2  B2  C2
  3  A3  B3  C3
  4  A4  B4  C4
```

If datasets with columns in common are concatenated, the resulting dataset is a union of the source datasets (default parameter `join='outer'`). Missing values (in uncommon columns) are filled up as `NaN`:

```
>>> a = create_df('ABC', range(3))
>>> b = create_df('BCD', range(3))
>>> pd.concat([a, b])
     A   B   C    D
0   A0  B0  C0  NaN
1   A1  B1  C1  NaN
```

```
2   A2  B2  C2  NaN
0  NaN  B0  C0   D0
1  NaN  B1  C1   D1
2  NaN  B2  C2   D2
```

If the resulting dataset should only consist of the columns in common of the source datasets, setting the parameter `join='inner'` will create a dataset as an intersection of the source columns:

```
>>> pd.concat([a, b], join='inner')
    B   C
0  B0  C0
1  B1  C1
2  B2  C2
0  B0  C0
1  B1  C1
2  B2  C2
```

For fine-grained control of the resulting columns, the parameter `join_axes` can be set to a `Index` object representing the output columns:

```
>>> pd.concat([a, b], join_axes=[pd.Index(['A', 'B', 'C'])])
     A   B   C
0   A0  B0  C0
1   A1  B1  C1
2   A2  B2  C2
0  NaN  B0  C0
1  NaN  B1  C1
2  NaN  B2  C2
```

An existing `Index` object of the source datasets can also be used:

```
>>> pd.concat([a, b], join_axes=[a.columns])
     A   B   C
0   A0  B0  C0
1   A1  B1  C1
2   A2  B2  C2
0  NaN  B0  C0
1  NaN  B1  C1
2  NaN  B2  C2
```

The `append()` method of a `DataFrame` is a shorthand for the `pd.concat()` function:

```
>>> a = create_df('ABC', range(3))
>>> b = create_df('ABC', [3, 4, 5])
>>> a.append(b)
    A   B   C
0  A0  B0  C0
```

```
1  A1  B1  C1
2  A2  B2  C2
3  A3  B3  C3
4  A4  B4  C4
5  A5  B5  C5
```

It should not be used when combining more than two datasets, because new indices and data buffers are created for every intermediary step.

3.7.2 Merge and Join

Pandas offers high-performance, in-memory join and merge operations. The `pd.merge()` function is the main interface, but `DataFrame` and `Series` also offer a `join()` method for higher convenience.

There are three types of joins:

1. one-to-one (1:1)
2. one-to-many (1:n)
3. many-to-many (n:m)

The type of join to be performed depends solely on the input data.

A one-to-one join is similar to column-wise concatenation. The datasets are automatically joined using a column common to both datasets:

```
>>> employees = pd.DataFrame(
        {'employee': ['Dilbert', 'Catbert', 'Pointy Haired Boss'],
         'department': ['Engineering', 'HR', 'Management']})
>>> employees
             employee   department
0             Dilbert  Engineering
1             Catbert           HR
2  Pointy Haired Boss   Management

>>> departments = pd.DataFrame(
        {'department': ['Management', 'HR', 'Engineering'],
         'location': ['upper floor', 'middle floor', 'basement']})
>>> departments
    department      location
0   Management   upper floor
1           HR  middle floor
2  Engineering      basement

>>> pd.merge(employees, departments)
             employee   department      location
```

```
0             Dilbert  Engineering      basement
1             Catbert           HR  middle floor
2  Pointy Haired Boss   Management   upper floor
```

The index of the input datasets is discarded; a new index is generated for the resulting dataset. The order of entries in the output may be different from the input.

If one of the key columns contains duplicates, a one-to-many join is performed. Using the same `departments`, but a extended `employees` dataset:

```
>>> employees = pd.DataFrame(
        {'employee': ['Dilbert', 'Wally', 'Catbert', 'Pointy Haired Boss'],
         'department': ['Engineering', 'Engineering', 'HR', 'Management']})
>>> employees
             employee   department
0             Dilbert  Engineering
1               Wally  Engineering
2             Catbert           HR
3  Pointy Haired Boss   Management

>>> pd.merge(employees, departments)
             employee   department      location
0             Dilbert  Engineering      basement
1               Wally  Engineering      basement
2             Catbert           HR  middle floor
3  Pointy Haired Boss   Management   upper floor
```

If the key columns on both sides contain duplicates, a many-to-many join is performed:

```
>>> employees = pd.DataFrame(
        {'name': ['Dilbert', 'Wally', 'Catbert'],
         'department': ['Engineering', 'Engineering', 'HR']})
>>> employees
      name   department
0  Dilbert  Engineering
1    Wally  Engineering
2  Catbert           HR

>>> skills = pd.DataFrame(
        {'skill': ['programming', 'thinking', 'thinking', 'manipulating'],
         'department': ['Engineering', 'Engineering', 'HR', 'HR']})
>>> skills
         skill   department
0  programming  Engineering
1     thinking  Engineering
2     thinking           HR
```

```
3  manipulating           HR

>>> pd.merge(employees, skills)
      name   department        skill
0  Dilbert  Engineering  programming
1  Dilbert  Engineering     thinking
2    Wally  Engineering  programming
3    Wally  Engineering     thinking
4  Catbert           HR     thinking
5  Catbert           HR  manipulating
```

These examples all assume *one column common to both datasets*, which is often not given in real-world datasets. The behaviour of `merge()` can be further specified to overcome this constraint.

If there are multiple common columns in both datasets, the column to be joined on can be defined using the on parameter:

```
>>> employees = pd.DataFrame(
        {'id': [1, 2, 3],
         'name': ['Dilbert', 'Wally', 'Catbert'],
         'department': ['Engineering', 'Engineering', 'HR']})
>>> employees
   id     name   department
0   1  Dilbert  Engineering
1   2    Wally  Engineering
2   3  Catbert           HR

>>> departments = pd.DataFrame(
        {'id': [1, 2],
         'department': ['Engineering', 'HR'],
         'location': ['basement', 'middle floor']})
>>> departments
   id   department      location
0   1  Engineering      basement
1   2           HR  middle floor

>>> pd.merge(employees, departments, on='department')
   id_x     name   department  id_y      location
0     1  Dilbert  Engineering     1      basement
1     2    Wally  Engineering     1      basement
2     3  Catbert           HR     2  middle floor
```

If the columns to be joined have a different name, the join can be defined using the `left_on` and `right_on` parameters:

```
>>> employees = pd.DataFrame(
        {'id': [1, 2, 3],
         'name': ['Dilbert', 'Wally', 'Catbert'],
         'department_id': [1, 1, 2]})
>>> employees
   id     name  department_id
0   1  Dilbert              1
1   2    Wally              1
2   3  Catbert              2

>>> departments = pd.DataFrame(
        {'id': [1, 2, 3],
         'department': ['Engineering', 'HR', 'Management']})
>>> departments
   id   department
0   1  Engineering
1   2           HR
2   3   Management

>>> pd.merge(employees, departments,
             left_on='department_id', right_on='id')
   id_x     name  department_id  id_y   department
0     1  Dilbert              1     1  Engineering
1     2    Wally              1     1  Engineering
2     3  Catbert              2     2           HR
```

Redundant columns can be removed from the output using the `drop()` method by providing the name of the column to be discarded, and the argument `axis=1` to specify that the column has to be dropped (as opposed to the row with `axis=0`):

```
>>> pd.merge(employees, departments,
             left_on='department_id', right_on='id').drop('id_x', axis=1)
      name  department_id  id_y   department
0  Dilbert              1     1  Engineering
1    Wally              1     1  Engineering
2  Catbert              2     2           HR
```

Joins can also be performed based on the index instead of on columns. Using the datasets `employees` and `department` from above with appropriate indices, the join can be performed by setting the `left_index` and `right_index` flags to `True`:

```
>>> employees = employees.set_index('id')
>>> employees
       name  department_id
id
1   Dilbert              1
```

```
2     Wally              1
3   Catbert              2

>>> departments = departments.set_index('id')
>>> departments
     department
id
1   Engineering
2            HR
3    Management

>>> pd.merge(employees, departments, left_index=True, right_index=True)
       name  department_id   department
id
1   Dilbert              1  Engineering
2     Wally              1           HR
3   Catbert              2   Management
```

Merging on the index is the default behaviour of the `join()` method:

```
>>> employees.join(departments)
       name  department_id   department
id
1   Dilbert              1  Engineering
2     Wally              1           HR
3   Catbert              2   Management
```

Merging on indices and columns can also be mixed, specifying either the `left_on`/`right_index` or the `left_index`/`right_on` parameter pairs:

```
>>> employees = pd.DataFrame({
        'id': [1, 2, 3],
        'name': ['Dilbert', 'Wally', 'Catbert'],
        'department_id': [1, 1, 2]})
>>> employees
   id     name  department_id
0   1  Dilbert              1
1   2    Wally              1
2   3  Catbert              2

>>> departments = pd.DataFrame({
        'id': [1, 2, 3],
        'department': ['Engineering', 'HR', 'Management']})
>>> departments = departments.set_index('id')
>>> departments
     department
```

```
id
1   Engineering
2            HR
3    Management

>>> pd.merge(employees, departments, left_on='department_id', right_index=True)
   id     name  department_id   department
0   1  Dilbert              1  Engineering
1   2    Wally              1  Engineering
2   3  Catbert              2           HR
```

The type of the join to be performed in terms of set arithmetic can be defined using the how keyword. The default option is inner; only entries common to both input datasets are contained in the result:

```
>>> employees = pd.DataFrame({
        'employee': ['Dilbert', 'Pointy Haired Boss', 'Dogbert'],
        'department': ['Engineering', 'Management', 'Evil Operations']})
>>> employees
             employee       department
0             Dilbert      Engineering
1  Pointy Haired Boss       Management
2             Dogbert  Evil Operations

>>> departments = pd.DataFrame({
        'department': ['Engineering', 'Management', 'Marketing'],
        'location': ['basement', 'upper floor', 'middle floor']})
>>> departments
    department      location
0  Engineering      basement
1   Management   upper floor
2    Marketing  middle floor

>>> pd.merge(employees, departments, how='inner')
             employee   department     location
0             Dilbert  Engineering     basement
1  Pointy Haired Boss   Management  upper floor
```

The option outer fills up missing entries (i.e. entries not common to both input datasets) with NaN in the result:

```
>>> pd.merge(employees, departments, how='outer')
             employee       department      location
0             Dilbert      Engineering      basement
1  Pointy Haired Boss       Management   upper floor
2             Dogbert  Evil Operations           NaN
```

```
3                 NaN        Marketing  middle floor
```

The options `left` and `right` preserve all values from the `left` resp. `right` side, and fill up all the missing entries on the other side with `NaN`:

```
>>> pd.merge(employees, departments, how='left')
             employee       department     location
0             Dilbert      Engineering     basement
1  Pointy Haired Boss       Management  upper floor
2             Dogbert  Evil Operations          NaN

>>> pd.merge(employees, departments, how='right')
             employee   department      location
0             Dilbert  Engineering      basement
1  Pointy Haired Boss   Management   upper floor
2                 NaN    Marketing  middle floor
```

If the two input datasets have columns with the same name that are not used to perform the join operation, a suffix (_x and _y) is added to both columns to prevent conflicts:

```
>>> employees.index.names = ['id']
>>> employees = employees.reset_index()
>>> employees
   id            employee       department
0   0             Dilbert      Engineering
1   1  Pointy Haired Boss       Management
2   2             Dogbert  Evil Operations

>>> departments.index.names = ['id']
>>> departments = departments.reset_index()
>>> departments
   id   department      location
0   0  Engineering      basement
1   1   Management   upper floor
2   2    Marketing  middle floor

>>> pd.merge(employees, departments, on='department')
   id_x            employee   department  id_y     location
0     0             Dilbert  Engineering     0     basement
1     1  Pointy Haired Boss   Management     1  upper floor
```

A list of custom suffixes can be set using the `suffixes` parameter:

