Factor Analysis Exercises

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- What is factor analysis
- CFA versus PCA
- Variance in factor analysis
- Considertations for factor analysis
- Identifying / extracting factors
- Rotation
- Cronbach's alpha

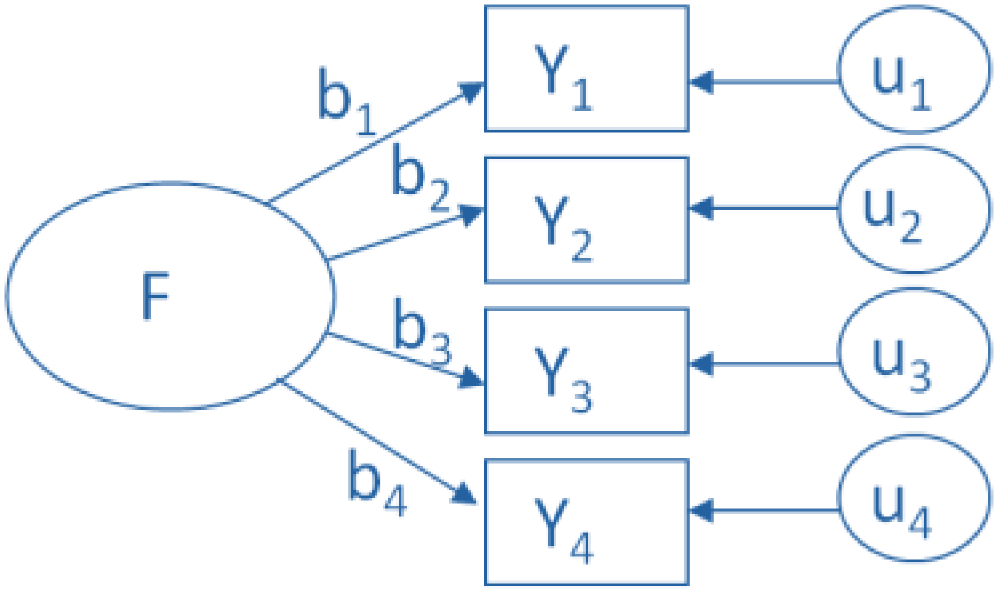
Exploratory Factor analysis

- Identify the relational structure between a set of variables in order to reduce them to a smaller set of factors
  - The process of **dimension reduction** (identify new variables) or **data summarisation** (summarise what is already there)

Dimension reducton

- **Latent Variables**: Not directly observable. Rather they are inferred from other responses
  - Many psychological constructs (e.g. anxiety) are latent variables that we cannot directly measure.
  - Rather, we can measure behaviours, cognitions and other variables that are related to the construct.

We might concptualise this as: "Responses to the questions are indicative of levels of underlying anxiety"



Data summarisation

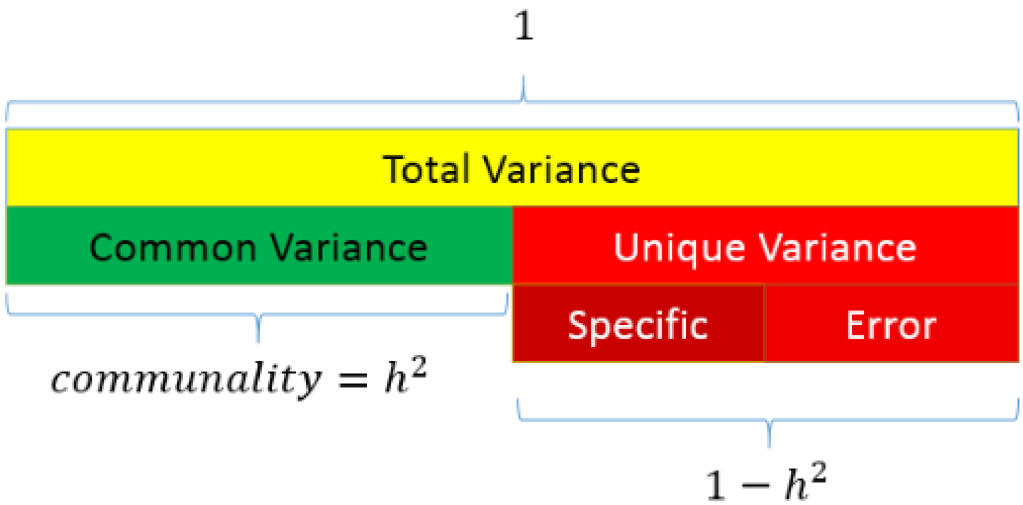
- **Index Variables** or **Components**: A weighted summary of measured variables that contribute to the component variable
- "Principal components are variables of maximal variance constructed from linear combinations of the input features"

We might conceptualise this as: "We can reduce these measures/questions to a smaller set of higher order, independent, composite variables"

Variance in exploratory factor analysis

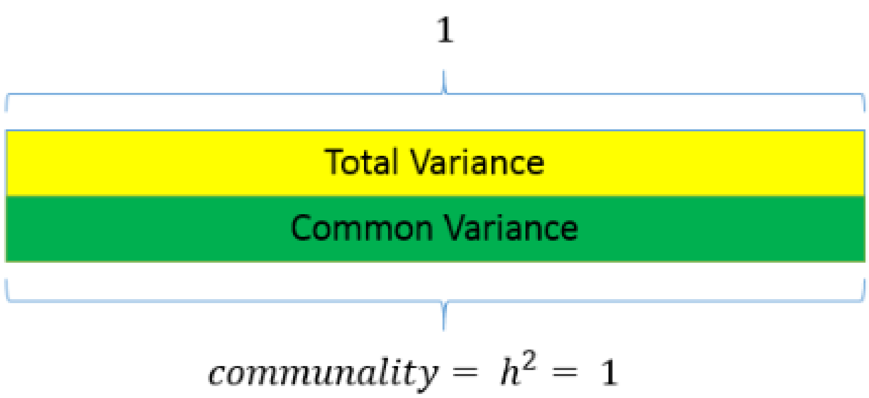
There are two common methods of exploratory factor analysis: **Common Factor analysis** and **Principal Component Analysis**

- CFA assumes that there are two types of variance: common and unique



Variance in exploratory factor analysis

- PCA only assumes common variance



Variance in exploratory factor analysis

- Due to these different approaches, PCA is considered to be reflective of the current sample but not generalisable to the wider population
- Whereas, CFA is considered appropriate for hypothesis testing and making inferences to the population

What is factor analysis?

- If we measure several variables (or questions), we can examine the correlation between sets of these variables
  - Such a correlation matrix is known as an **R Matrix** (*r* because correlation)

- If there are clusters of correlations between a number of the variables (or questions), this indicates that they might be linked to the same underlying dimension (or latent variable)
- The researcher should use informed judgement when assessing the appropriateness of variables for inclusion

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Correlations								
	1	2	3	4	5	6	7	8
1	1							
2	-.099**	1						
3	-.337**	.318**	1					
4	.436**	-.112**	-.380**	1				
5	.402**	-.119**	-.310**	.401**	1			
6	.217**	-.074**	-.227**	.278**	.257**	1		
7	.305**	-.159**	**-.382****	.409**	.339**	**.514****	1	
8	.331**	-.050*	-.259**	.349**	.269**	.223**	.297**	1
**. Correlation is significant at the 0.01 level (2-tailed).								
*. Correlation is significant at the 0.05 level (2-tailed).								

An r matrix example

Considerations with factor analysis

- Sample size:
  - Must be more data points than variables being measured
  - A common rule of thumb is at least 10 per variable
  - There are tests to assess sample size adequacy (e.g. Kaiser-Meyer test should be greater than 0.5)
- Inter-correlation:
  - There must be sufficient correlation between the variables being measured
  - A high number of correlations over 0.3
  - Can be tested using Bartlett test of sphericity (sig. result means factor analysis can be used)

