A Backbone for Simulating Quantities of Interest from Generalized Linear Models

by Christopher Gandrud

Abstract Simulating quantities of interest estimated from generalized linear models (GLM) can be an effective way to explore and communicate statistical results. **coreSim** provides core functions that can serve as a reliable and extensible "backbone" to new packages for simulating quantaties of interest for finding and plotting simulated quantities of interest from GLMs. I demonstrate how to use **coreSim** as a backbone by showing how it is incorporated into the new **pltesim** package for simulating and displaying probabilistic long-term effects in models with temporal dependence.

Introduction

There has been a recent trend in the social sciences to improve the interpretation of results from generalized linear models (GLM) by presenting estimated quantities of interest from substantively meaningful scenarios (e.g. Gandrud, 2015; Licht, 2011; Williams and Whitten, 2012). King et al. (2000) advanced a post-estimation simulation technique for estimating the uncertainty around these estimates. A number of R packages implement this approach [CITE]. In particular, the **Zelig** package (Choirat et al., 2016) implements this technique for a wide range of model types in an attempt to be "everyone's statistical software".

While all of these packages based on the same simulation procedure, all of these packages rely on different, custom built ways of doing these simulations. This presents unnecessarily increases the labor needed to use post-estimation simulations for showing results in new modelling situations. Current implementations of these packages have also proven to be unreliable in ways directly related to their architecture.

In this paper I introduce a new R package–**coreSim**–that aims to be an extensible and reliable "backbone" for future analyses and R packages that find post-estimation simulations for substantively relevant scenarios. I give an example of this by showing how **coreSim** forms the basis of the new **pltesim** package for simulating and displaying probabilistic long-term effects in models with temporal dependence.

Post-estimation simulations from GLMs

One way to estimate substantively meaningful quantities of interest for complex scenarios from with their associated uncertainty is through post-estimation simulations.[1] The procedure is straightforward. Remember that a generalized linear models can be summarized by two equations. One describes the stochastic component:

$$Y_i \sim f(\theta_i, \alpha) \tag{1}$$

Here the dependent variable Y is a random drawn from the probability density function $f(\theta_i, \alpha)$. The other equation describes the systematic component:

$$\theta_i = g(X_i, \beta), \tag{2}$$

where X is a vector of explanatory variables and β is a vector of effect parameters. The function g is often referred to as the link function, which specifies how the variables and effect parameters are translated into θ.

We can use post-estimation simulations to find and communicate substantively meaningful results from these models. To do this we first estimate $\hat{\beta}$ and $\hat{\alpha}$. Second, we drawn n number of values of these parameters from the multivariate normal distribution using the parameter point estimates and their variance covariance matrix with a mean of $\hat{\gamma}$–a vector created by stacking $\hat{\beta}$ and $\hat{\alpha}$–and variance of $\hat{\mathrm{V}}(\hat{\gamma})$:

$$\widetilde{\gamma} \sim \mathrm{N}(\hat{\gamma}, \hat{\mathrm{V}}(\hat{\gamma})) \tag{3}$$

Drawing these simulations is straightforward using two functions from the **stats** package–`coef`

[1] See King et al. (2000) for a comparision with related fully Bayesisan Markov chain Monte Carlo and Bootstrapping methods.

and vcov–as well as the `mvrnorm` function from the **MASS** package. Both **stats** and **MASS** are included with the default R installation. For example:

```
# Load package that contains the Prestige data set
library(car)

library(MASS)

# Estimate normal linear model
m1 <- lm(prestige ~ education + type, data = Prestige)

# Extract coeficient estimates
m1_coef <- coef(m1)

# Extract variance-covariance matrix
m1_vcov <- vcov(m1)

# Draw 1000 simulations from multivariate normal distribution
m1_sims <- mvrnorm(n = 1000, mu = m1_coef, Sigma = m1_vcov)

head(m1_sims)

#>       (Intercept) education  typeprof      typewc
#> [1,]  -2.46299473  4.479084  6.791089  -1.1362359
#> [2,]  -3.37548512  4.822845  1.722389  -6.1467418
#> [3,]  -9.23523872  5.116772  7.931571  -4.9137734
#> [4,] -10.82754562  5.565439 -2.061336 -10.0247785
#> [5,]  -0.06587696  3.996145 10.140270  -0.5249211
#> [6,]   4.38447086  3.891146  7.054453  -5.1137317
```