```
>>> pd.merge(employees, departments, on='department', suffixes=['_emp', '_dep'])
   id_emp            employee   department  id_dep     location
0       0             Dilbert  Engineering       0     basement
1       1  Pointy Haired Boss   Management       1  upper floor
```

3.8 Aggregation

Computing aggregations is an essential technique for efficient summarization of data sets. The `planets` dataset of the `seaborn` package is useful for practicing aggregations:

```
>>> import seaborn as sns
>>> planets = sns.load_dataset('planets')
```

A good starting point is to get an overview over the dataset using the `describe()` function, which is a convenience method that performs a couple of aggregations for the purpose of understanding rather than further processing the data:

```
>>> planets.describe()
            number  orbital_period       mass     distance         year
count  1035.000000      992.000000  513.000000   808.000000  1035.000000
mean      1.785507     2002.917596    2.638161   264.069282  2009.070531
std       1.240976    26014.728304    3.818617   733.116493     3.972567
min       1.000000        0.090706    0.003600     1.350000  1989.000000
25%       1.000000        5.442540    0.229000    32.560000  2007.000000
50%       1.000000       39.979500    1.260000    55.250000  2010.000000
75%       2.000000      526.005000    3.040000   178.500000  2012.000000
max       7.000000   730000.000000   25.000000  8500.000000  2014.000000
```

Important aggregation functions are:

Function	Returns
`count()`	number of entries (`NaN` not counted)
`min()`	minimum value
`max()`	maximum value
`sum()`	sum (addition)
`prod()`	product (multiplication)
`mean()`	mean (arithmetic average)
`median()`	median (middle value)
`std()`	standard deviation
`var()`	variance
`mad()`	mean absolute deviation

Aggregations on a `DataFrame` result in summarized columns. To aggregate rows instead of columns, the `axis` parameter ban be set accordingly:

```
>>> planets.mean(axis='columns')
```

The `axis` parameters describe what is to be aggregated (the *columns* of each row), not what the result should be!

3.9 Grouping

Grouping allows to split a dataset up based on its values or index, perform computations within the groups and combine the group results together to overall results. Grouping is a three-step process:

1. split: breaking up and grouping a `DataFrame` (based on the values of a specified key or other property)
2. apply: perform computations within each group:
   1. filter: remove or retain values for further processing
   2. transform: map the input values to output values
   3. aggregate: reduce the multitude of values to a single value (or a smaller amount of values)
   4. apply: perform computations on the aggregation result(s)
3. combine: merge the results to a single resulting dataset

The `groupby()` method allows to perform those three steps together in an efficient way. When called on a `DataFrame`, it returns a `DataFrameGroupBy` object, which is a special (grouped) view onto the underlying `DataFrame`:

```
>>> import seaborn as sns
>>> planets = sns.load_dataset('planets')
>>> planets.groupby('year')
<pandas.core.groupby.groupby.DataFrameGroupBy object at 0x7f9db32f2eb8>
```

A `DataFrameGroupBy` is a collection of `DataFrame`s that allows for the operations filter, transform, aggregate and apply. No computation is performed until an aggregation is applied (lazy evaluation), which returns a new `DataFrame`:

```
>>> planets.groupby('year').sum()
      number  orbital_period       mass  distance
year
1989       1       83.888000   11.68000     40.57
1992       6       91.803900    0.00000      0.00
1994       3       98.211400    0.00000      0.00
...
```

Selecting a column on a `DataFrameGroupBy` object returns a `SeriesGroupBy` object, which can be also used for aggregations and the like:

```
>>> planets.groupby('year')['distance']
<pandas.core.groupby.groupby.SeriesGroupBy object at 0x7f9db3224f60>
```

A `GroupBy` object allows to iterate over the individual groups, yielding the group key and the `DataFrame`:

```
>>> for (key, df) in planets.groupby('year'):
        print(key, ', '.join(df.columns))
```

```
1989 method, number, orbital_period, mass, distance, year
1992 method, number, orbital_period, mass, distance, year
1994 method, number, orbital_period, mass, distance, year
...
```

However, the apply() method is usually faster and more convenient than an explicit iteration.

When a method of a DataFrame is called on a GroupBy object, it is dispatched to each of the underlying DataFrame objects:

```
>>> planets.groupby('year').first()
               method  number  orbital_period     mass  distance
year
1989  Radial Velocity       1       83.888000  11.6800     40.57
1992    Pulsar Timing       3       25.262000      NaN       NaN
1994    Pulsar Timing       3       98.211400      NaN       NaN
...
```

As mentioned earlier, after grouping and before combining the data, different operations can be performed on the grouped data.

The filter() method executes a predicate function (or lambda expression) on every entry, retains it in the dataset (matching condition) or discards it from the dataset (not matching condition). The predicate function/lambda expression expects a DataFrame and returns a boolean:

```
>>> teams = ['Mercedes', 'Mercedes', 'Ferrari', 'Ferrari']
>>> drivers = ['Hamilton', 'Bottas', 'Vettel', 'Raikkoenen']
>>> points = [408, 247, 320, 251]
>>> championship = df.DataFrame(
        {'team': teams, 'driver': drivers, 'points': points})
>>> championship
       team       driver  points
0  Mercedes     Hamilton     408
1  Mercedes       Bottas     247
2   Ferrari       Vettel     320
3   Ferrari  Raikkoenen     251

>>> championship.groupby('team').filter(lambda x: x['points'].mean() > 300)
       team    driver  points
0  Mercedes  Hamilton     408
1  Mercedes    Bottas     247
```

The DataFrame is grouped by team. For every team the mean of points scored is calculated, and only entries with a team's point mean above 300 are retained. This filtering uses a predicate function:

```
>>> def below_600(x):
        return x['points'].sum() < 600
>>> championship.groupby('team').filter(below_600)
      team       driver  points
2  Ferrari       Vettel     320
3  Ferrari  Raikkoennen     251
```

The `transform()` method allows to map the input data record by record to output data of the same shape:

```
>>> championship.groupby('team')['points'].transform(lambda x: x / x.mean())
0    1.245802
1    0.754198
2    1.120841
3    0.879159
```

Each driver's ratio of points scored to the team is computed in terms of mean points per team. Notice that the `points` column was selected, so `x` refers to a `Series`, not to a `DataFrame`.

The `aggregate()` method allows to reduce a group in two fundamental ways:

First, by applying one or more aggregation functions that are passed either as a function or as a function name (string):

```
>>> championship.groupby('team').aggregate([min, 'max'])
               driver           points
                  min       max    min  max
team
Ferrari   Raikkoennen    Vettel    251  320
Mercedes       Bottas  Hamilton    247  408
```

Second, by applying different aggregation functions for each column, by providing a dictionary that maps a function to every column:

```
>>> championship['position'] = [1, 5, 2, 3]
>>> championship.groupby('team').aggregate({'points': max, 'position': min})
          points  position
team
Ferrari      320         2
Mercedes     408         1
```

The `apply()` method allows to execute a function on every group result. It takes a `DataFrame`/`Series` and returns either a `DataFrame`/`Series` object, or the function reduces the group results further to a single scalar:

```
>>> championship.groupby('team')['points'].apply(sum)
team
Ferrari     571
```

```
Mercedes    655
Name: points, dtype: int64
```

The grouping of the data is not limited to a single column name. Different alternatives are available.

First, provide a list/array/series/index of group keys, telling every entry in which group to go:

```
>>> names = ['Harry Potter', 'Draco Malfoy', 'Hermine Granger', 'Ron Weasley']
>>> students = pd.Series(names)
>>> houses = ['Griffindor', 'Slytherin', 'Griffindor', 'Griffindor']
>>> students.groupby(houses).apply(lambda s: ', '.join(s))
Griffindor    Harry Potter, Hermine Granger, Ron Weasley
Slytherin                                   Draco Malfoy
dtype: object
```

Second, provide a dictionary that maps the index keys to groups:

```
>>> courses = ['Math', 'English', 'History', 'Geography', 'Music', 'Biology']
>>> results = ['A', 'C', 'E', 'B', 'D', 'F']
>>> grouping = {'A': 'good', 'B': 'good', 'C': 'ok', 'D': 'ok', 'E': 'bad', 'F': 'bad'}
>>> marks = pd.DataFrame({'course': courses, 'result': results})
>>> marks = marks.set_index('result')
>>> marks
           course
result
A            Math
C         English
E         History
B       Geography
D           Music
F         Biology

>>> marks.groupby(grouping).aggregate(lambda c: ', '.join(c))
                course
bad   History, Biology
good   Math, Geography
ok      English, Music
```

Third, provide any function that maps a input (index) to a output (group):

```
>>> lectures = ['Math: Calculus', 'Math: Statistics',
        'Computer Science: Algorithms', 'Computer Science: Data Structures']
>>> professors = ['Smith', 'Myers', 'Dijkstra', 'Kernighan']
>>> plan = pd.DataFrame({'lecture': lectures, 'professor': professors})
>>> plan = plan.set_index('lecture')
```

```
>>> plan
                                   professor
lecture
Math: Calculus                         Smith
Math: Statistics                       Myers
Computer Science: Algorithms        Dijkstra
Computer Science: Data Structures  Kernighan

>>> plan.groupby(lambda l: l.split(':')[0]).aggregate(lambda p: ', '.join(p))
                            professor
Computer Science  Dijkstra, Kernighan
Math                     Smith, Myers
```

And fourth, use a combination thereof, which results in a MultiIndex:

```
>>> marks.groupby([str.lower, grouping]).aggregate(lambda m: ' '.join(m))
           course
a good       Math
b good  Geography
c ok      English
d ok        Music
e bad     History
f bad     Biology
```

3.10 Pivot Tables

Pivot Tables are essentially a multidimensional version of the GroupBy aggregation. A DataFrame can be analyzed in two dimensions. In terms of GroupBy, the split and combine steps are performed along a two-dimensional grid, and the two dimensions can be defined (as index and columns).

The "titanic" dataset of the Seaborn package is a good example for a multidimensional analysis. This GroupBy operation aggregates the survival rates by both sex *and* class:

```
>>> titanic.groupby(['sex', 'class'])['survived'].aggregate('mean').unstack()
class      First    Second     Third
sex
female  0.968085  0.921053  0.500000
male    0.368852  0.157407  0.135447
```

The instruction reads as "group by sex and class, select the survived column, calculate the mean thereof, and display the result in a two-dimensional view".

The same result can be achieved with less typing using the pivot_table() method:

```
>>> titanic.pivot_table('survived', index='sex', columns='class')
class      First    Second     Third
```

```
sex
female  0.968085  0.921053  0.500000
male    0.368852  0.157407  0.135447
```

Calculating the mean is the default aggregation of the `pivot_table()` method. The instruction reads as "calculate the mean of the `survived` column by sex and class".

Grouping is not restricted to single values. More dimensions can be brought in by providing a list of criteria.

The `cut()` method categorizes a series of values using the given boundaries. The age categories are then used as an additional (third) dimension:

```
>>> age = pd.cut(titanic['age'], [0, 18, 80])
>>> titanic.pivot_table('survived', ['sex', age], 'class')
class                First    Second     Third
sex    age
female (0, 18]   0.909091  1.000000  0.511628
       (18, 80]  0.972973  0.900000  0.423729
male   (0, 18]   0.800000  0.600000  0.215686
       (18, 80]  0.375000  0.071429  0.133663
```

The `qcut()` method splits up a series of values to the given number of quantiles. The fare quantiles are then used as an additional (fourth) dimension:

```
>>> fare = pd.qcut(titanic['fare'], 2)
>>> titanic.pivot_table('survived', ['sex', age], [fare, 'class'])
fare            (-0.001, 14.454]                     (14.454, 512.329]
class                      First    Second     Third             First    Second     Third
sex    age
female (0, 18]               NaN  1.000000  0.714286          0.909091  1.000000  0.318182
       (18, 80]              NaN  0.880000  0.444444          0.972973  0.914286  0.391304
male   (0, 18]               NaN  0.000000  0.260870          0.800000  0.818182  0.178571
       (18, 80]              0.0  0.098039  0.125000          0.391304  0.030303  0.192308
```

The `pivot_table()` method has a lot of additional parameters. Its signature looks as follows:

```
DataFrame.pivot_table(values=None, index=None, columns=None, aggfunc='mean',
                      fill_values=None, margins=False, dropna=True,
                      margins_name='All')
```

The parameters have the following meaning:

- `values`: the column of interest (to be aggregated)
- `index`: the y-axis group keys
- `columns`: the x-axis group keys
- `aggfunc`: the aggregation to be performed on `values`
  - accepts either a list of functions
  - or a dictionary specifying column/aggregation pairs (`values` can be omitted)

- `fill_value`: value to use for empty fields
- `margins`: whether or not to compute totals
- `dropna`: whether or not to ignore `NaN` entries
- `margins_name`: labels for the margin totals (default: `'All'`)

Example:

```
>>> titanic.pivot_table(values='survived', index='embark_town', columns='alone',
                        aggfunc='mean', fill_value=False, margins=True,
                        dropna=True, margins_name='survival rate')
alone              False      True  survival rate
embark_town
Cherbourg       0.674699  0.435294       0.553571
Queenstown      0.350000  0.403509       0.389610
Southampton     0.462151  0.256997       0.336957
survival rate   0.505650  0.300935       0.382452
```

3.11 Vectorized String Operations

Real-world datasets often contain a lot of messy string data. Pandas supports vectorized string operations that can easily be applied on entire columns or datasets without worrying about the shape of the data or missing values. Vectorized operations are also more efficient than explicitly iterating over the values and calling the operation on each value.

`Series` and `Index` objects have a `str` attribute that provides functionality to deal with the underlying strings. (A column of a `DataFrame` is a `Series` and therefore also has a `str` attribute.)

Pandas implements a good deal of Python's native string and regular expression functions as methods of the `str` attribute, which are demonstrated on the following dataset:

```
>>> names = ['Dilbert', 'Alice', 'Wally', 'Pointy Haired Boss']
>>> notes = ['nerdy, whiny', 'aggressive, grumpy', 'lazy, dorky', 'clueless, cocky']
>>> review = pd.DataFrame({'employees': names, 'properties': notes})
>>> review
            employees          properties
0             Dilbert        nerdy, whiny
1               Alice  aggressive, grumpy
2               Wally         lazy, dorky
3  Pointy Haired Boss     clueless, cocky
```

Predicate methods check a property of a string and return a `boolean` value indicating whether or not the property in question applies to it:

Method	Description
`startswith(prefix)`	begins with `prefix`?

Method	Description
`endswith(suffix)`	begins with `suffix`?
`isalnum()`	consists of letters and digits only?
`isalpha()`	consists of letters only?
`isdigit()`	consists of digits only? (like 3, 2^2)
`isnumeric()`	is a numeric expression? (like ½, 2^2)
`isdecimal()`	is a numeric expression? (like 123)
`isspace()`	consists of spaces only?
`istitle()`	is every word written in title case?
`islower()`	consists of lower case letters only?
`isupper()`	consists of upper case letters only?

These methods perform a transformation on the underlying string and return the result of that tranformation:

Method	Description
`ljust(width)`	left align to `width`
`rjust(width)`	right align to `width`
`center(width)`	center align to `width`
`pad(width, side)`	justify to `width` with `side` ('left', 'right', 'both')
`zfill(width)`	fill up with `0` from left to `width`
`strip()`	remove trailing whitespace
`lstrip()`	remove trailing whitespace on the left
`rstrip()`	remove trailing whitespace on the right
`wrap(n)`	add newline after `n` characters
`join(s)`	separate characters with string `s`
`cat()`	concatenate the strings
`upper()`	all upper case letters
`lower()`	all lower case letters
`capitalize()`	first letter of first word upper case
`swapcase()`	upper to lower, and lower to upper case
`translate(table)`	apply map of translation rules in `table`
`normalize(form)`	'NFC', 'NFKC', 'NFD' or 'NFKD' unicode normalization
`repeat(n)`	repeats the string `n` times
`slice_replace(a, z, repl)`	replaces the slice `[a:z]` with `repl`
`get(i)/[i]`	get character at index `i`
`slice(a, z, s)/[a:z:s]`	slice (from `a` to `z` with step `s`)

The `translate` method requires a table, which can be created using the `string` method `maketrans`:

```
>>> table = str.maketrans({'t': 'th', 'i': 'y'})
```

```
>>> review['employees'].str.translate(table)
0              Dylberth
1                 Alyce
2                 Wally
3    Poynthy Hayred Boss
```

The following miscellaneous methods return neither a `boolean` value nor a modified string, but either a number or other data structure:

Method	Description
`len()`	length in characters
`find(s)`	start index of substring `s` (-1 if not contained)
`rfind(s)`	like `find()`, but starts from the end
`index(s, a, z)`	like `find()` with range `a:z` (`ValueError` if not contained)
`rindex(s, a, z)`	like `index()`, but starts from the end
`partition(sep)`	split into three parts: before, `sep`, after (default `sep`: whitespace)
`rpartition(sep)`	like `partition()`, but starts from the end
`get_dummies(sep)`	transform encoded string into `DataFrame` using `sep` to split values

The `get_dummies()` method is especially useful when meaning is encoded into a string using multiple, separated values:

```
>>> review['properties'].str.get_dummies(', ')
   aggressive  clueless  cocky  dorky  grumpy  lazy  nerdy  whiny
0           0         0      0      0       0     0      1      1
1           1         0      0      0       1     0      0      0
2           0         0      0      1       0     1      0      0
3           0         1      1      0       0     0      0      0
```

These methods implement functionality from Python's regular expression library (`re`):

Method	Description
`match(pat)`	does the pattern `pat` match? (see `re.match`)
`contains(str)`	is the string `str` contained? (see `re.search`)
`extract(pat)`	extracts the groups from the pattern `pat`
`findall(pat)`	returns all occurences matching `pat`
`replace(pat, repl)`	replaces occurences of `pat` with `repl`
`count(pat)`	number of matches of `pat`
`split(pat)`	split at matches of `pat`
`rsplit(pat)`	like `split()`, but starts from the end

3.12 Time Series

Pandas has strong capabilities to deal with dates, times and data indexed by date and time. The notion of time can be expressed in different concepts:

- *Time stamps* refer to a particular moment, like June 24th 1987, 8:25 a.m.
- *Time intervals* and *periods* express a length of time between a beginning and an end point, like the year 2019 or the second week of 2019.
  - *Periods* are a special kind of interval: They do not overlap with other intervals and are of uniform length, like a day or an hour.
- *Time deltas* or *durations* express an exact length of time, like 9.87 seconds.

Pandas capabilities for dealing with date and time set up on Python's native date and time tools.

Python's built-in `datetime` module with the `datetime` type is useful for expressing single dates:

```
>>> from datetime import datetime
>>> birth = datetime(year=1987, month=6, day=24, hour=8, minute=25)
>>> birth
datetime.datetime(1987, 6, 24, 8, 25)

>>> birth.strftime('%A') # %A: day of week
'Wednesday'
```

The third-party `dateutil` module can parse dates of various string formats:

```
>>> from dateutil import parser
>>> birth = parser.parse("24th of June, 1987 at 8:25 a.m")
>>> birth
datetime.datetime(1987, 6, 24, 8, 25)

>>> birth.strftime('%A') # %A: day of week
'Wednesday'
```

The third-party `pytz` module helps to deal with time zones.

Those tools are convenient, but do not scale for big data sets consisting of date and time information. One alternative is NumPy's `datetime64` type.

A better alternative in the context of Pandas is the `Timestamp` object, which combines the comfort of Python's native `datetime` and third-party `dateutil` with the efficiency of NumPy's `datetime64`.

Dates can be parsed as with `dateutil`:

```
>>> birth = pd.to_datetime("24th of June, 1987 at 8:25 a.m.")
>>> birth
```

```
Timestamp('1987-06-24 08:25:00')

>>> birth.strftime('%A')
'Wednesday'
```

Vectorized operations on dates can be performed as efficiently as with NumPy's `datetime64` type:

```
>>> date = pd.to_datetime("1st of January 2019")
>>> date + pd.to_timedelta(range(3), 'D')
DatetimeIndex(['2019-01-01', '2019-01-02', '2019-01-03'], dtype='datetime64[ns]', freq=None)
```

A `DatetimeIndex` is used to index `Timestamp` objects in a `Series` or `DataFrame`. It offers powerful slicing and indexing operations:

```
>>> index = pd.DatetimeIndex(['2015-01-01', '2016-04-01', '2017-07-01', '2018-10-01'])
>>> dates = pd.Series(range(4), index=index)
>>> dates
2015-01-01    0
2016-04-01    1
2017-07-01    2
2018-10-01    3
dtype: int64

>>> dates['2016-01-01':'2017-12-31'] # slicing
2016-04-01    1
2017-07-01    2
dtype: int64

>>> dates['2016'] # indexing
2016-04-01    1
dtype: int64
```

Pandas implements the different time concepts with different data types and indices:

Concept	Type	Index Type	Python/NumPy Type
Time Stamp	`Timestamp`	`DatetimeIndex`	`datetime/datetime64`
Time Period	`Period`	`PeriodIndex`	`-/datetime64`
Time Delta/Duration	`Timedelta`	`TimedeltaIndex`	`timedelta/timedelta64`

These types and indices can be used directly, but Pandas offers convenience functions for easier parsing and handling of entire `Series`.

The `pd.to_datetime()` function yields a `Timestamp` if a single date is passed, and a `DatetimeIndex` if a series of dates (in any format) is passed:

```
>>> date pd.to_datetime('2018-12-24')
>>> date
Timestamp('2018-12-24 00:00:00')

>>> index = pd.to_datetime(['2018-03-17', '25th of March 1992',
                            datetime(2019, 6, 24), '1984-Jul-20', '20190101'])
DatetimeIndex(['2018-03-17', '1992-03-25', '2019-06-24', '1984-07-20',
               '2019-01-01'],
              dtype='datetime64[ns]', freq=None)
```

A `DatetimeIndex` can be converted to a `PeriodIndex` using the `to_period()` method by indicating a frequency code, like `'D'` for days:

```
>>> periods = index.to_period('D')
PeriodIndex(['2018-03-17', '1992-03-25', '2019-06-24', '1984-07-20',
             '2019-01-01'],
            dtype='period[D]', freq='D')
```

A `timedeltaIndex`, describing the difference between dates, can be created by a subtraction, for example:

```
>>> deltas = index - index[0]
>>> deltas
TimedeltaIndex(['0 days', '-9488 days', '464 days', '-12293 days', '290 days'],
               dtype='timedelta64[ns]', freq=None)
```

3.12.1 Sequences

Pandas offers convenience functions to create regular date sequences. Like Python's `range()` and NumPy's `np.arange()`, they accept a beginning and end point, and an optional frequency.

A sequence of dates can be created using the `pd.date_range()` function:

```
>>> pd.date_range('2018-01-01', '2018-01-08')
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-06', '2018-01-07', '2018-01-08'],
              dtype='datetime64[ns]', freq='D')
```

Instead of defining an end date, the number of periods can be defined:

```
>>> pd.date_range('2018-01-01', periods=8)
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-06', '2018-01-07', '2018-01-08'],
              dtype='datetime64[ns]', freq='D')
```

Any combination of two indications (start, end, frequency) is enough to create a sequence:

```
>>> pd.date_range(start='2018-01-01', end='2018-01-08') # start and end
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-06', '2018-01-07', '2018-01-08'],
              dtype='datetime64[ns]', freq='D')

>>> pd.date_range(start='2018-01-01', periods=8) # start and periods
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-06', '2018-01-07', '2018-01-08'],
              dtype='datetime64[ns]', freq='D')

>>> pd.date_range(end='2018-01-08', periods=8) # end and periods
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-06', '2018-01-07', '2018-01-08'],
              dtype='datetime64[ns]', freq='D')