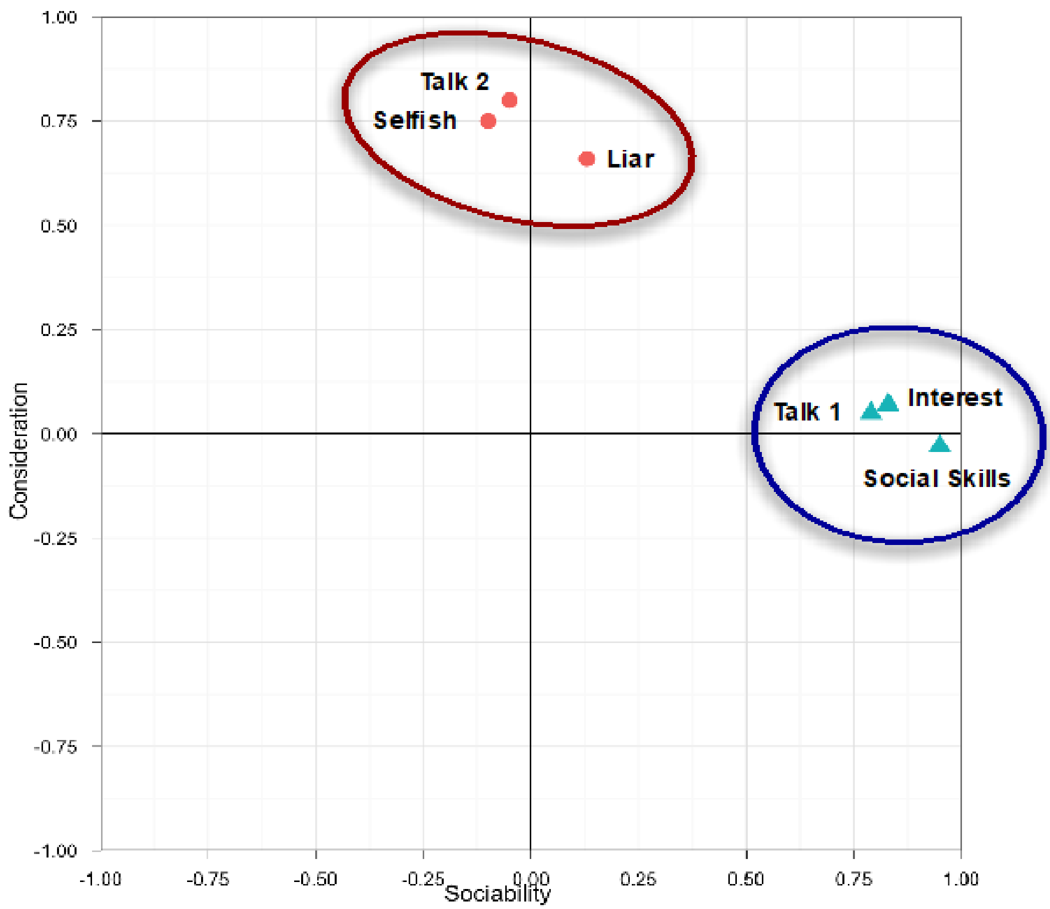
Other things to check (see Field, 2018)

- The quality of analysis depends upon the quality of the data (GI□GO).
- Avoid multicollinearity:

  - several variables highly correlated, r > .80.
  - Determinent: should be greater than 0.00001
- Avoid singularity:
  - some variables perfectly correlated, r = 1.
- Screen the correlation matrix, eliminate any variables that obviously cause concern.

Representing factor analysis

We can represent factors visually based on the strength of their inter-correlations - Here, the axis of the graph represents a factor or latent variable



We can also represent factor analysis using a regression equation - Here the beta values represent the extent to which the variable “loads onto” a particular factor

$$Y = b_1X_1 + b_2X_2 + \ldots + b_nX_n$$
$$\text{Factor}_i = b_1\text{Variable}_1 + b_2\text{Variable}_2 + \ldots + b_n\text{Variable}_n$$

$$Y = b_1X_1 + b_2X_2 + \ldots + b_nX_n$$
$$\begin{aligned}\text{Sociability} &= b_1\text{Talk1} + b_2\text{Social Skills} + b_3\text{Interest} \\ &\quad + b_4\text{Talk2} + b_5\text{Selfish} + b_6\text{Liar} \\ \text{Consideration} &= b_1\text{Talk1} + b_2\text{Social Skills} + b_3\text{Interest} \\ &\quad + b_4\text{Talk2} + b_5\text{Selfish} + b_6\text{Liar}\end{aligned}$$

Example: Statistics anxiety

- Many people get anxious about statistics
- We can ask them about their experience in a number of ways (e.g. questions compiled by students in a stats class)
- Their responses might indicate that stats anxiety has a number of dimensions
  - i.e. it is a multi-dimensional construct, as opposed to a unitary construct

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SD = Strongly Disagree, D = Disagree, N = Neither, A = Agree, SA = Strongly Agree

		SD	D	N	A	SA
1	Statistics make me cry	○	○	○	○	○
2	My friends will think I'm stupid for not being able to cope with R	○	○	○	○	○
3	Standard deviations excite me	○	○	○	○	○
4	I dream that Pearson is attacking me with correlation coefficients	○	○	○	○	○
5	I don't understand statistics	○	○	○	○	○
6	I have little experience of computers	○	○	○	○	○
7	All computers hate me	○	○	○	○	○
8	I have never been good at mathematics	○	○	○	○	○
9	My friends are better at statistics than me	○	○	○	○	○
10	Computers are useful only for playing games	○	○	○	○	○
11	I did badly at mathematics at school	○	○	○	○	○
12	People try to tell you that R makes statistics easier to understand but it doesn't	○	○	○	○	○
13	I worry that I will cause irreparable damage because of my incompetence with computers	○	○	○	○	○
14	Computers have minds of their own and deliberately go wrong whenever I use them	○	○	○	○	○
15	Computers are out to get me	○	○	○	○	○
16	I weep openly at the mention of central tendency	○	○	○	○	○
17	I slip into a coma whenever I see an equation	○	○	○	○	○
18	R always crashes when I try to use it	○	○	○	○	○
19	Everybody looks at me when I use R	○	○	○	○	○
20	I can't sleep for thoughts of eigenvectors	○	○	○	○	○
21	I wake up under my duvet thinking that I am trapped under a normal distribution	○	○	○	○	○
22	My friends are better at R than I am	○	○	○	○	○

