A Table of Commutation Relations

Nathaniel Beaver

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1 A table of commutation relations for operators of a single particle.

1.1 Operators to use.

- Observables (Hermitian): $x, p_x, H = \frac{p_x^2}{2m}, N = a^\dagger a, J_x, J_y, J_z, J, \Pi$
- Non-observables: $a, a^\dagger, T,$

1.2 The table.

$$[B, B] = 0 \tag{1}$$
$$[p_x, x] = -i\hbar \tag{2}$$
$$[p_x, V(x)] = -i\hbar \frac{\partial}{\partial_x} V(x) \tag{3}$$
$$[N, a] = -a \tag{4}$$
$$[a, a^\dagger] = 1 \tag{5}$$
$$\tag{6}$$

1.2.1 Canonical commutation relations

Relation 2 is an example of a canonical commutation relation. In general if q_i are coordinates and π_i are the canonically conjugate momenta to q_i, then

$$[\pi_i, q_j] = -i\hbar\delta_{ij} \tag{7}$$

1.2.2 Spin

In general,

$$[S_i, S_j] = i\hbar\varepsilon_{ijk} S_k \tag{8}$$
$$[S_i, \mathbf{S}^2] = 0 \tag{9}$$

In addition, for spin-1/2 particles

$$\{S_a, S_b\} = \hbar^2/2\delta_{ab}. \tag{10}$$

1.3 Operator definitions.

- $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ in the x-basis. Townsend p.158.
- $\Pi\,|x\rangle = |{-x}\rangle$ Townsend p.213.
- $H = \frac{p^2}{2m} + V(x)$ Sakurai p.97, (2.4.2).

1.4 References.

0 Any operator commutes with itself.

$-i\hbar$ See Townsend eq. 6.31, p. 155 or Griffiths eq. 2.51, p.55. This leads to the uncertainty principle $\Delta x \Delta p_x \geq \frac{\hbar}{2}$.

$-a$ See Sakurai p.90, (2.3.10).

$-a^\dagger$ See Sakurai p.90, (2.3.11).

0 By the product rule.

2 A table of commutation relations for operators of two particles.