>>> pd.date_range(start='2018-01-01', end='2018-01-08', periods=4) # all three
DatetimeIndex(['2018-01-01 00:00:00', '2018-01-03 08:00:00',
               '2018-01-05 16:00:00', '2018-01-08 00:00:00'],
              dtype='datetime64[ns]', freq=None)
```

The frequency defaults to one day. In the last example, where start, end *and* periods were given, no fixed frequency is used, but calculated to evenly distribute the dates between start and end.

A frequency can be defined using the `freq` parameter:

```
>>> pd.date_range(start='2018-01-01', periods=4, freq='H')
DatetimeIndex(['2018-01-01 00:00:00', '2018-01-01 01:00:00',
               '2018-01-01 02:00:00', '2018-01-01 03:00:00'],
              dtype='datetime64[ns]', freq='H')

>>> pd.date_range(start='2018-01-01', periods=4, freq='M')
DatetimeIndex(['2018-01-31', '2018-02-28', '2018-03-31', '2018-04-30',
               '2018-05-31', '2018-06-30', '2018-07-31', '2018-08-31'],
              dtype='datetime64[ns]', freq='M')
```

Regular sequences of periods can be created using the `period_range()` function:

```
>>> pd.period_range('2018-01', periods=12, freq='M')
PeriodIndex(['2018-01', '2018-02', '2018-03', '2018-04', '2018-05', '2018-06',
             '2018-07', '2018-08', '2018-09', '2018-10', '2018-11', '2018-12'],
            dtype='period[M]', freq='M')
```

Regular sequences of durations/time deltas can be created using the `timedelta_range()` function:

```
>>> pd.timedelta_range(0, periods=10, freq='H')
```

```
TimedeltaIndex(['00:00:00', '01:00:00', '02:00:00', '03:00:00', '04:00:00',
                '05:00:00', '06:00:00', '07:00:00', '08:00:00', '09:00:00'],
               dtype='timedelta64[ns]', freq='H')
```

Pandas offers the following *date* frequencies (at either the start or end of each period):

Code	Frequency	Code	Frequency
AS	year start	A	year end
BAS	business year start	BA	business year end
QS	quarter start	Q	quarter end
BQS	business quarter start	BQ	business quarter end
MS	month start	M	month end
BMS	business month start	BM	business month end

And these *time* frequencies:

Code	Frequency	Code	Frequency
W	week	T	minute
D	day	S	second
B	business day	L	millisecond
H	hour	U	microsecond
BH	business hour	N	nanosecond

Quarter and year frequencies can be marked with a month suffix, weekly frequencies can be marked with a day suffix in order to specify the split points:

```
>>> pd.date_range('2018-01-01', periods=8, freq='QS-JAN')
DatetimeIndex(['2018-01-01', '2018-04-01', '2018-07-01', '2018-10-01',
               '2019-01-01', '2019-04-01', '2019-07-01', '2019-10-01'],
              dtype='datetime64[ns]', freq='QS-JAN')

>>> pd.date_range('2018-01-01', periods=8, freq='AS-JUL')
DatetimeIndex(['2018-07-01', '2019-07-01', '2020-07-01', '2021-07-01',
               '2022-07-01', '2023-07-01', '2024-07-01', '2025-07-01'],
              dtype='datetime64[ns]', freq='AS-JUL')

>>> pd.date_range('2018-01-01', periods=8, freq='W-SUN')
DatetimeIndex(['2018-01-07', '2018-01-14', '2018-01-21', '2018-01-28',
               '2018-02-04', '2018-02-11', '2018-02-18', '2018-02-25'],
              dtype='datetime64[ns]', freq='W-SUN')
```

The frequency codes refer to instances of the module `pandas.tseries.offsets` and can used as functions:

```
>>> pd.date_range('2018-01-01', periods=8, freq=BDay())
DatetimeIndex(['2018-01-01', '2018-01-02', '2018-01-03', '2018-01-04',
               '2018-01-05', '2018-01-08', '2018-01-09', '2018-01-10'],
              dtype='datetime64[ns]', freq='B')
```

Frequency codes can be combined with additional numbers to create custom periods, such as 1 hour and 45 minutes:

```
>>> pd.date_range('2018-01-01', periods=8, freq='23H15T')
DatetimeIndex(['2018-01-01 00:00:00', '2018-01-01 23:15:00',
               '2018-01-02 22:30:00', '2018-01-03 21:45:00',
               '2018-01-04 21:00:00', '2018-01-05 20:15:00',
               '2018-01-06 19:30:00', '2018-01-07 18:45:00'],
              dtype='datetime64[ns]', freq='1395T')
```

3.12.2 Resampling, Shifting, Windowing

Resampling, Shifting and Windowing are useful operations to analyze time series. Analyzing stock prices is a important use case, and stock prices can be conveniently loaded with the `pandas-datareader` package from Yahoo Finance, for example the closing price of the Microsoft stock:

```
>>> from pandas_datareader import data
>>> msft = data.DataReader('MSFT', start='1986', end='2019', data_source='yahoo')
>>> msft = msft['Close']
>>> msft.describe()
count    8269.000000
mean       25.047959
std        22.397970
min         0.090278
25%         2.992188
50%        25.930000
75%        32.345001
max       115.610001
Name: Close, dtype: float64
```

The stock price over time can be visualized using the matplot library, using the optics from the seaborn package:

```
>>> import matplotlib.pyplot as plt
>>> import seaborn
>>> seaborn.set()
>>> msft.plot();
>>> plt.show();
```

The time series can be resampled to a higher or lower frequency using the `resample()`

Figure 1: Microsoft Stock Price

method, which can be used to perform a data aggregation. The simpler `asfreq()` converts the frequency by simply selecting data (as opposed to aggregating them).

Both methods are used here to visualize the stock price by business year compared to the daily closing prices:

```
>>> msft.plot(style='-', alpha=0.5)
>>> msft.resample('BA').mean().plot(style=':') # mean of business year
>>> msft.asfreq('BA').plot(style='--') # business year's closing price
>>> plt.legend(['original', 'resample', 'asfreq'], loc='upper left')
>>> plt.show()
```

Figure 2: Resampling and Frequency Conversion

Time shifts are useful to compute differences over time. The method `tshift()` can be used to shift the index values, whereas the method `shift()` shifts the data itself. The shift is specified in multiples of the underlying frequency:

```
>>> cs = data.DataReader('CS', start='2000', end='2019', data_source='yahoo')
>>> cs = cs['Close'].asfreq('D')
```

```
>>> cs.plot()
>>> cs.shift(365).plot()
>>> plt.legend(['original', 'shift(365)'], loc='upper left')
>>> plt.show()
```

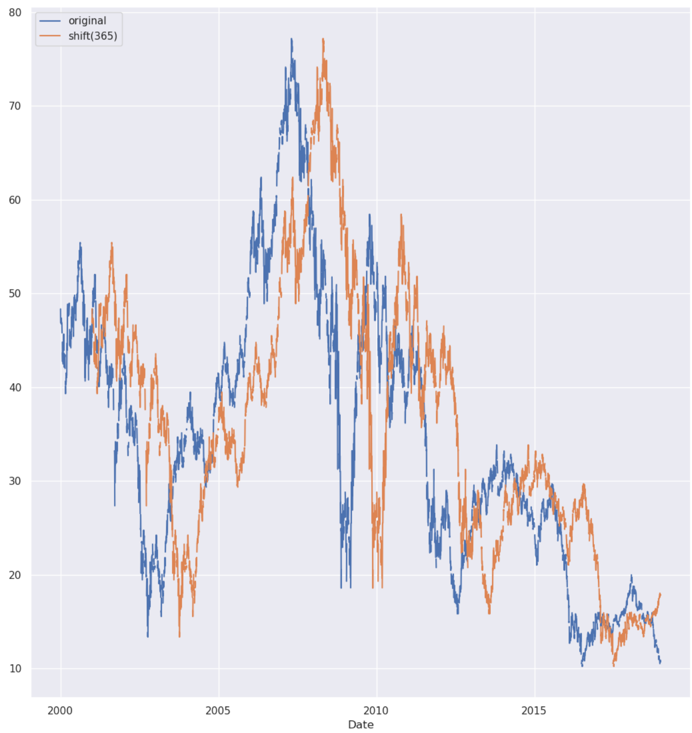

Figure 3: Shifting

Rolling statistics can be used to perform different aggregations over a rolling data window, like the mean of the last 365 days relative to every day.

```
>>> aapl = data.DataReader('AAPL', start='2000', end='2019', data_source='yahoo')
>>> aapl = aapl['Close']
>>> rolling = aapl.rolling(365, center=True)
>>> aapl.plot()
>>> rolling.mean().plot()
>>> plt.legend(['original', 'mean over 365 days'], loc='upper left')
>>> plt.show()
```

Figure 4: Rolling Window

3.13 High-Performance Pandas: `eval()` and `query()`

Even though vectorized operations in NumPy and Pandas are much more efficient than explicit iterations, compound expressions still cause a big memory overhead to store the intermediate steps.

Consider this masking operation:

```
>>> mask = (x > 0.5) & (y < 0.5)
```

Every intermediate step allocates memory, which becomes more obvious if the above expression is written as such:

```
>>> tmp1 = (x > 0.5)
>>> tmp2 = (< < 0.5)
>>> mask = tmp1 & tmp2
```

Pandas `eval()` and `query()` methods, which are based on the Numexpr package, can do without full-sized temporary arrays and hence are much lighter on memory consumtion then vectorized operations.

The `eval()` function accepts a string expression describing an operation on `DataFrames`:

```
>>> %load_ext memory_profiler
>>> n = 100_000_000
>>> cols = 10
>>> df1, df2, df3, df4 = (pd.DataFrame(np.random.random(n).reshape(n//cols, cols))
                          for i in range(4))
>>> %memit df1 + df2 + df3 + df4 # vectorized operation
peak memory: 5175.80 MiB, increment: 1450.38 MiB

>>> %memit sum = pd.eval('df1 + df2 + df3 + df4') # numeric expression
peak memory: 4323.87 MiB, increment: 1159.55 MiB
```

Supported are arithmetic (+, -, *, /), `comparison (==,!=,>,>=,<,<=), bitwise resp. element-wise (&,|) and logical (and,or) operators, as well as indexing (df['col']) and attribute access (df.attr). Constructs like loops and function calls aren't available witheval(), but need direct use of theNumexpr'` package.

`DataFrame` as its own `eval()` method. In addition to the features of the `pd.eval()` function, it supports direct column access by their names and access to variables:

```
>>> n = 3_000_000
>>> df = pd.DataFrame(np.random.random(n).reshape(n//3, 3),
                      columns=['A', 'B', 'C'])

>>> %memit (df['A'] + df['B']) / (df['C'] - 1) # vectorized operation
peak memory: 123.24 MiB, increment: 19.62 MiB
```

```
>>> %memit pd.eval('(df.A + df.B) / (df.C - 1)') # columns as attributes
peak memory: 112.56 MiB, increment: 8.15 MiB

>>> %memit df.eval('(A + B) / (C - 1)') # direct column access
peak memory: 143.60 MiB, increment: 8.21 MiB

>>> %memit df.eval('D = (A + B) / (C - 1)', inplace=True) # create new column
peak memory: 166.62 MiB, increment: 30.73 MiB

>>> %memit df.eval('D = (A + B) / C', inplace=True)
peak memory: 166.66 MiB, increment: 0.00 MiB # overwrite existing column
```

Variables from the enclosing scope can be used with the `@` prefix (in order to distinguish them from columns):

```
>>> mean = df['A'].mean()
>>> %memit df.eval('D = (A + B) / (C - @mean)')
peak memory: 227.79 MiB, increment: 61.04 MiB
```

Masking and filtering expressions cannot be expressed using the `DataFrame.eval()` method. The method `DataFrame.query()` makes this possible:

```
>>> %memit df[(df.A > mean) & (df.B < mean)] # vectorized operation
peak memory: 228.57 MiB, increment: 0.00 MiB

>>> %memit pd.eval('df[(df.A > mean) & (df.B < mean)]') # pd.eval()
peak memory: 230.36 MiB, increment: 1.88 MiB