Step 1: Create a correlation matrix

```
raq.matrix <- cor(raq)

raq.matrix
```

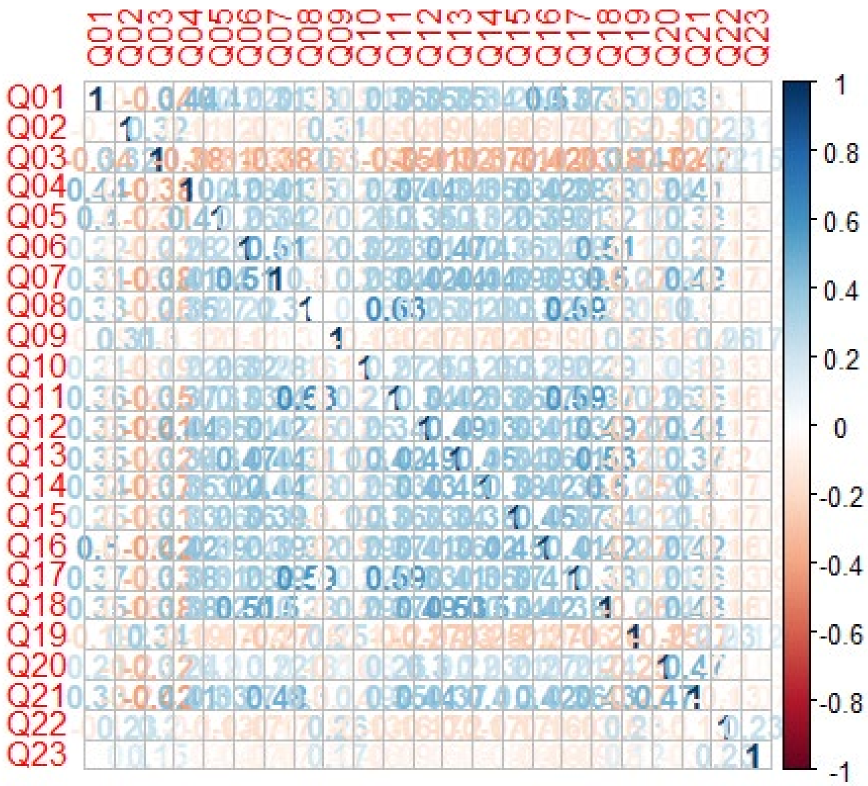
```
##              Q01         Q02        Q03         Q04         Q05
## Q01  1.000000000 -0.09872403 -0.3366489  0.43586018  0.40243992
## Q02 -0.098724032  1.00000000  0.3183902 -0.11185965 -0.11934658
## Q03 -0.336648879  0.31839020  1.0000000 -0.38046016 -0.31030879
## Q04  0.435860179 -0.11185965 -0.3804602  1.00000000  0.40067225
## Q05  0.402439917 -0.11934658 -0.3103088  0.40067225  1.00000000
## Q06  0.216733985 -0.07420968 -0.2267405  0.27820154  0.25746014
## Q07  0.305365139 -0.15917448 -0.3819533  0.40861502  0.33939179
## Q08  0.330737608 -0.04962257 -0.2586342  0.34942939  0.26862697
## Q09 -0.092339458  0.31464054  0.2998036 -0.12454637 -0.09570151
## Q10  0.213681706 -0.08400316 -0.1933887  0.21581010  0.25820925
## Q11  0.356786290 -0.14382984 -0.3506397  0.36865655  0.29782882
## Q12  0.345381133 -0.19486946 -0.4099513  0.44164706  0.34674325
## Q13  0.354646283 -0.14274026 -0.3179193  0.34429168  0.30182159
## Q14  0.337879655 -0.16469991 -0.3707551  0.35080964  0.31533810
## Q15  0.245752635 -0.16499581 -0.3123968  0.33423089  0.26137190
## Q16  0.498618057 -0.16755228 -0.4186478  0.41586725  0.39491795
## Q17  0.370550512 -0.08699527 -0.3273715  0.38273945  0.31041722
## Q18  0.347118037 -0.16389415 -0.3752329  0.38200149  0.32209148
## Q19 -0.189011027  0.20329748  0.3415737 -0.18597751 -0.16532210
## Q20  0.213897945 -0.20159437 -0.3248338  0.24291796  0.19966945
## Q21  0.329153138 -0.20461730 -0.4171878  0.41029317  0.33461494
## Q22 -0.104408664  0.23087487  0.2036569 -0.09838349 -0.13253593
## Q23 -0.004480593  0.09967828  0.1502065 -0.03381815 -0.04165684
##             Q06         Q07         Q08         Q09         Q10
## Q01  0.21673399  0.30536514  0.33073761 -0.09233946  0.21368171
## Q02 -0.07420968 -0.15917448 -0.04962257  0.31464054 -0.08400316
## Q03 -0.22674048 -0.38195325 -0.25863421  0.29980362 -0.19338871
## Q04  0.27820154  0.40861502  0.34942939 -0.12454637  0.21581010
## Q05  0.25746014  0.33939179  0.26862697 -0.09570151  0.25820925
## Q06  1.00000000  0.51358048  0.22283175 -0.11264384  0.32223023
## Q07  0.51358048  1.00000000  0.29749696 -0.12829828  0.28372299
## Q08  0.22283175  0.29749696  1.00000000  0.01573316  0.15860850
## Q09 -0.11264384 -0.12829828  0.01573316  1.00000000 -0.13418658
## Q10  0.32223023  0.28372299  0.15860850 -0.13418658  1.00000000
## Q11  0.32807072  0.34474770  0.62929768 -0.11552479  0.27143657
## Q12  0.31250937  0.42298591  0.25198582 -0.16739436  0.24582591
## Q13  0.46640487  0.44211926  0.31424716 -0.16743882  0.30196707
## Q14  0.40224407  0.44070276  0.28058958 -0.12150197  0.25468730
## Q15  0.35989309  0.39136675  0.29968600 -0.18657099  0.29523438
## Q16  0.24433888  0.38854534  0.32149420 -0.18886556  0.29058576
## Q17  0.28226121  0.39074283  0.59014022 -0.03681556  0.21832214
## Q18  0.51332164  0.50086685  0.27974433 -0.14957782  0.29250304
## Q19 -0.16675017 -0.26912031 -0.15947671  0.24931170 -0.12723487
## Q20  0.10092489  0.22095420  0.17515089 -0.15864747  0.08406520
## Q21  0.27233273  0.48300388  0.29571756 -0.13594310  0.19313633
## Q22 -0.16513541 -0.16820488 -0.07917265  0.25684622 -0.13090831
## Q23 -0.06868743 -0.07029016 -0.05023839  0.17077441 -0.06191796
##             Q11         Q12         Q13         Q14         Q15
## Q01  0.35678629  0.34538113  0.35464628  0.33787966  0.24575263
```

```
## Q02 -0.14382984 -0.19486946 -0.14274026 -0.16469991 -0.16499581
## Q03 -0.35063969 -0.40995127 -0.31791928 -0.37075510 -0.31239678
## Q04  0.36865655  0.44164706  0.34429168  0.35080964  0.33423089
## Q05  0.29782882  0.34674325  0.30182159  0.31533810  0.26137190
## Q06  0.32807072  0.31250937  0.46640487  0.40224407  0.35989309
## Q07  0.34474770  0.42298591  0.44211926  0.44070276  0.39136675
## Q08  0.62929768  0.25198582  0.31424716  0.28058958  0.29968600
## Q09 -0.11552479 -0.16739436 -0.16743882 -0.12150197 -0.18657099
## Q10  0.27143657  0.24582591  0.30196707  0.25468730  0.29523438
## Q11  1.00000000  0.33529466  0.42316548  0.32532025  0.36482687
## Q12  0.33529466  1.00000000  0.48871303  0.43270398  0.33179910
## Q13  0.42316548  0.48871303  1.00000000  0.44978632  0.34219704
## Q14  0.32532025  0.43270398  0.44978632  1.00000000  0.38011484
## Q15  0.36482687  0.33179910  0.34219704  0.38011484  1.00000000
## Q16  0.36907763  0.40805908  0.35837775  0.41841820  0.45427861
## Q17  0.58683495  0.33269383  0.40837657  0.35374183  0.37310235
## Q18  0.37341373  0.49296482  0.53293713  0.49830615  0.34287045
## Q19 -0.19965203 -0.26665953 -0.22697105 -0.25405813 -0.20980230
## Q20  0.25533736  0.29802585  0.20396327  0.22592173  0.20625622
## Q21  0.34643407  0.44063832  0.37443078  0.39938896  0.29971557
## Q22 -0.16198921 -0.16728557 -0.19535632 -0.16983754 -0.16790617
## Q23 -0.08637256 -0.04642506 -0.05298304 -0.04847418 -0.06200665
##             Q16         Q17         Q18        Q19         Q20         Q21
## Q01  0.49861806  0.37055051  0.34711804 -0.1890110  0.21389794  0.32915314
## Q02 -0.16755228 -0.08699527 -0.16389415  0.2032975 -0.20159437 -0.20461730
## Q03 -0.41864780 -0.32737145 -0.37523290  0.3415737 -0.32483385 -0.41718781
## Q04  0.41586725  0.38273945  0.38200149 -0.1859775  0.24291796  0.41029317
## Q05  0.39491795  0.31041722  0.32209148 -0.1653221  0.19966945  0.33461494
## Q06  0.24433888  0.28226121  0.51332164 -0.1667502  0.10092489  0.27233273
## Q07  0.38854534  0.39074283  0.50086685 -0.2691203  0.22095420  0.48300388
## Q08  0.32149420  0.59014022  0.27974433 -0.1594767  0.17515089  0.29571756
## Q09 -0.18886556 -0.03681556 -0.14957782  0.2493117 -0.15864747 -0.13594310
## Q10  0.29058576  0.21832214  0.29250304 -0.1272349  0.08406520  0.19313633
## Q11  0.36907763  0.58683495  0.37341373 -0.1996520  0.25533736  0.34643407
## Q12  0.40805908  0.33269383  0.49296482 -0.2666595  0.29802585  0.44063832
## Q13  0.35837775  0.40837657  0.53293713 -0.2269710  0.20396327  0.37443078
## Q14  0.41841820  0.35374183  0.49830615 -0.2540581  0.22592173  0.39938896
## Q15  0.45427861  0.37310235  0.34287045 -0.2098023  0.20625622  0.29971557
## Q16  1.00000000  0.40976309  0.42197911 -0.2670470  0.26514025  0.42054273
## Q17  0.40976309  1.00000000  0.37560681 -0.1628810  0.20523013  0.36349147
## Q18  0.42197911  0.37560681  1.00000000 -0.2566318  0.23518040  0.43010427
## Q19 -0.26704702 -0.16288096 -0.25663183  1.0000000 -0.24859386 -0.27489793
## Q20  0.26514025  0.20523013  0.23518040 -0.2485939  1.00000000  0.46770448
## Q21  0.42054273  0.36349147  0.43010427 -0.2748979  0.46770448  1.00000000
## Q22 -0.15579385 -0.12629066 -0.15982631  0.2339226 -0.09970186 -0.12902148
## Q23 -0.08152195 -0.09167243 -0.08041698  0.1224344 -0.03466529 -0.06766437
##             Q22          Q23
## Q01 -0.10440866 -0.004480593
## Q02  0.23087487  0.099678285
## Q03  0.20365686  0.150206522
```

```
## Q04 -0.09838349 -0.033818152
## Q05 -0.13253593 -0.041656841
## Q06 -0.16513541 -0.068687430
## Q07 -0.16820488 -0.070290157
## Q08 -0.07917265 -0.050238392
## Q09  0.25684622  0.170774410
## Q10 -0.13090831 -0.061917956
## Q11 -0.16198921 -0.086372565
## Q12 -0.16728557 -0.046425059
## Q13 -0.19535632 -0.052983042
## Q14 -0.16983754 -0.048474181
## Q15 -0.16790617 -0.062006650
## Q16 -0.15579385 -0.081521950
## Q17 -0.12629066 -0.091672426
## Q18 -0.15982631 -0.080416984
## Q19  0.23392259  0.122434401
## Q20 -0.09970186 -0.034665293
## Q21 -0.12902148 -0.067664367
## Q22  1.00000000  0.230369402
## Q23  0.23036940  1.000000000
```

