

# A Table of Commutation Relations

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## 1 A table of commutation relations for operators of a single particle.

### 1.1 Operators to use.

- Observables (Hermitian):  $x, p_x, H = \frac{p_x^2}{2m}, N = a^\dagger a, J_x, J_y, J_z, J, \Pi$
- Non-observables:  $a, a^\dagger, T,$

### 1.2 The table.

[row, column]	$x$	$p_x$	H	N	J	$J_x$	$J_y$	$J_z$	$\Pi$	$a$	$a^\dagger$	$T$
$x$	0	$i\hbar$										
$p_x$	$-i\hbar$	0	0									
H		0	0									
N				0						$-a$	$-a^\dagger$	
J					0							
$J_x$						0						
$J_y$							0					
$J_z$								0				
$\Pi$									0			
$a$										0	1	
$a^\dagger$										-1	0	
$T$												0
[row, column]	$S_x$	$S_y$	$S_z$									
$S_x$	0	$i\hbar S_z$	$-i\hbar S_y$									
$S_y$	$-i\hbar S_z$	0	$i\hbar S_x$									
$S_z$	$i\hbar S_y$	$-i\hbar S_x$	0									

### 1.3 Operator definitions.

- $p_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$  in the x-basis. Townsend p.158.
- $\Pi |x\rangle = |-x\rangle$  Townsend p.213.
- $H = \frac{p^2}{2m} + V(x)$  Sakurai p.97, (2.4.2).

### 1.4 References.

- 0 Any operator commutes with itself.
- $-i\hbar$  See Townsend eq. 6.31, p. 155 or Griffiths eq. 2.51, p.55. This leads to the uncertainty principle  $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ .
- $-a$  See Sakurai p.90, (2.3.10).
- $-a^\dagger$  See Sakurai p.90, (2.3.11).
- 0  $H$  and  $p_x$  commute since  $H = \frac{p_x^2}{2m}$ .

## 2 A table of commutation relations for operators of two particles.