>>> %memit df.query('A > @mean and B < @mean') # DataFrame.query()
peak memory: 230.98 MiB, increment: 0.00 MiB
```

Notice that the bitwise (element-wise) & operator has to be translated to `and` in the expression for the `query()` method.

`eval()` and `query()` have some downsides:

1. They deal with strings as opposed to Python syntax, which makes it harder to detect syntax errors for both the human eye and tools.
2. They have some computational overhead, which might outweigh the possible savings on temporary memory usage by far.

A good starting point in the decision between vectorized operations and `eval()`/`query()` is the size of a `DataFrame`:

```
>>> n = 300_000
>>> df = pd.DataFrame(np.random.random(n).reshape(n//3, 3),
                      columns=['A', 'B', 'C'])
```

```
>>> df.values.nbytes / (1024*1024) # size in megabytes
2.288818359375
```

If a DataFrame doesn't fit into the CPU cache, heavy vectorized operations may cause the DataFrame to be moved from the ultra-fast cache to the slower memory. Using eval() and query() are potentially more efficient in those cases, but even then the gain in performance and saving in memory is marginal.

The benefit becomes more obvious for big datasets (gigabytes). The intermediate steps create full copies of the underlying DataFrame, so that the data may not even fit into the memory and needs to be swapped on the disk. The computation might not even terminate if the computer runs out of swap space. In those cases, eval() and query() not only help saving memory, but also make some operations possible in the first place.

3.14 Miscellaneous

Pandas allows to read CSV files into a DataFrame. Given the CSV file countries.csv, it can be read as follows:

```
Country, Population, Area
USA, 326625792, 9147593
Russia, 142257520, 16377742
Germany, 80594016, 348672
Switzerland, 8236303, 39997
```

```
>>> countries = pd.read_csv('countries.csv')
>>> countries
       Country  Population      Area
0          USA   326625792   9147593
1       Russia   142257520  16377742
2      Germany    80594016    348672
3  Switzerland     8236303     39997
```

Data can also be read from JSON files, like countries.json, which can be read as follows:

```
{
  "country": [
    "USA",
    "Russia",
    "Germany",
    "Switzerland"
  ],
  "population": [
    326625792,
    142257520,
    80594016,
```

```
    8236303
  ],
  "area": [
    9147593,
    16377742,
    348672,
    39997
  ]
}
```

```
>>> countries = pd.read_json('countries.json')
>>> countries
       country  population      area
0          USA   326625792   9147593
1       Russia   142257520  16377742
2      Germany    80594016    348672
3  Switzerland     8236303     39997
```

4 Matplotlib

Matplotlib is a multiplatform data visualization library built on NumPy arrays. It supports different graphic backends and output styles, and works on virtually any platform. Some projects, including Pandas, offer wrappers around the API of Matplotlib. It is, however, still useful to know how to deal directly with Matplotlib.

Conventionally, Matplotlib is imported as follows:

```
>>> import matplotlib as mpl
>>> import matplotlib.pyplot as plt
```

The plot style can be set on the `plt` object:

```
>>> plt.style.use('classic')
```

Depending on the context, there are different ways of opening the plots for display.

From a script, the method `plt.show()` opens all figures plotted so far:

```
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 100)
plt.plot(x, np.sin(x))
plt.plot(x, np.cos(x))

plt.show()
```

The method `plt.show()` must onle be used once per script or session.

Plots created in a IPython shell can be displayed automatically by calling the `%matplotlib` magic command before calling methods on the `plt` object. The plot will be displayed in a separate window. The method `plt.draw()` forces the output to be updated.

```
>>> import matplotlib as mpl
>>> import matplotlib.pyplot as plt
>>> import numpy as np

>>> %matplotlib
Using matplotlib backend: Qt5Agg

>>> x = np.linspace(0, 10, 100)
>>> plt.plot(x, np.sin(x))
```

From within a Jupyter Notebook, there are two options to display plots:

1. `%matplotlib inline`: display plots as static images
2. `%matplotlib notebook`: display interactive plots

The latter option will draw every plot output in the most recent figure, which can be created using the `plt.figure()` method:

```
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 100)

%matplotlib notebook

plt.figure()
plt.plot(x, np.sin(x))
plt.plot(x, np.cos(x))
```

A figure object can be saved using its `savefig()` method, which requires a file name. Notice that the `plot()` method only draws into the most recent figure object created, if the magic command `%matplotlib` hasn't been used before:

```
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np

fig = plt.figure()
x = np.linspace(0, 10, 100)
plt.plot(x, np.sin(x))
```

```
plt.plot(x, np.cos(x))
fig.savefig('sin-x-cos-x.png')
```

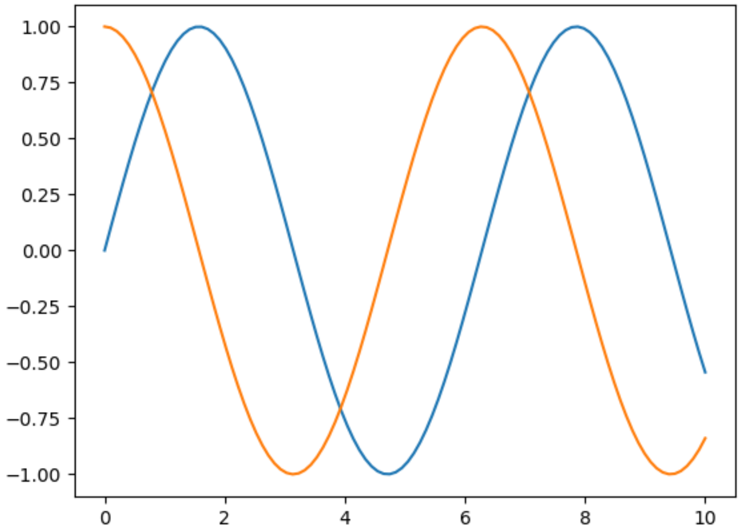

Figure 5: Plot of `sin(x)` and `cos(x)`

An image–no longer a plot!–can be loaded using IPython's `Image` object:

```
>>> from IPython.display import Image, display
>>> img = Image('sin-x-cos-x.png')
>>> display(img)
```

For both saving and loading, the file format is inferred from the file's extension. The formats supported by the graphics backend in use can be retrieved as a dictionary from a `figure` object:

```
>>> import matplotlib as mpl
>>> import matplotlib.pyplot as plt

>>> fig = plt.figure()
>>> fig.canvas.get_supported_filetypes()
{'ps': 'Postscript',
 'eps': 'Encapsulated Postscript',
 'pdf': 'Portable Document Format',
 'pgf': 'PGF code for LaTeX',
 'png': 'Portable Network Graphics',
 'raw': 'Raw RGBA bitmap',
 'rgba': 'Raw RGBA bitmap',
```

```
 'svg': 'Scalable Vector Graphics',
 'svgz': 'Scalable Vector Graphics'}
```

4.1 Interfaces: MATLAB-style and Object Oriented

Matplotlib started out as a Python alternative for MATLAB. The `plt` object represents the stateful interface known to MATLAB users. Plots created on the `plt` object are drawn to the figure and axes objects that have been created most recently.

In this example, two subplots on a single figure are created:

```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 100)
plt.figure() # create a new figure
plt.subplot(2, 1, 1) # (row, column, panel): first panel on a 2*1 field
plt.plot(x, np.sin(x)) # plot to the first subplot
plt.subplot(2, 1, 2) # second panel on the same 2*1 field
plt.plot(x, np.cos(x)) # plot to the second subplot
plt.show()
```

It is possible to plot on other figures/axes than the current active, but only if their references have been retrieved and stored using `plt.gcf()` (get current figure) and `plt.gca()` (get current axes):

```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 100)
plt.figure()
plt.subplot(2, 1, 1)
plt.plot(x, np.sin(x))
first = plt.gca() # store reference to first aces
plt.subplot(2, 1, 2)
plt.plot(x, np.cos(x))
first.plot(x, np.cos(x)) # also draw cosine on first axes
plt.show()
```

"Going back" is not possible if one fails to store the such references, especially in an interactive session. The object-oriented interface of Matplotlib doesn't rely on a *current state*, but requires the user to always explicitly refer to the figure/axes to be dealt with:

```
import matplotlib.pyplot as plt
import numpy as np
```

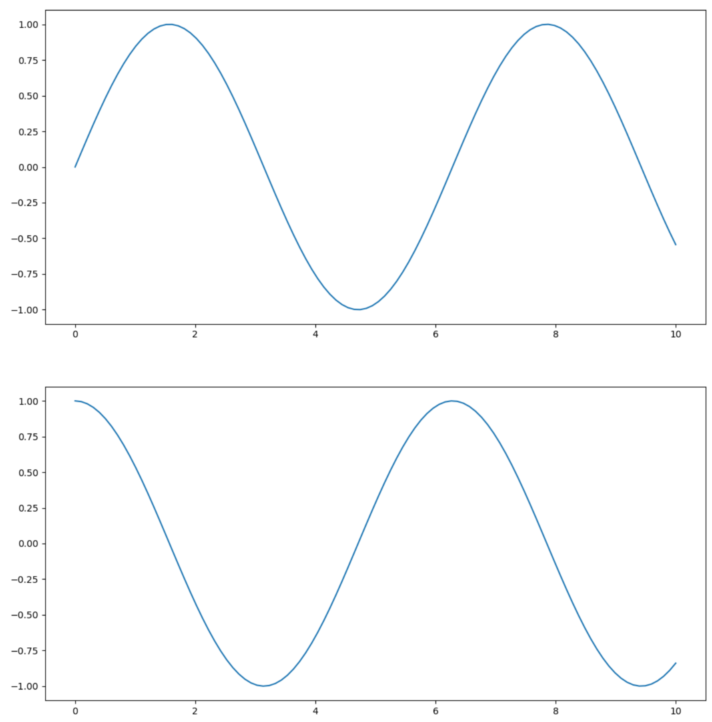

Figure 6: MATLAB-style interface: Subplots

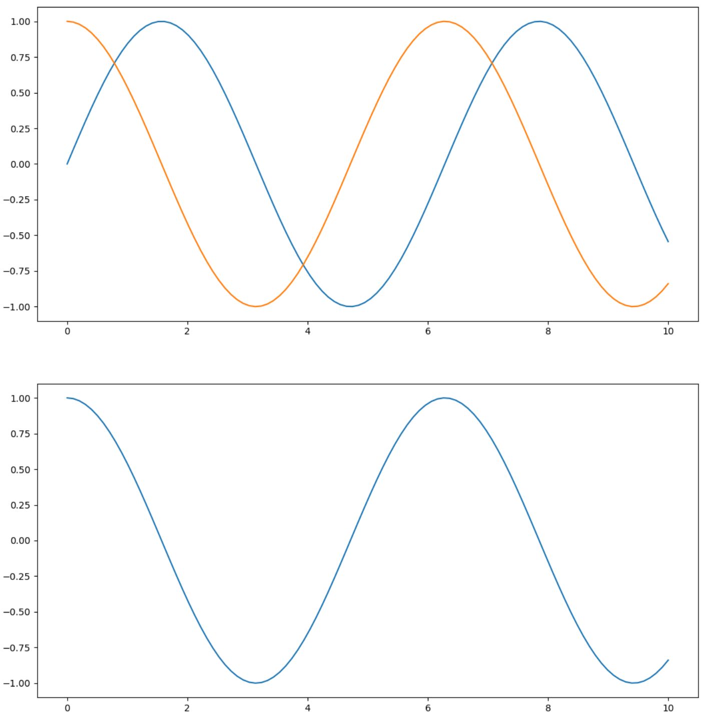

Figure 7: MATLAB-style interface: Draw to “inactive” Axes

```
x = np.linspace(0, 10, 100)
fig, ax = plt.subplots(2)
ax[0].plot(x, np.sin(x))
ax[1].plot(x, np.cos(x))
ax[0].plot(x, np.cos(x))
plt.show()
```

The choice between the two interfaces is mostly a matter of preference for simple tasks. More complicated plots, however, do require the object-oriented approach.

4.2 Line Plots

Simple functions of the form `y=f(x)` can be visualized using line plots. The following examples require this boilerplate code:

```
import matplotlib.pyplot as plt
import numpy as np

%matplotlib inline

plt.style.use('seaborn-whitegrid') # simple style

x = np.linspace(0, 10, 100) # 100 points in range 0..10
```

A figure, implemented by `plt.Figure`, contains all the graphics objects, like text, labels – and the axes. A axes, implemented by `plt.Aces`, is a bounding box with ticks and labels, which contains the plotted lines. Conventionally, the objects are called `fig` and `ax`:

```
fig = plt.figure()
ax = plt.axes()
```

The sine function of the `x` values computed before can be drawn using the aces `plot()` method:

```
ax.plot(x, np.sin(x)) # plot x and y=sin(x)
```

The MATLAB-style interface can be used alongsinde, plotting to the figure/axes used most recently:

```
plt.plot(x, np.cos(x)) # plot x and y=cos(x)
```

The lines get a color assigned automatically from a predefined set. The colors can also be assigned manually using the `color` keyword of the `plot()` method. The following options are supported:

- HTML color name: `blue`, `green`, `fuchsia` etc. (common HTML color names)
- RGB/CMYK short code: `r`, `g`, `b`, `c`, `m`, `y`, `k`
- Grayscale value: floating point number between `0` (black) and `1` (white)

- RGB hex code: `#ff00aa`, `#efefef`
- RGB tuple with floating point numbers between `0` and `1`: `(0.1, 0.75, 0.66)`

The line style can be adjusted using the `linestyle` keyword. The following options are supported, both having a short and a long form:

- `-/solid`
- `--/dashed`
- `-./dashdot`
- `:/dotted`

```
ax.plot(x, np.sin(x-0), color='fuchsia', linestyle='-')
ax.plot(x, np.sin(x-1), color='m', linestyle='--')
ax.plot(x, np.sin(x-2), color='0.25', linestyle='-.')
ax.plot(x, np.sin(x-3), color='#0a123b', linestyle=':')
ax.plot(x, np.sin(x-4), color=(0.1, 0.75, 0.66))
```

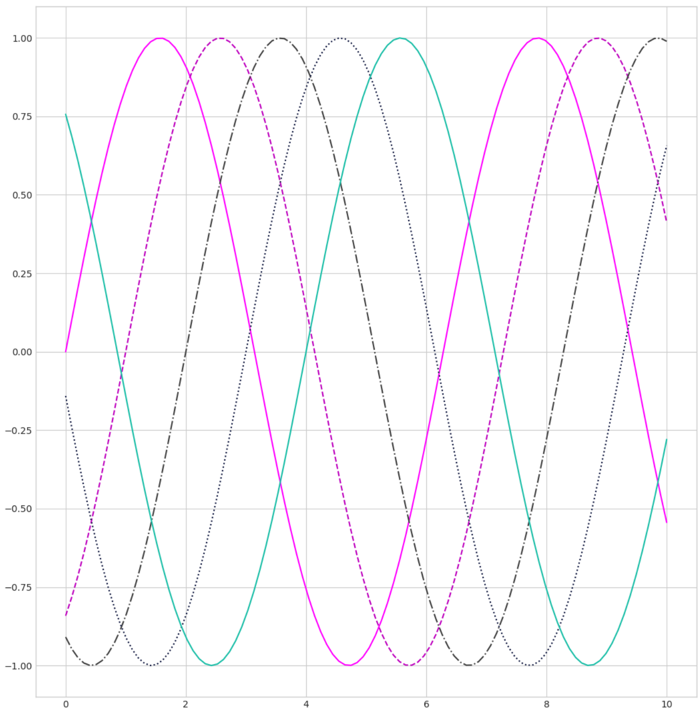

Figure 8: Line Colors and Styles

The MATLAB-style interface `plt.plot()` accepts a shorthand style indicator as a third nonkeyword argument, combining a line style with a a RGB/CMYK color code:

```
plt.plot(x, np.sin(x-5), ':y') # dotted yellow line
```

4.3 Limits, Labels, Legends

The axes limits can be set using the `plt.xlim()` and `plt.ylim()` function (MATLAB-style) or using the `ax.set_xlim()` and `ax.set_ylim()` method (OO-style) by passing a lower and a upper bound:

```
plt.xlim(-5, 5)
plt.ylim(-1, 1)
```

A plot can be flipped along both axis by passing the arguments in reverse order (using an `axes` object here):

```
ax.set_xlim(5, -5)
ax.set_ylim(1, -1)
```

The `plt.axis()` method allows to set both ranges at once by providing a list of the form `[xmin, xmax, ymin, ymax]`:

```
plt.axis([-5, 5, -1, 1])
```

The ranges can be set automatically to just fit in the plot by using the `'tight'` parameter:

```
plt.axis('tight')
```

The `'equal'` parameter makes sure the plot fits in and that the x and y axis are scaled equally:

```
plt.axis('equal')
```

The axes object supports the same method: `ax.axis()`.

Both axis and the plot as a whole can be labeled using the `plt.xlabel()`, `plt.ylabel()` and `plt.title()` function (MATLAB-style) or the `ax.set_xlabel()`, `ax.set_ylabel()`, `ax.set_title()` method (OO-style):

```
plt.xlabel('x')
plt.ylabel('y=sin(x)')
plt.title('A Sine Curve')

ax.set_xlabel('x')
ax.set_ylabel('y=cos(x)')
ax.set_title('A Cosine Curve')
```

Lines with different styles and colors can be labeled with a legend by calling the `plt.legend()` function or the `ax.legend()` method, which requires the individual plots (as opposed to its axis) to be labeled with the `plot()` call (keyword `label`):

```python
plt.plot(x, np.sin(x), '-g', label='sin(x)')
plt.plot(x, np.cos(x), ':b', label='cos(x)')
plt.legend()

ax.plot(x, np.sin(x), color='green', linestyle='-', label='sin(x)')
ax.plot(x, np.cos(x), color='blue', linestyle=':', label='cos(x)')
ax.legend()
```

The `ax.set()` method is a convenient interface for setting limits (using tuples), labels and a title all at once:

```python
ax.set(xlim=(0, 10), ylim=(-1, 1),
       xlabel='x', ylabel='sin(x)',
       title='A Sine Curve')
```

Bringing it all together (in a script):

```python
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 100)

ax = plt.axes()
ax.set(xlim=(0, 10), ylim=(-1, 1),
       xlabel='x', ylabel='y',
       title='Sine and Cosine')

ax.plot(x, np.sin(x), color='green', linestyle='-', label='sin(x)')
ax.plot(x, np.cos(x), color='blue', linestyle=':', label='cos(x)')
ax.legend()

plt.show()
```

4.4 Scatter Plots

Scatter plots represetn the data points individually instead of joining them with a line. The `plt.plot()` function is capable of producing scatter plots, if the third argumetnis a character representing an according symbol, such as `'o'`, `'.'`, `','`, `'x'`, `'+'`, `'v'`, `'^'`, `'<'`, `'>'`, `'s'`, `'d'`:

```python
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(0, 10, 50)
plt.plot(x, np.sin(x), 'o', label='sin(x)')
plt.plot(x, np.cos(x), 'x', label='cos(x)')
```

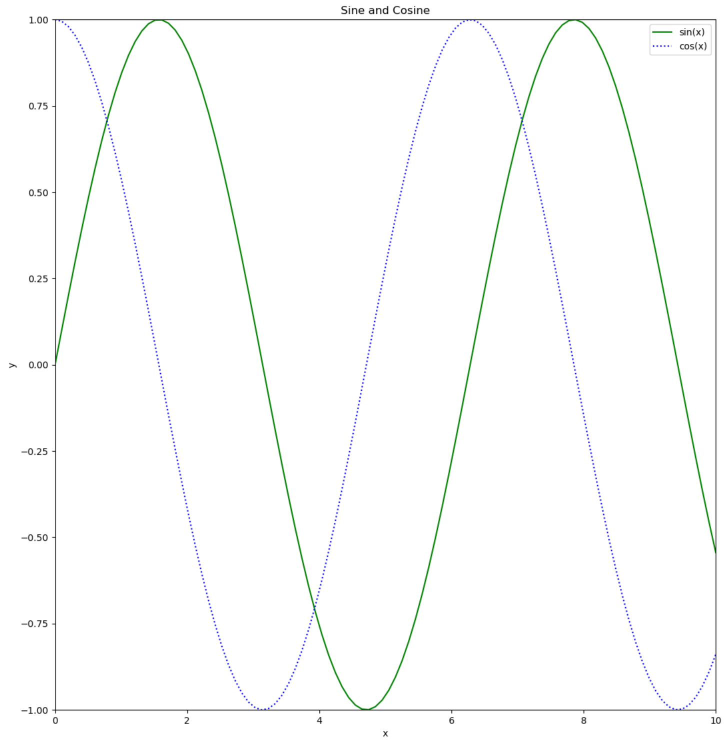

Figure 9: Limits, Labels, Legends

```
plt.legend()
plt.show()
```

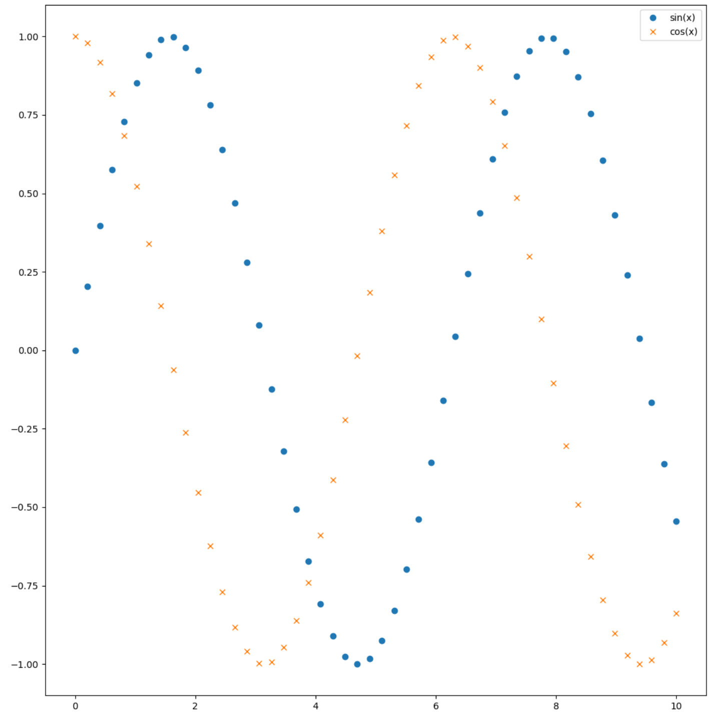

Figure 10: Sine and Cosine curve as a scatter plot

The dots can be connected when combining the style parameter with a line style:

```
plt.plot(x, np.sin(x), 'o-c', label='sin(x)') # dots & solid cyan line
plt.plot(x, np.cos(x), 'x:m', label='cos(x)') # crosses & dotted magenta line
```

The lines and markers (points) can be further specified using the following arguments of the `plt.plot()` function:

- `markersize`
- `markerfacecolor`
- `markeredgecolor`
- `markeredgewidth`
- `linewidth`

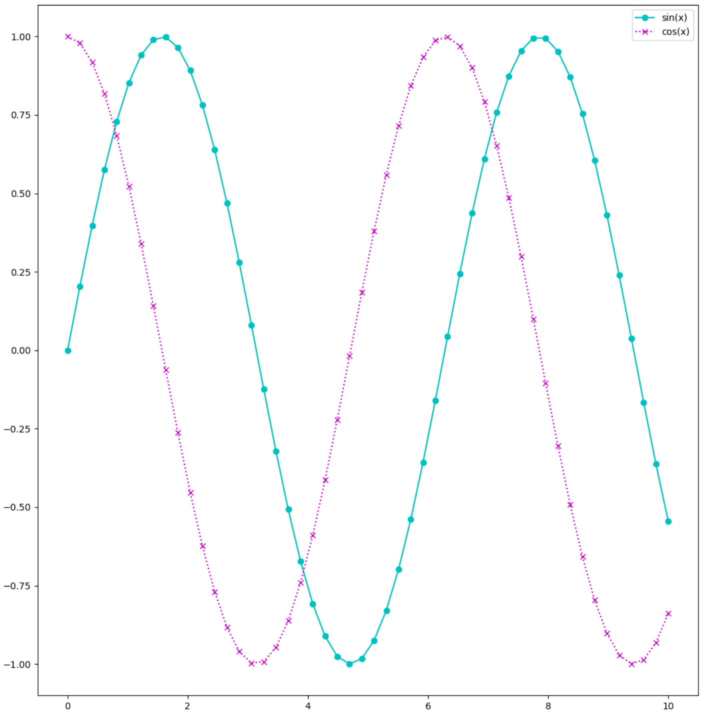

Figure 11: Points connected with a line