Step 2: Let's check for Inter-correlation

```
library(corrplot)

## Warning: package 'corrplot' was built under R version 3.6.1

## corrplot 0.84 loaded

corrplot(raq.matrix, method = "number")
```



Step 2: Let's check for Inter-correlation

- We can use bartlett's test from the psych package

```
library(psych)

## Warning: package 'psych' was built under R version 3.6.1

##
## Attaching package: 'psych'

## The following objects are masked from 'package:ggplot2':
##
##     %+%, alpha

cortest.bartlett(raq.matrix, n=2571)

## $chisq
## [1] 19334.49
##
## $p.value
## [1] 0
##
## $df
## [1] 253
```

Step 3: Check sampling adequacy

- Overall should be > 0.5

```
KMO(raq)

## Kaiser-Meyer-Olkin factor adequacy
## Call: KMO(r = raq)
## Overall MSA =  0.93
## MSA for each item = 
##  Q01  Q02  Q03  Q04  Q05  Q06  Q07  Q08  Q09  Q10  Q11  Q12  Q13  Q14  Q15 
## 0.93 0.87 0.95 0.96 0.96 0.89 0.94 0.87 0.83 0.95 0.91 0.95 0.95 0.97 0.94 
##  Q16  Q17  Q18  Q19  Q20  Q21  Q22  Q23 
## 0.93 0.93 0.95 0.94 0.89 0.93 0.88 0.77
```

Step 4: Identify number of factors

Based on Eigenvalues:

- Kaiser (1960) – retain factors with eigen values > 1.
- Joliffe (1972) – retain factors with eigen values > .70.

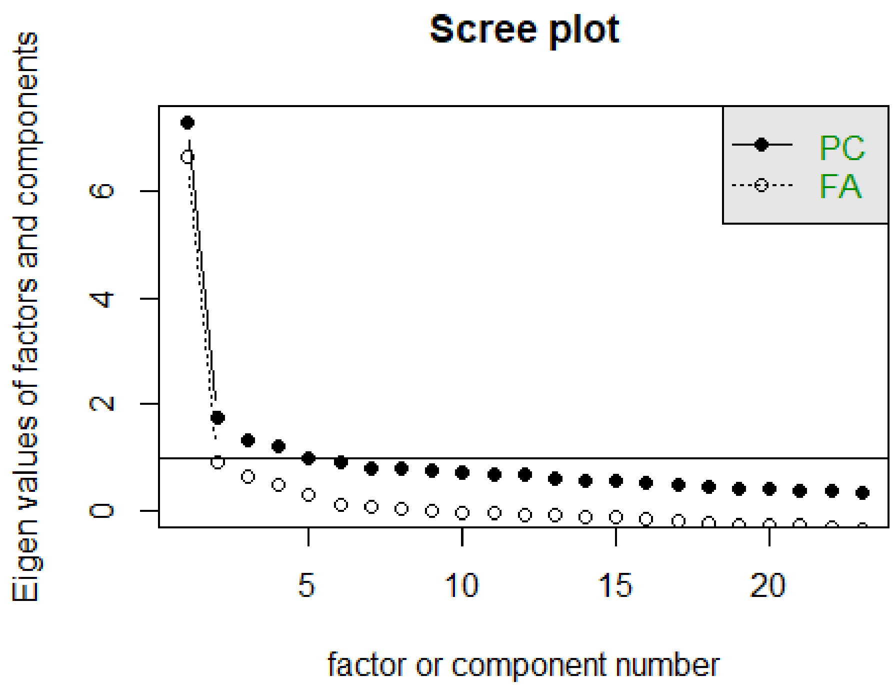
Use a scree plot: Cattell (1966): use 'point of inflexion'.

Which rule?

- Use Kaiser's extraction when
  - Less than 30 variables, communalities after extraction > 0.7
  - Sample size > 250 and mean communality ≥ 0.6
- Scree plot is good if sample size is > 200

Scree plot

```
scree(raq)
```



- We are looking for the point of inflection
- Where there is a drop-off

One approach: See how many factors we can draw a line through

We can get another scee plot from the parallel analysis output (next slide)

Step 4: Identify number of factors

How many dimensions of stats anxiety are captured in the questionnaire?

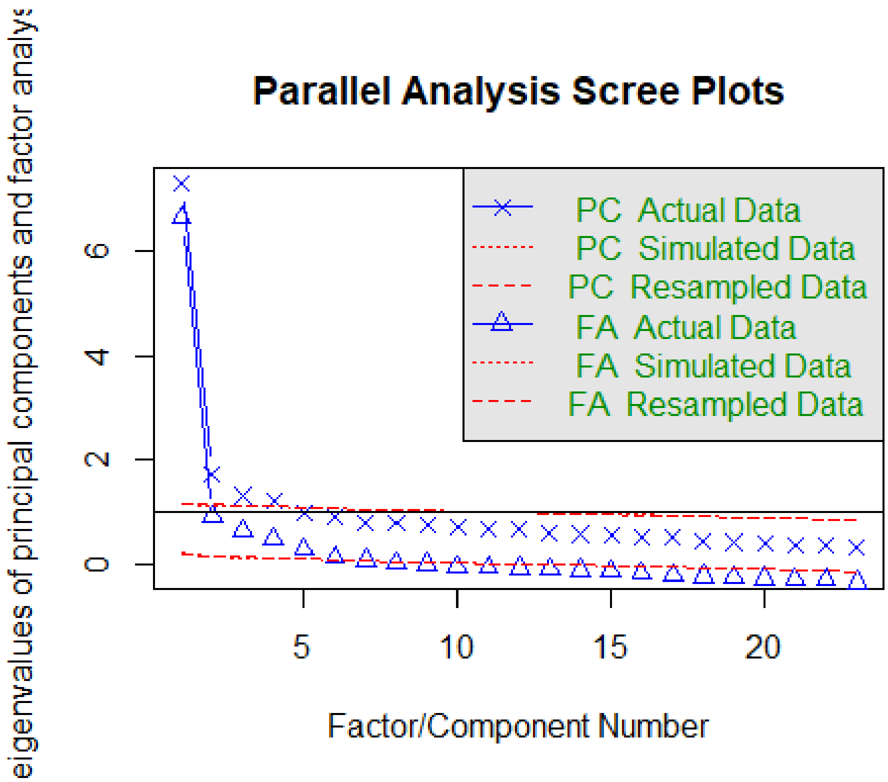
- We can run a **parallel analysis** to get an indication of the number of factors contained within the data
- Parallel Analysis:
  - Simulates data within the same range of values as our data set
  - Suggests that we retain, at maximum, the factors with eigenvalues larger than those extracted from simulated data.

SD = Strongly Disagree, D = Disagree, N = Neither, A = Agree, SA = Strongly Agree

		SD	D	N	A	SA
1	Statistics make me cry	○	○	○	○	○
2	My friends will think I'm stupid for not being able to cope with R	○	○	○	○	○
3	Standard deviations excite me	○	○	○	○	○
4	I dream that Pearson is attacking me with correlation coefficients	○	○	○	○	○
5	I don't understand statistics	○	○	○	○	○
6	I have little experience of computers	○	○	○	○	○
7	All computers hate me	○	○	○	○	○
8	I have never been good at mathematics	○	○	○	○	○
9	My friends are better at statistics than me	○	○	○	○	○
10	Computers are useful only for playing games	○	○	○	○	○
11	I did badly at mathematics at school	○	○	○	○	○
12	People try to tell you that R makes statistics easier to understand but it doesn't	○	○	○	○	○
13	I worry that I will cause irreparable damage because of my incompetence with computers	○	○	○	○	○
14	Computers have minds of their own and deliberately go wrong whenever I use them	○	○	○	○	○
15	Computers are out to get me	○	○	○	○	○
16	I weep openly at the mention of central tendency	○	○	○	○	○
17	I slip into a coma whenever I see an equation	○	○	○	○	○
18	R always crashes when I try to use it	○	○	○	○	○
19	Everybody looks at me when I use R	○	○	○	○	○
20	I can't sleep for thoughts of eigenvectors	○	○	○	○	○
21	I wake up under my duvet thinking that I am trapped under a normal distribution	○	○	○	○	○
22	My friends are better at R than I am	○	○	○	○	○

Step 4: Identify number of factors

```
library(psych)

 parallel_analysis <- fa.parallel(raq)
```




```
## Parallel analysis suggests that the number of factors =  6  and the number
of components =  4
```