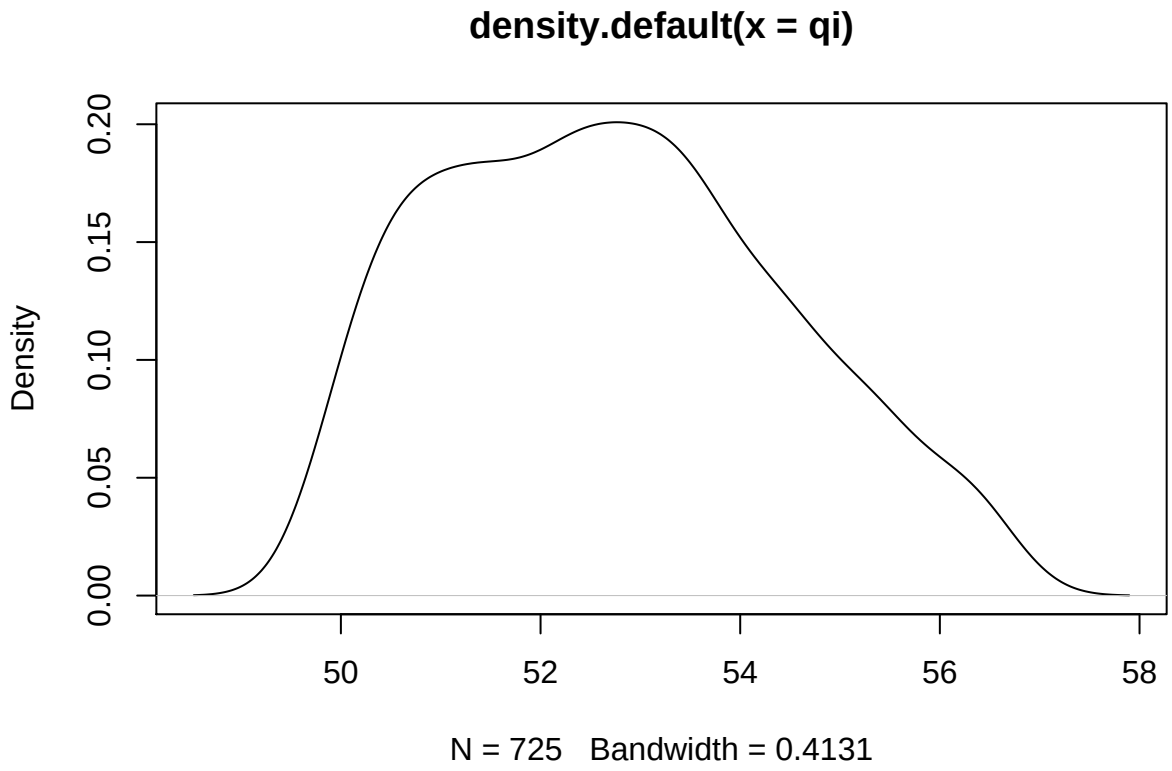
Now that we have the simulated effect coefficients, we can find our quantity of interest for a given scenario. In this example, our quantity of interest is simply the predicted value of the `prestige` dependent variable. We simply need to find the systematic component from the linear form for each simulation: $g(X_i, \beta) = X_i\beta_i = \beta_0 + X_{i1}\beta_1 + X_{i2}\beta_2 + \ldots$. For example imagine that we want to find the predicted level of `prestige` when `education` is at its median (10.54) and the type of profession is `prof`. We could calculate `prestige` for this scenario using:

```
# Create a data frame with the scenario values
fitted <- data.matrix(data.frame(education = 10.54, typeprof = 1))

# Reformat simulations
intercept <- m1_sims[, 1] # extract intercept
other_betas <- data.matrix(m1_sims[, colnames(fitted)])

# Find quantitiy of interest
qi <- intercept + (other_betas %*% c(fitted))

head(qi)

#>          [,1]
#> [1,] 51.53764
#> [2,] 49.17970
#> [3,] 52.62711
#> [4,] 45.77085
#> [5,] 52.19376
#> [6,] 52.45161
```

Finally, we can trim of outliers by keeping the central 95% interval and then visualise the results.

```
# Find lower and upper bound of the simulations' central 95% interval
lower_bound <- quantile(qi, probs = 0.25)
upper_bound <- quantile(qi, probs = 0.975)

# Restrict the simulations to the central 95% interval
qi <- qi[qi > lower_bound]
qi <- qi[qi < upper_bound]
```

```
# Plot
plot(density(qi))
```



This is a trivial example that shows the basic steps required to find quantities of interest from post-estimation simulations. The use of this technique in this context does not provide us with much more information than we could relatively easily find using the point estimates themselves and their confidence intervals. Where post-estimation simulations are much more useful is for exploring more complex, contrasting scenarios, often with interactions and non-linearities. These relationships can be difficult to meaningfully interpret with individual point estimates and their associated confidence intervals.

The process for finding these scenarios with post-estimation simulations is in practice much more difficult as it requires calculating quantities of interest for each scenario.

Previous post-estimation simulation packages

The **Zelig** package could potentially help streamline this process and in so doing form the basis of R packages that generate post-estimation simuliations in novel areas.

Zelig has a number of issues that have made it unreliable over time. By aiming to be everyone's statistical software, **Zelig** relies on many (13 imported) dependencies.

Summary

Bibliography

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