```python
plt.plot(x, np.sin(x), 'o-c', markersize=10, markerfacecolor='blue',
         markeredgecolor='white', markeredgewidth=2, linewidth=3)
```

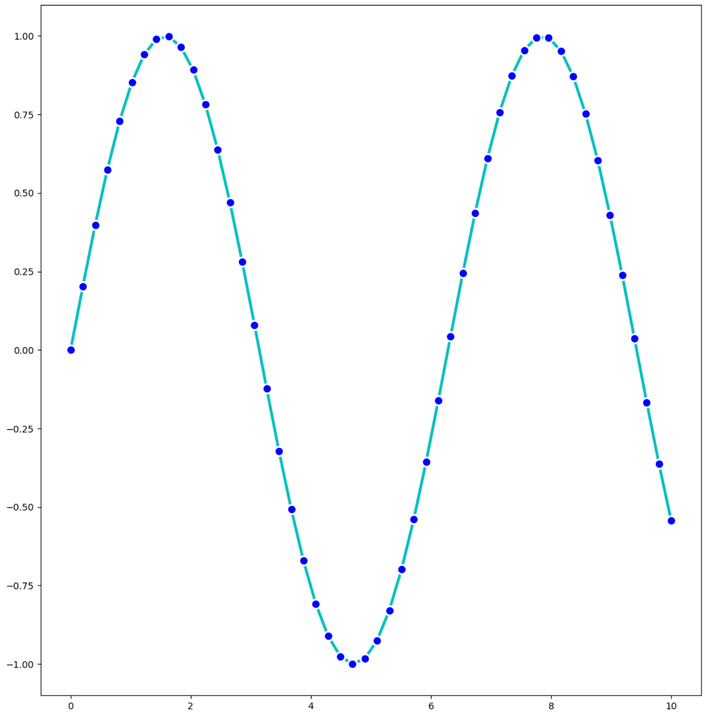

Figure 12: Marker options

The function `plt.scatter()` can set the individual properties of each point by passing a list instead of a single value as the size and color parameters (`s` and `c`):

```python
n_points = 100
rng = np.random.RandomState(0)
x = rng.randn(n_points)
y = rng.randn(n_points)

colors = rng.randn(n_points)
sizes = 500 * rng.randn(n_points)

plt.scatter(x, y, c=colors, s=sizes, alpha=0.3)

plt.colorbar()
```

```
plt.show()
```

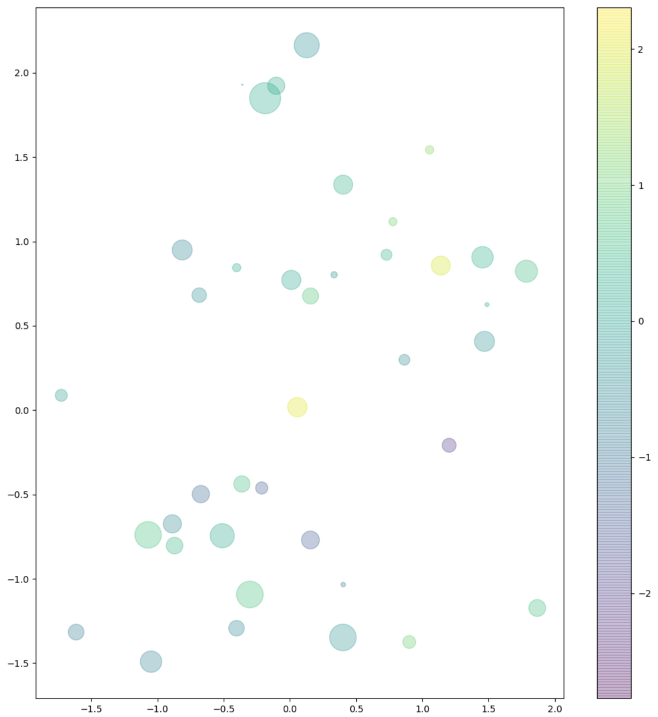

Figure 13: Scatter plot with individual marker sizes and colors

This is useful for visualizing multi-dimensional data (four dimensions: x and y value, color and size).

Because `plt.scatter()` figures out the rendering for each individual point separately, it can be slower than `plt.plot()`, especially when dealing with big data sets. If all the scatter points are to be drawn alike, `plt.plot()` should be prefered to `plt.scatter()`.

The OO-style interface (`ax.plot()`, `ax.scatter()`) works with the same parameters.

4.5 Visualizing Errors

In many applications, reporting the range of possible error is just as important as reporting the value itself.

For discrete values, Matplotlib can plot error bars using the `plt.errorbar()` function. The error range, either on the x- or y-axis, can be set using the parameter `xerr` or `yerr`, respectively. The `fmt` parameter accepts a format specifier consisting of style and color code:

```
rng = np.random.RandomState(0)
points = 20
dy = 0.5

x = np.linspace(0, 10, points)
err = dy * rng.randn(points)
y = np.sin(x) + err

plt.errorbar(x, y, yerr=dy, fmt='.')
plt.show()
```

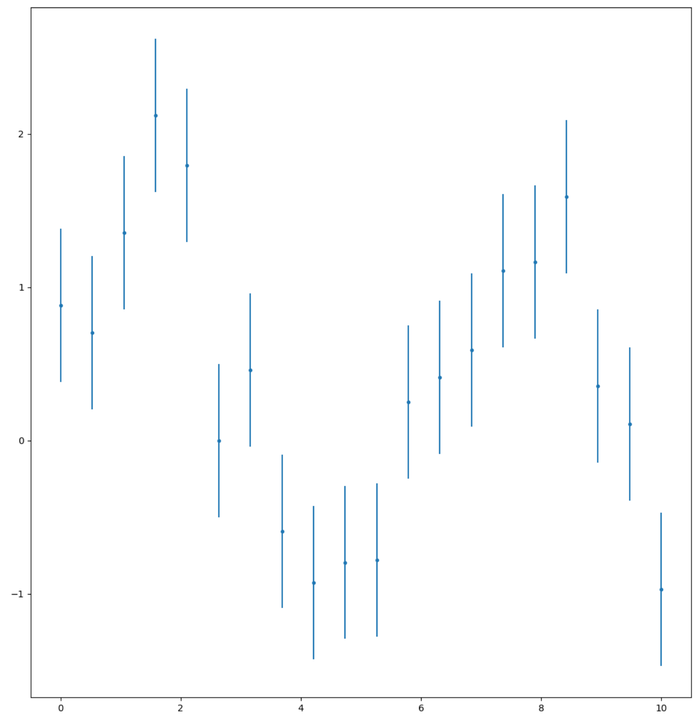

Figure 14: Vertical Error Bars

The error bar can be further fine-tuned by specifying the `ecolor` (color of the bar), the `elinewidth` (the width of the error bar) and the `capsize` (the size of the ticks orthogonal to the error bar) parameters:

```
plt.errorbar(x, y, yerr=dy, fmt='.', ecolor='#cccccc', elinewidth=2, capsize=5)
```

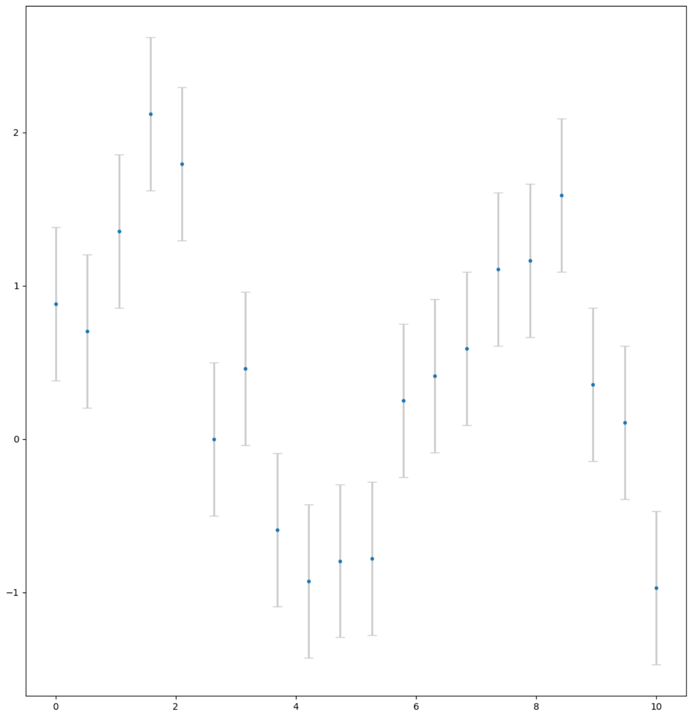

Figure 15: Customized Error Bars

The error of continuous quantities can be indicated by filling a area around the graph displaying the values. This can be achieved by using a combination of the `plt.plot` (indicating the values) and the `plt.fill_between` (indicating the area of error) function.

```
rng = np.random.RandomState(0)
points = 100
dy = 0.25

x = np.linspace(0, 10, points)
err = dy * rng.randn(points)
y = np.sin(x) + err

plt.plot(x, y)
plt.fill_between(x, y-dy, y+dy, color='#cccccc', alpha=.5)
plt.show()
```

The second argument (y-dy) is the lower, the third argument (y+dy) the upper bound of the error area.

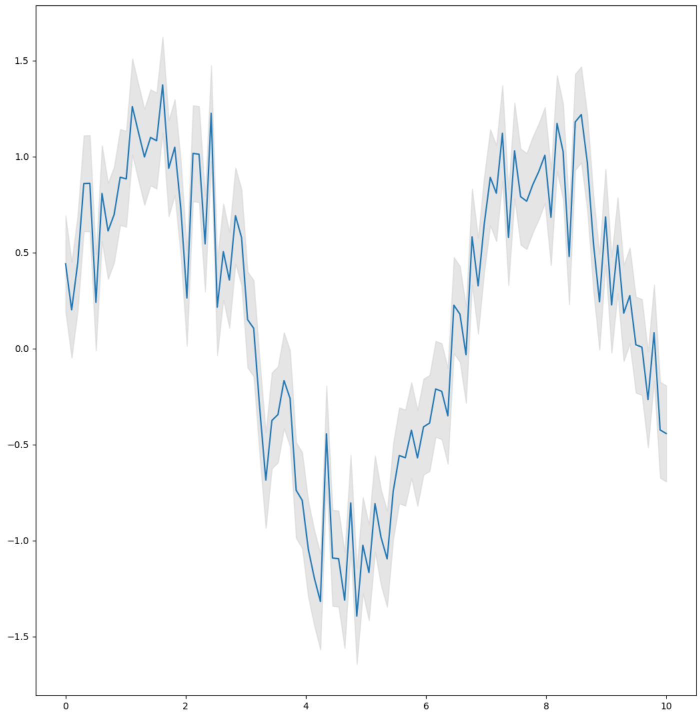

Figure 16: Error Area for Continuous Values

The methods errorbar and fill_between are also available in the axes' OO-style interface:

```
fig, ax = plt.subplots(2)
ax[0].errorbar(x, y, yerr=err, fmt='.')
ax[1].plot(x, y)
ax[1].fill_between(x, y-dy, y+dy, color='#cccccc', alpha=.5)
plt.show()
```

4.6 Density and Contour Plots

Three-dimensional data can be displayed in two dimensions using contours or color-coded regions. A function z=f(x, y) can be visualized by using x and y as the positions on the grid, and z for the contour level:

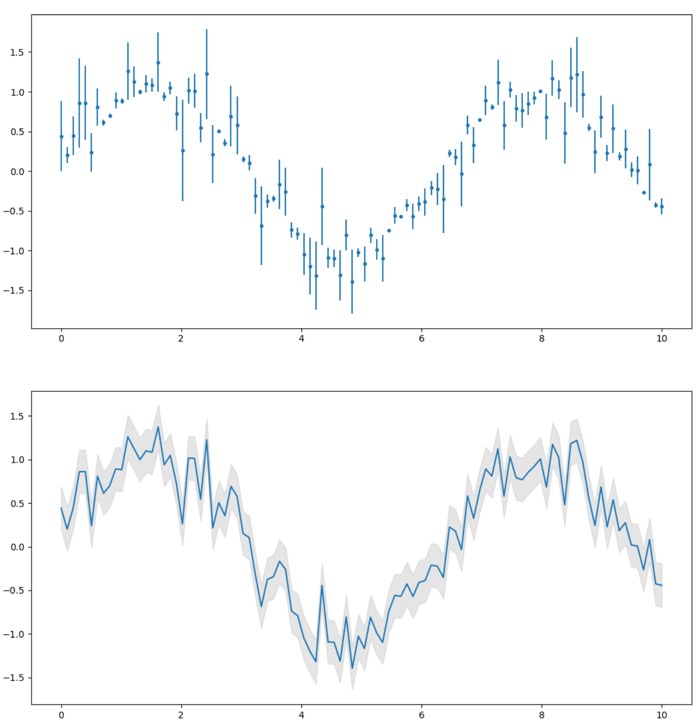

Figure 17: Error Bar and Area Combined

```
def f(x, y):
    return np.sin(x) + np.cos(x * y) * np.cos(x)
```

The `z` values are broadcasted into a two-dimensional grid. For the `x` and `y` values, broadcasting can be done using the `np.meshgrid` function:

```
x = np.linspace(0, 5, 50)
y = np.linspace(0, 5, 40)
X, Y = np.meshgrid(x, y)
Z = f(X, Y)
```

The contour plot can be created using the `plt.contour` function:

```
plt.contour(X, Y, Z, colors='black')
plt.show()
```

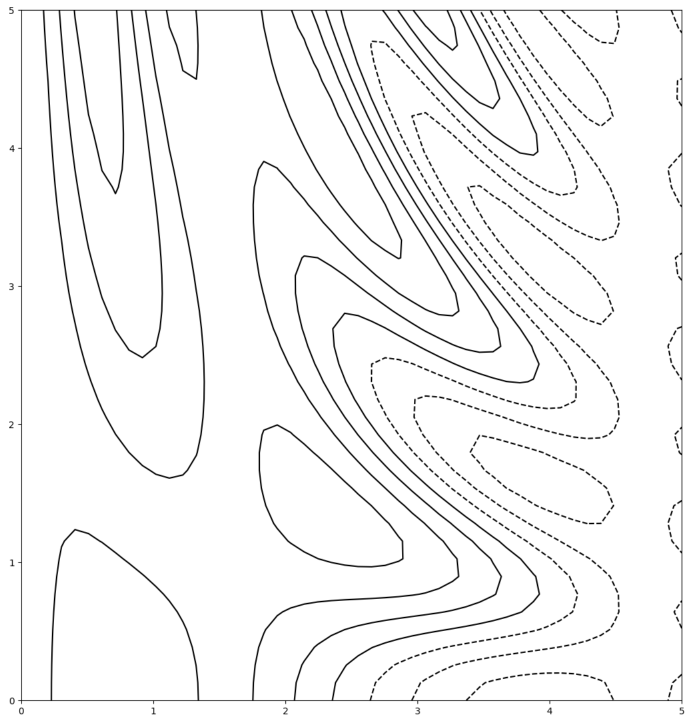

Figure 18: Contour Plot of a `z=f(x,y)` Function

Negative `z` values are represented by dashed, positive `z` values by solid lines. A color code with a number of intervals can be used in conjuncton with a colormap instead:

```
plt.contour(X, Y, Z, 20, cmap='RdGy')
```

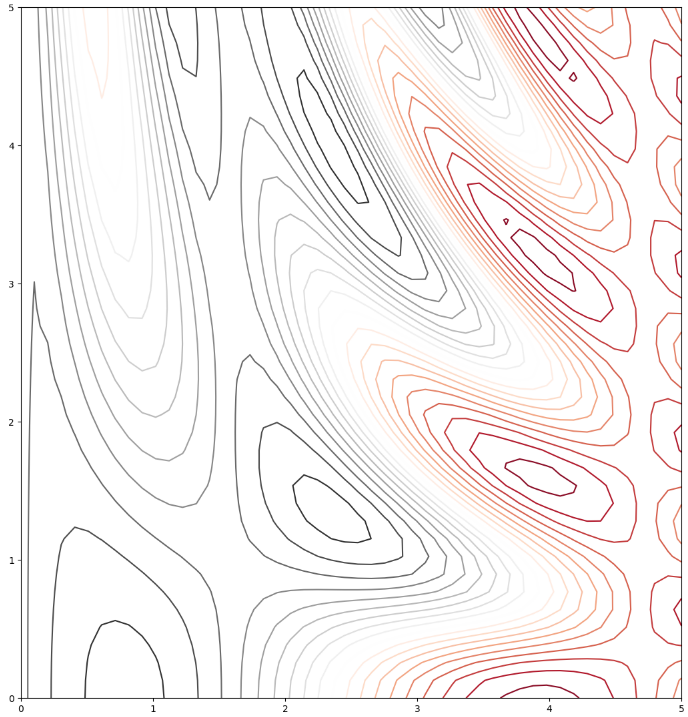

Figure 19: Contour Plot Using a Color Map

`RdGy` is a red-green colormap, with red indicating negative and grey positive values. More colormaps are available under `plt.cm`.

Instead of a contour plot with its distracting gaps, a *filled* contour plot can be created using the `plt.contourf` function:

```
plt.contourf(X, Y, Z, 20, cmap='RdGy')
plt.colorbar()
```

The colorbar helps to identify peaks and valleys. The color steps are discrete (20 contours) rather than continuous. The number of contours could be increases, which would be rather inefficient. The `plt.imshow` function is a faster option for that purpose:

```
plt.imshow(Z, extent=[0, 5, 0, 5], origin='lower', cmap='RdGy')
plt.colorbar()
plt.axis(aspect='image')
```

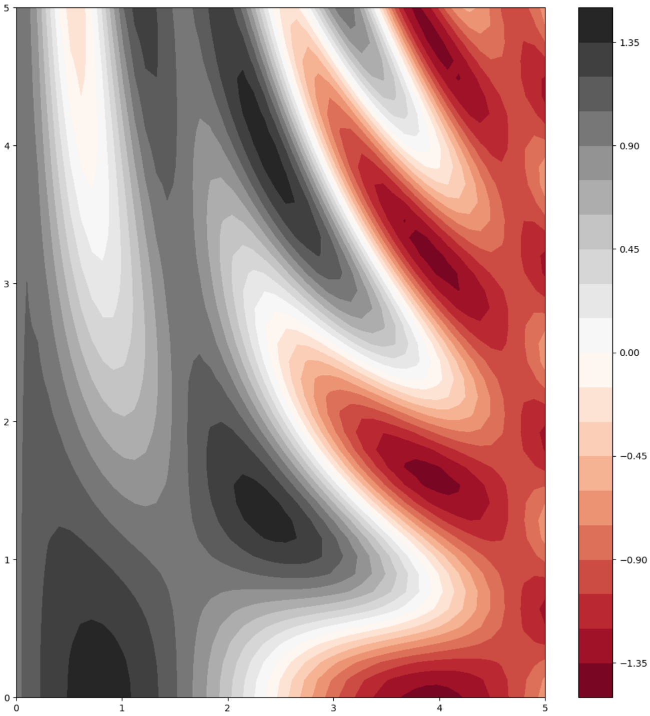

Figure 20: Filled Contour Plot

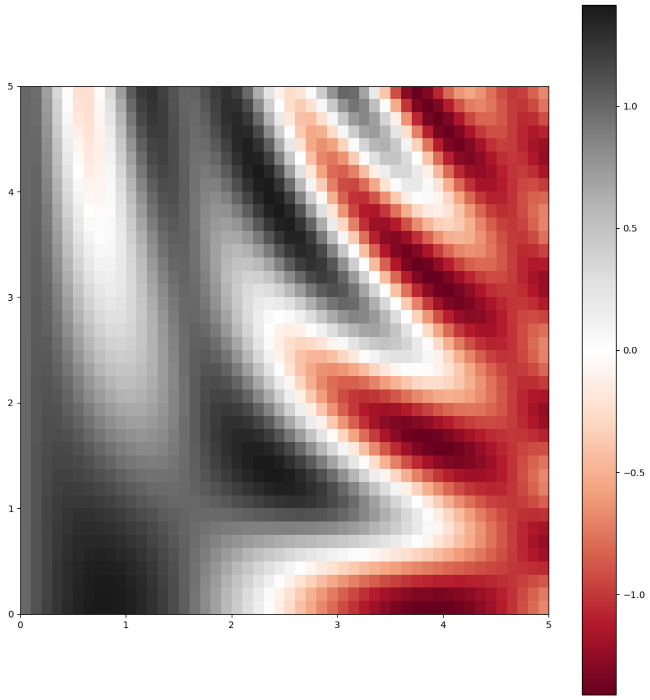

Figure 21: Contour Plot as an Image

- Instead of a grid, the value range on the x and y axis is defined as the `extent` of the form `[xmin, xmax, ymin, ymax]`.
- The image is drawn from the lower left (like a function), not from the upper left (like an image) by setting the `origin` argument `lower`.
- To match the x and y units, the `aspect` argument is set to `image` to prevent automatic aspect ratio adjustment.

Image and contour plots can also be combined, and the contours can also be labeled with their value:

```
contours = plt.contour(X, Y, Z, 3, colors='black')
plt.clabel(contours, inline=True, fontsize=8)
plt.imshow(Z, extent=[0, 5, 0, 5], origin='lower', cmap='RdGy', alpha=0.5)
plt.colorbar()
```

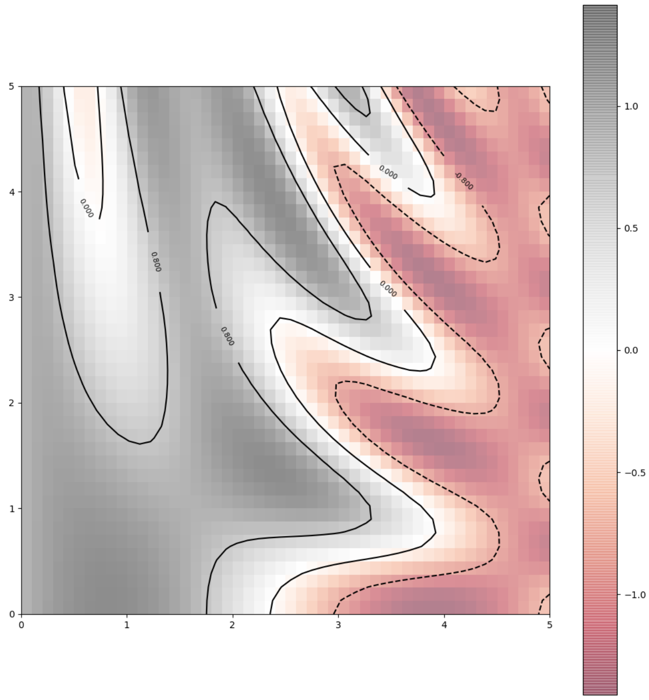

Figure 22: Partially Transparent Background Image, Over-Plotted with Contours and Labels