Step 4: Identify number of factors

```
parallel_analysis

## Call: fa.parallel(x = raq)
## Parallel analysis suggests that the number of factors =  6  and the number
of components =  4
##
##  Eigen Values of
##   Original factors Resampled data Simulated data Original components
## 1             6.64           0.20           0.21                7.29
## 2             0.91           0.15           0.15                1.74
## 3             0.63           0.13           0.13                1.32
## 4             0.48           0.12           0.11                1.23
## 5             0.29           0.10           0.10                0.99
## 6             0.13           0.08           0.08                0.90
##   Resampled components Simulated components
## 1                 1.17                 1.17
## 2                 1.15                 1.14
## 3                 1.13                 1.12
## 4                 1.11                 1.10
## 5                 1.09                 1.09
## 6                 1.08                 1.08
```

Step 5: Perform factor analysis (with initial recommended # factors)

```
paf <- fa(raq,
nfactors = 6,
fm="pa",
max.iter = 100,
rotate = "none")

paf

## Factor Analysis using method =  pa
## Call: fa(r = raq, nfactors = 6, rotate = "none", max.iter = 100, fm = 
"pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##       PA1   PA2   PA3   PA4   PA5   PA6   h2   u2 com
## Q01  0.57  0.13 -0.12  0.23 -0.28 -0.19 0.52 0.48 2.3
## Q02 -0.28  0.37  0.17  0.12 -0.03  0.01 0.26 0.74 2.6
## Q03 -0.60  0.25  0.20 -0.02 -0.01  0.03 0.46 0.54 1.6
## Q04  0.61  0.08 -0.06  0.18 -0.09 -0.03 0.42 0.58 1.3
## Q05  0.52  0.04 -0.02  0.15 -0.17 -0.08 0.33 0.67 1.5
## Q06  0.55  0.02  0.49 -0.17  0.07 -0.01 0.57 0.43 2.2
## Q07  0.66 -0.03  0.22  0.03  0.11  0.06 0.50 0.50 1.3
## Q08  0.55  0.49 -0.27 -0.21  0.10 -0.02 0.66 0.34 2.9
## Q09 -0.27  0.46  0.12  0.21  0.10  0.03 0.35 0.65 2.4
## Q10  0.40 -0.01  0.17 -0.09 -0.15  0.02 0.22 0.78 1.8
## Q11  0.64  0.31 -0.20 -0.27  0.08 -0.04 0.63 0.37 2.1
## Q12  0.64 -0.10  0.06  0.15  0.05 -0.07 0.45 0.55 1.2
## Q13  0.65  0.02  0.22 -0.06  0.06 -0.13 0.50 0.50 1.4
## Q14  0.63 -0.04  0.16  0.06  0.01  0.01 0.42 0.58 1.2
## Q15  0.58 -0.01  0.07 -0.15 -0.19  0.44 0.59 0.41 2.3
## Q16  0.66 -0.02 -0.11  0.14 -0.28  0.09 0.56 0.44 1.6
## Q17  0.63  0.36 -0.15 -0.15  0.04  0.01 0.57 0.43 1.9
## Q18  0.68 -0.04  0.28  0.04  0.09 -0.10 0.57 0.43 1.4
## Q19 -0.40  0.27  0.11  0.06 -0.05  0.02 0.25 0.75 2.0
## Q20  0.41 -0.17 -0.25  0.19  0.24  0.11 0.37 0.63 3.5
## Q21  0.64 -0.10 -0.11  0.27  0.28  0.10 0.60 0.40 2.0
## Q22 -0.28  0.29  0.05  0.28  0.05  0.11 0.26 0.74 3.4
## Q23 -0.13  0.18  0.08  0.23  0.01  0.08 0.12 0.88 3.1
##
##                        PA1  PA2  PA3  PA4  PA5  PA6
## SS loadings           6.79 1.14 0.83 0.67 0.45 0.32
## Proportion Var        0.30 0.05 0.04 0.03 0.02 0.01
## Cumulative Var        0.30 0.34 0.38 0.41 0.43 0.44
## Proportion Explained  0.67 0.11 0.08 0.07 0.04 0.03
## Cumulative Proportion 0.67 0.78 0.86 0.92 0.97 1.00
##
## Mean item complexity =  2
## Test of the hypothesis that 6 factors are sufficient.
##
## The degrees of freedom for the null model are  253  and the objective 
function was  7.55 with Chi Square of  19334.49
```

```
## The degrees of freedom for the model are 130  and the objective function
was  0.23
##
## The root mean square of the residuals (RMSR) is  0.02
## The df corrected root mean square of the residuals is  0.02
##
## The harmonic number of observations is  2571 with the empirical chi square
364.66  with prob <  3.9e-24
## The total number of observations was  2571  with Likelihood Chi Square =
578.65  with prob <  7.6e-58
##
## Tucker Lewis Index of factoring reliability =  0.954
## RMSEA index =  0.037  and the 90 % confidence intervals are  0.034 0.04
## BIC =  -442.12
## Fit based upon off diagonal values = 1
## Measures of factor score adequacy
##                                                    PA1  PA2  PA3  PA4
## Correlation of (regression) scores with factors   0.97 0.83 0.80 0.75
## Multiple R square of scores with factors          0.93 0.68 0.64 0.56
## Minimum correlation of possible factor scores     0.87 0.37 0.27 0.12
##                                                     PA5   PA6
## Correlation of (regression) scores with factors    0.70  0.65
## Multiple R square of scores with factors           0.48  0.42
## Minimum correlation of possible factor scores     -0.03 -0.17
```

Check the factor matrix

- We are looking high levels of variance explained with SS loadings > 1

```
print(paf$loadings, cutoff=0, digits=3)
```

```
##
## Loadings:
##        PA1    PA2    PA3    PA4    PA5    PA6
## Q01  0.567  0.129 -0.120  0.229 -0.275 -0.188
## Q02 -0.280  0.369  0.172  0.115 -0.029  0.009
## Q03 -0.603  0.245  0.199 -0.022 -0.006  0.030
## Q04  0.606  0.082 -0.056  0.184 -0.090 -0.033
## Q05  0.523  0.043 -0.020  0.154 -0.167 -0.083
## Q06  0.548  0.024  0.488 -0.166  0.073 -0.006
## Q07  0.662 -0.026  0.223  0.030  0.107  0.057
## Q08  0.545  0.488 -0.272 -0.214  0.096 -0.020
## Q09 -0.266  0.462  0.124  0.210  0.097  0.032
## Q10  0.405 -0.005  0.172 -0.090 -0.148  0.024
## Q11  0.644  0.312 -0.199 -0.270  0.085 -0.037
## Q12  0.641 -0.099  0.063  0.154  0.047 -0.067
## Q13  0.650  0.024  0.223 -0.058  0.061 -0.134
## Q14  0.626 -0.036  0.161  0.056  0.011  0.013
## Q15  0.580 -0.007  0.072 -0.152 -0.188  0.436
## Q16  0.661 -0.016 -0.109  0.138 -0.283  0.094
```

```
## Q17  0.629  0.355 -0.155 -0.150  0.038  0.006
## Q18  0.683 -0.039  0.277  0.041  0.092 -0.099
## Q19 -0.395  0.267  0.110  0.060 -0.052  0.022
## Q20  0.412 -0.171 -0.250  0.190  0.241  0.114
## Q21  0.644 -0.099 -0.110  0.270  0.283  0.099
## Q22 -0.279  0.291  0.050  0.284  0.047  0.114
## Q23 -0.130  0.182  0.081  0.235  0.011  0.077
##
##                  PA1   PA2   PA3   PA4   PA5   PA6
## SS loadings    6.786 1.140 0.827 0.667 0.452 0.324
## Proportion Var 0.295 0.050 0.036 0.029 0.020 0.014
## Cumulative Var 0.295 0.345 0.381 0.410 0.429 0.443
```

Check the structure matrix

======

```
print(paf$Structure, cutoff=0, digits=3)
```

```
##
## Loadings:
##     PA1    PA2    PA3    PA4    PA5    PA6
## Q01  0.567  0.129 -0.120  0.229 -0.275 -0.188
## Q02 -0.280  0.369  0.172  0.115 -0.029  0.009
## Q03 -0.603  0.245  0.199 -0.022 -0.006  0.030
## Q04  0.606  0.082 -0.056  0.184 -0.090 -0.033
## Q05  0.523  0.043 -0.020  0.154 -0.167 -0.083
## Q06  0.548  0.024  0.488 -0.166  0.073 -0.006
## Q07  0.662 -0.026  0.223  0.030  0.107  0.057
## Q08  0.545  0.488 -0.272 -0.214  0.096 -0.020
## Q09 -0.266  0.462  0.124  0.210  0.097  0.032
## Q10  0.405 -0.005  0.172 -0.090 -0.148  0.024
## Q11  0.644  0.312 -0.199 -0.270  0.085 -0.037
## Q12  0.641 -0.099  0.063  0.154  0.047 -0.067
## Q13  0.650  0.024  0.223 -0.058  0.061 -0.134
## Q14  0.626 -0.036  0.161  0.056  0.011  0.013
## Q15  0.580 -0.007  0.072 -0.152 -0.188  0.436
## Q16  0.661 -0.016 -0.109  0.138 -0.283  0.094
## Q17  0.629  0.355 -0.155 -0.150  0.038  0.006
## Q18  0.683 -0.039  0.277  0.041  0.092 -0.099
## Q19 -0.395  0.267  0.110  0.060 -0.052  0.022
## Q20  0.412 -0.171 -0.250  0.190  0.241  0.114
## Q21  0.644 -0.099 -0.110  0.270  0.283  0.099
## Q22 -0.279  0.291  0.050  0.284  0.047  0.114
## Q23 -0.130  0.182  0.081  0.235  0.011  0.077
##
##                  PA1   PA2   PA3   PA4   PA5   PA6
## SS loadings    6.786 1.140 0.827 0.667 0.452 0.324
## Proportion Var 0.295 0.050 0.036 0.029 0.020 0.014
## Cumulative Var 0.295 0.345 0.381 0.410 0.429 0.443
```

Check eigenvalues

```
paf$e.values[1:6]

## [1] 7.2900471 1.7388287 1.3167515 1.2271982 0.9878779 0.8953304
```

Check communalities

- Communality for each variable: the percentage of variance that can be explained by the retained factors.
- Retained factors should explain more of the variance in each variable.

```
paf$communality

##       Q01       Q02       Q03       Q04       Q05       Q06       Q07
## 0.5170176 0.2585136 0.4643374 0.4196524 0.3341637 0.5720655 0.5042725
##       Q08       Q09       Q10       Q11       Q12       Q13       Q14
## 0.6649413 0.3542281 0.2240464 0.6328967 0.4544862 0.4973541 0.4223263
##       Q15       Q16       Q17       Q18       Q19       Q20       Q21
## 0.5902303 0.5571656 0.5700891 0.5655104 0.2467731 0.3686202 0.5991875
##       Q22       Q23
## 0.2606533 0.1178839
```

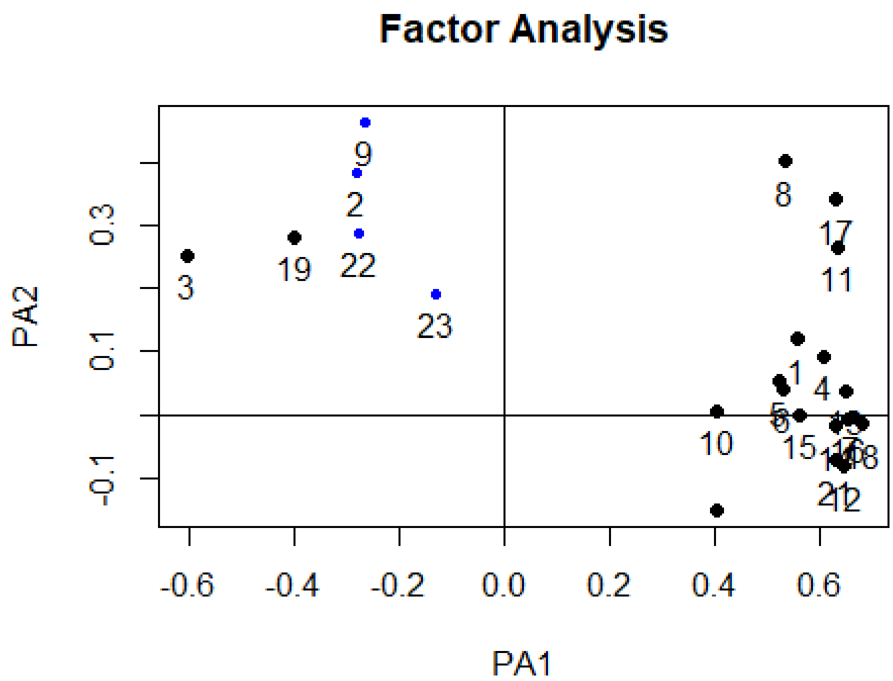
Step 6: Perform factor analysis (with reduced number of factors)

```
paf1 <- fa(raq,
nfactors = 2,
fm="pa",
max.iter = 100,
rotate = "none")

paf1

## Factor Analysis using method =  pa
## Call: fa(r = raq, nfactors = 2, rotate = "none", max.iter = 100, fm =
"pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##       PA1   PA2    h2   u2 com
## Q01  0.56  0.12 0.324 0.68 1.1
## Q02 -0.28  0.39 0.228 0.77 1.8
## Q03 -0.61  0.25 0.430 0.57 1.3
## Q04  0.61  0.09 0.377 0.62 1.0
## Q05  0.52  0.05 0.276 0.72 1.0
## Q06  0.53  0.04 0.282 0.72 1.0
## Q07  0.66 -0.01 0.437 0.56 1.0
## Q08  0.53  0.40 0.445 0.56 1.9
## Q09 -0.27  0.46 0.287 0.71 1.6
## Q10  0.40  0.00 0.163 0.84 1.0
## Q11  0.63  0.27 0.472 0.53 1.3
## Q12  0.64 -0.08 0.421 0.58 1.0
```

```
## Q13  0.65  0.04 0.421 0.58 1.0
## Q14  0.63 -0.02 0.396 0.60 1.0
## Q15  0.56  0.00 0.315 0.68 1.0
## Q16  0.65 -0.01 0.428 0.57 1.0
## Q17  0.63  0.34 0.511 0.49 1.5
## Q18  0.68 -0.02 0.461 0.54 1.0
## Q19 -0.40  0.28 0.238 0.76 1.8
## Q20  0.40 -0.15 0.187 0.81 1.3
## Q21  0.63 -0.07 0.403 0.60 1.0
## Q22 -0.28  0.29 0.161 0.84 2.0
## Q23 -0.13  0.19 0.053 0.95 1.8
##
##                        PA1  PA2
## SS loadings           6.67 1.04
## Proportion Var        0.29 0.05
## Cumulative Var        0.29 0.34
## Proportion Explained  0.86 0.14
## Cumulative Proportion 0.86 1.00
##
## Mean item complexity =  1.3
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  253  and the objective
function was  7.55 with Chi Square of  19334.49
## The degrees of freedom for the model are 208  and the objective function
was  1.23
##
## The root mean square of the residuals (RMSR) is  0.05
## The df corrected root mean square of the residuals is  0.05
##
## The harmonic number of observations is  2571 with the empirical chi square
3114.53  with prob <  0
## The total number of observations was  2571  with Likelihood Chi Square =
3155.34  with prob <  0
##
## Tucker Lewis Index of factoring reliability =  0.812
## RMSEA index =  0.074  and the 90 % confidence intervals are  0.072 0.077
## BIC =  1522.12
## Fit based upon off diagonal values = 0.97
## Measures of factor score adequacy
##                                                    PA1  PA2
## Correlation of (regression) scores with factors   0.96 0.78
## Multiple R square of scores with factors          0.92 0.61
## Minimum correlation of possible factor scores     0.83 0.23
```

Step 6: Perform factor analysis (with reduced number of factors)

```
plot(paf1)
```



Factor analysis rotation

What is rotation?

- It is possible that variables load “highly” onto one factor and “medium” onto another
- By rotating the factor axes, the variables are aligned with the factors that they load onto most
- This helps us discriminate between factors

There are different methods of rotation

- **Orthogonal rotation:** Assumes that factors are unrelated and keeps them that way
- **Oblique rotation:** Assumes that factors might be related and allows them to be correlated after rotation

Are factors related?

-Theoretical: Do we have logical reason for thinking they could be connected?

-Based on data: Does the factor plot suggest independence or relatedness?

Step 7: Rotation

- Perform factor analysis (with rotation)

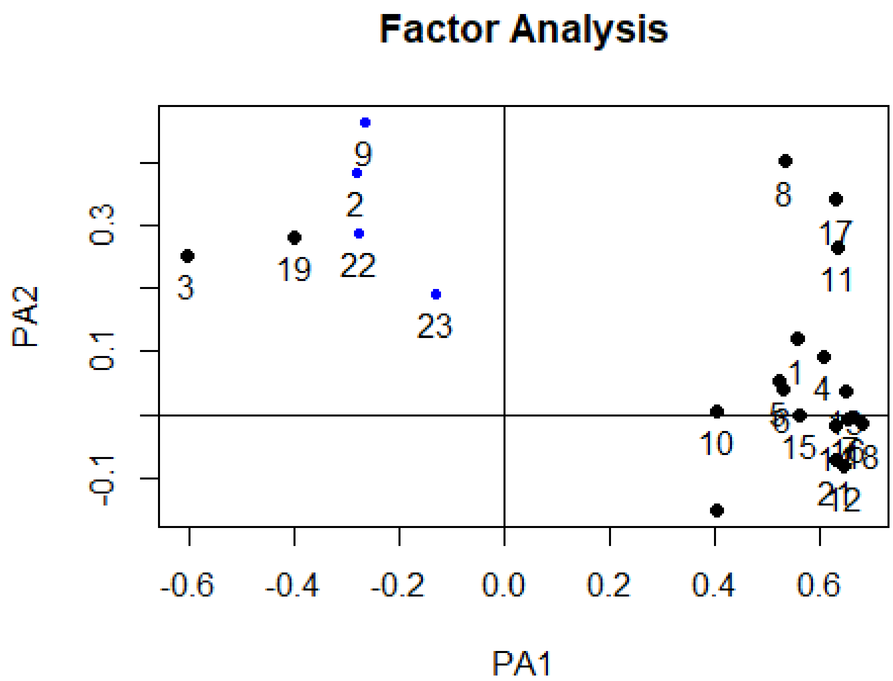
```
paf2 <- fa(raq,
nfactors = 2,
```

```
fm="pa",
max.iter = 100,
rotate = "oblimin")

## Loading required namespace: GPArotation

paf2

## Factor Analysis using method =  pa
## Call: fa(r = raq, nfactors = 2, rotate = "oblimin", max.iter = 100,
##     fm = "pa")
## Standardized loadings (pattern matrix) based upon correlation matrix
##       PA1   PA2    h2   u2 com
## Q01  0.57  0.03 0.324 0.68 1.0
## Q02 -0.12  0.44 0.228 0.77 1.2
## Q03 -0.48  0.36 0.430 0.57 1.8
## Q04  0.61 -0.01 0.377 0.62 1.0
## Q05  0.52 -0.03 0.276 0.72 1.0
## Q06  0.52 -0.05 0.282 0.72 1.0
## Q07  0.63 -0.11 0.437 0.56 1.1
## Q08  0.66  0.32 0.445 0.56 1.4
## Q09 -0.08  0.51 0.287 0.71 1.0
## Q10  0.39 -0.06 0.163 0.84 1.1
## Q11  0.70  0.16 0.472 0.53 1.1
## Q12  0.58 -0.19 0.421 0.58 1.2
## Q13  0.63 -0.07 0.421 0.58 1.0
## Q14  0.59 -0.12 0.396 0.60 1.1
## Q15  0.53 -0.09 0.315 0.68 1.1
## Q16  0.62 -0.12 0.428 0.57 1.1
## Q17  0.73  0.24 0.511 0.49 1.2
## Q18  0.64 -0.13 0.461 0.54 1.1
## Q19 -0.27  0.35 0.238 0.76 1.9
## Q20  0.33 -0.22 0.187 0.81 1.8
## Q21  0.57 -0.18 0.403 0.60 1.2
## Q22 -0.15  0.34 0.161 0.84 1.4
## Q23 -0.05  0.21 0.053 0.95 1.1
##
##                        PA1  PA2
## SS loadings           6.33 1.39
## Proportion Var        0.28 0.06
## Cumulative Var        0.28 0.34
## Proportion Explained  0.82 0.18
## Cumulative Proportion 0.82 1.00
##
##  With factor correlations of
##       PA1   PA2
## PA1  1.00 -0.22
## PA2 -0.22  1.00
##
## Mean item complexity =  1.2
```

```
## Test of the hypothesis that 2 factors are sufficient.
##
## The degrees of freedom for the null model are  253  and the objective
function was  7.55 with Chi Square of  19334.49
## The degrees of freedom for the model are 208  and the objective function
was  1.23
##
## The root mean square of the residuals (RMSR) is  0.05
## The df corrected root mean square of the residuals is  0.05
##
## The harmonic number of observations is  2571 with the empirical chi square
3114.53  with prob <  0
## The total number of observations was  2571  with Likelihood Chi Square =
3155.34  with prob <  0
##
## Tucker Lewis Index of factoring reliability =  0.812
## RMSEA index =  0.074  and the 90 % confidence intervals are  0.072 0.077
## BIC =  1522.12
## Fit based upon off diagonal values = 0.97
## Measures of factor score adequacy
##                                                    PA1  PA2
## Correlation of (regression) scores with factors   0.96 0.81
## Multiple R square of scores with factors          0.91 0.65
## Minimum correlation of possible factor scores     0.82 0.30
```

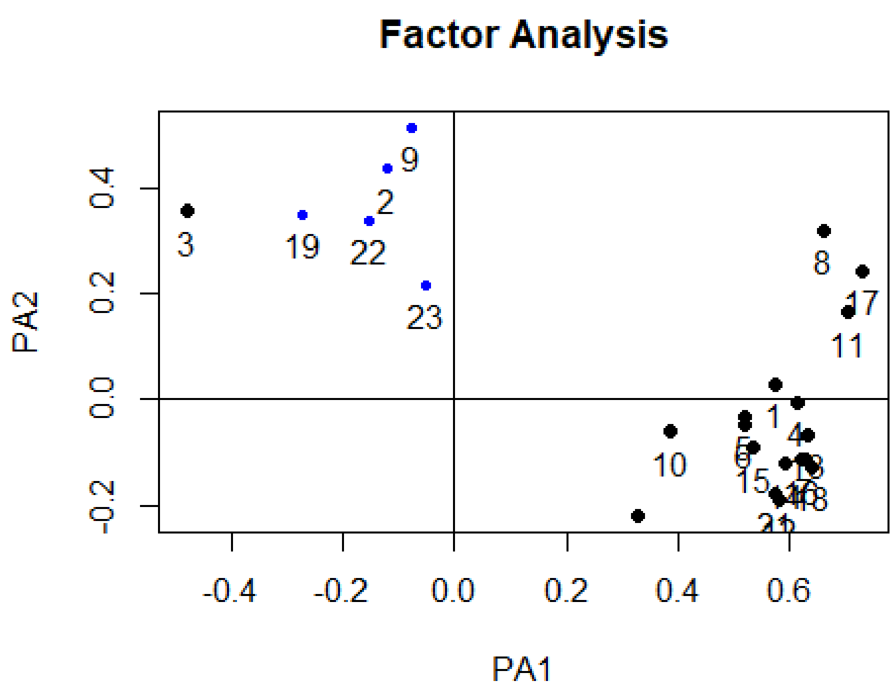
Step 7: Rotation

```
plot(paf1)
```


Factor Analysis
PA2
PA1
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
-0.1
0.1
0.3

***

```
plot(paf2)
```


Factor Analysis
PA2
PA1
-0.4
-0.2
0.0
0.2
0.4
0.6
-0.2
0.0
0.2
0.4

Reliability / internal consistency

type: section

Cronbach’s Alpha

- An expansion of the split-half reliability concept
- Alpha takes all possible combination of items and assesses their relationship to each other
- High values above 0.7 suggest internal consistency among items

Chronbach’s Alpha in R

- We can use the *alpha()* function in the psych package

```
library(psych)

alpha(raq)

## Warning in alpha(raq): Some items were negatively correlated with the
total scale and probably
## should be reversed.
## To do this, run the function again with the 'check.keys=TRUE' option

## Some items ( Q02 Q03 Q09 Q19 Q22 Q23 ) were negatively correlated with the
total scale and
## probably should be reversed.
## To do this, run the function again with the 'check.keys=TRUE' option

##
## Reliability analysis
## Call: alpha(x = raq)
##
##   raw_alpha std.alpha G6(smc) average_r S/N    ase mean   sd median_r
##       0.75      0.77    0.83      0.13 3.4 0.0065  3.3 0.39     0.23
##
##  lower alpha upper     95% confidence boundaries
## 0.74 0.75 0.77
##
##  Reliability if an item is dropped:
##     raw_alpha std.alpha G6(smc) average_r S/N alpha se var.r med.r
## Q01      0.73      0.76    0.82      0.12 3.1   0.0071 0.071  0.23
## Q02      0.77      0.79    0.84      0.15 3.8   0.0061 0.071  0.25
## Q03      0.79      0.81    0.85      0.16 4.2   0.0055 0.059  0.25
## Q04      0.73      0.75    0.82      0.12 3.0   0.0072 0.070  0.22
## Q05      0.74      0.76    0.82      0.12 3.1   0.0071 0.072  0.22
## Q06      0.73      0.76    0.82      0.12 3.1   0.0072 0.072  0.23
## Q07      0.73      0.75    0.82      0.12 3.0   0.0074 0.069  0.22
## Q08      0.73      0.76    0.82      0.12 3.1   0.0071 0.072  0.23
```

```
## Q09      0.78      0.79    0.84      0.15 3.8   0.0058 0.071  0.25
## Q10      0.74      0.76    0.83      0.13 3.3   0.0068 0.074  0.23
## Q11      0.73      0.75    0.81      0.12 3.0   0.0072 0.069  0.22
## Q12      0.73      0.75    0.82      0.12 3.1   0.0072 0.069  0.22
## Q13      0.73      0.75    0.82      0.12 3.0   0.0073 0.069  0.22
## Q14      0.73      0.75    0.82      0.12 3.1   0.0072 0.070  0.22
## Q15      0.73      0.76    0.82      0.12 3.1   0.0071 0.071  0.22
## Q16      0.73      0.75    0.82      0.12 3.0   0.0072 0.069  0.22
## Q17      0.73      0.75    0.81      0.12 3.0   0.0072 0.070  0.22
## Q18      0.72      0.75    0.81      0.12 3.0   0.0074 0.068  0.22
## Q19      0.78      0.80    0.85      0.15 4.0   0.0057 0.067  0.26
## Q20      0.75      0.77    0.83      0.13 3.3   0.0067 0.073  0.25
## Q21      0.73      0.75    0.82      0.12 3.1   0.0072 0.069  0.22
## Q22      0.77      0.79    0.84      0.15 3.8   0.0059 0.071  0.26
## Q23      0.77      0.79    0.84      0.14 3.7   0.0061 0.074  0.26
##
##  Item statistics
##         n   raw.r  std.r  r.cor r.drop mean   sd
## Q01 2571  0.5598  0.581  0.564  0.492  3.6 0.83
## Q02 2571 -0.0116 -0.018 -0.114 -0.105  4.4 0.85
## Q03 2571 -0.3356 -0.361 -0.465 -0.435  3.4 1.08
## Q04 2571  0.6064  0.618  0.606  0.533  3.2 0.95
## Q05 2571  0.5365  0.546  0.516  0.454  3.3 0.96
## Q06 2571  0.5709  0.560  0.547  0.478  3.8 1.12
## Q07 2571  0.6409  0.636  0.635  0.560  3.1 1.10
## Q08 2571  0.5646  0.582  0.578  0.493  3.8 0.87
## Q09 2571  0.0587  0.020 -0.068 -0.081  3.2 1.26
## Q10 2571  0.4300  0.442  0.391  0.346  3.7 0.88
## Q11 2571  0.6078  0.628  0.633  0.540  3.7 0.88
## Q12 2571  0.5909  0.602  0.593  0.519  2.8 0.92
## Q13 2571  0.6288  0.637  0.634  0.559  3.6 0.95
## Q14 2571  0.6056  0.609  0.596  0.528  3.1 1.00
## Q15 2571  0.5433  0.550  0.526  0.457  3.2 1.01
## Q16 2571  0.5965  0.615  0.612  0.525  3.1 0.92
## Q17 2571  0.6329  0.650  0.653  0.568  3.5 0.88
## Q18 2571  0.6534  0.653  0.656  0.578  3.4 1.05
## Q19 2571 -0.1316 -0.157 -0.264 -0.248  3.7 1.10
## Q20 2571  0.3705  0.375  0.326  0.265  2.4 1.04
## Q21 2571  0.5922  0.598  0.591  0.514  2.8 0.98
## Q22 2571 -0.0063 -0.027 -0.127 -0.121  3.1 1.04
## Q23 2571  0.1030  0.084 -0.014 -0.013  2.6 1.04
##
## Non missing response frequency for each item
##         1    2    3    4    5 miss
## Q01 0.02 0.07 0.29 0.52 0.11    0
## Q02 0.01 0.04 0.08 0.31 0.56    0
## Q03 0.03 0.17 0.34 0.26 0.19    0
## Q04 0.05 0.17 0.36 0.37 0.05    0
## Q05 0.04 0.18 0.29 0.43 0.06    0
## Q06 0.06 0.10 0.13 0.44 0.27    0
```

```
## Q07 0.09 0.24 0.26 0.34 0.07    0
## Q08 0.03 0.06 0.19 0.58 0.15    0
## Q09 0.08 0.28 0.23 0.20 0.20    0
## Q10 0.02 0.10 0.18 0.57 0.14    0
## Q11 0.02 0.06 0.22 0.53 0.16    0
## Q12 0.09 0.23 0.46 0.20 0.02    0
## Q13 0.03 0.12 0.25 0.48 0.12    0
## Q14 0.07 0.18 0.38 0.31 0.06    0
## Q15 0.06 0.18 0.30 0.39 0.07    0
## Q16 0.06 0.16 0.42 0.33 0.04    0
## Q17 0.03 0.10 0.27 0.52 0.08    0
## Q18 0.06 0.12 0.31 0.37 0.14    0
## Q19 0.02 0.15 0.22 0.33 0.29    0
## Q20 0.22 0.37 0.25 0.15 0.02    0
## Q21 0.09 0.29 0.34 0.26 0.02    0
## Q22 0.05 0.26 0.34 0.26 0.10    0
## Q23 0.12 0.42 0.27 0.12 0.06    0
```

Cronbach's Alpha in R

- Here we get a warning that some of the items are negatively correlated and we should probably reverse them.
- The decision to do so should be based on the logic of the questions themselves - check first
- However, since cronbach's alpha is designed to check internal consistency related to a single construct, we would expect that negative correlations would only result from:
  - Items that are designed to be reverse-scored
  - Questions that are related to another factor or construct

Cronbach's Alpha in R

- Let's check the questionnaire
  - (Q02, Q03, Q09, Q19, Q22, Q23):

SD = Strongly Disagree, D = Disagree, N = Neither, A = Agree, SA = Strongly Agree

		SD	D	N	A	SA
1	Statistics make me cry	○	○	○	○	○
2	My friends will think I'm stupid for not being able to cope with R	○	○	○	○	○
3	Standard deviations excite me	○	○	○	○	○
4	I dream that Pearson is attacking me with correlation coefficients	○	○	○	○	○
5	I don't understand statistics	○	○	○	○	○
6	I have little experience of computers	○	○	○	○	○
7	All computers hate me	○	○	○	○	○
8	I have never been good at mathematics	○	○	○	○	○
9	My friends are better at statistics than me	○	○	○	○	○
10	Computers are useful only for playing games	○	○	○	○	○
11	I did badly at mathematics at school	○	○	○	○	○
12	People try to tell you that R makes statistics easier to understand but it doesn't	○	○	○	○	○
13	I worry that I will cause irreparable damage because of my incompetence with computers	○	○	○	○	○
14	Computers have minds of their own and deliberately go wrong whenever I use them	○	○	○	○	○
15	Computers are out to get me	○	○	○	○	○
16	I weep openly at the mention of central tendency	○	○	○	○	○
17	I slip into a coma whenever I see an equation	○	○	○	○	○
18	R always crashes when I try to use it	○	○	○	○	○
19	Everybody looks at me when I use R	○	○	○	○	○
20	I can't sleep for thoughts of eigenvectors	○	○	○	○	○
21	I wake up under my duvet thinking that I am trapped under a normal distribution	○	○	○	○	○
22	My friends are better at R than I am	○	○	○	○	○

Cronbach's Alpha in R

- It is possible to run the analysis with automatic reversal of negatively-correlated items

```
alpha(raq, check.keys=TRUE)
```

```
## Warning in alpha(raq, check.keys = TRUE): Some items were negatively
correlated with total scale and were automatically reversed.
##  This is indicated by a negative sign for the variable name.

##
## Reliability analysis
## Call: alpha(x = raq, check.keys = TRUE)
##
##   raw_alpha std.alpha G6(smc) average_r S/N    ase mean   sd median_r
##       0.89      0.89    0.91      0.27 8.3 0.0031  3.1 0.54     0.27
##
##  lower alpha upper     95% confidence boundaries
## 0.88 0.89 0.9
##
##  Reliability if an item is dropped:
##      raw_alpha std.alpha G6(smc) average_r S/N alpha se var.r med.r
## Q01       0.88      0.89    0.90      0.26 7.9   0.0032 0.016  0.27
## Q02-      0.89      0.89    0.91      0.28 8.4   0.0031 0.016  0.28
## Q03-      0.88      0.89    0.90      0.26 7.8   0.0033 0.017  0.26
## Q04       0.88      0.89    0.90      0.26 7.8   0.0033 0.016  0.26
## Q05       0.89      0.89    0.90      0.27 8.0   0.0032 0.017  0.27
## Q06       0.88      0.89    0.90      0.27 8.0   0.0032 0.016  0.27
## Q07       0.88      0.89    0.90      0.26 7.7   0.0034 0.016  0.26
## Q08       0.89      0.89    0.90      0.27 8.0   0.0032 0.016  0.27
## Q09-      0.89      0.89    0.91      0.28 8.4   0.0030 0.016  0.28
## Q10       0.89      0.89    0.90      0.27 8.2   0.0032 0.017  0.28
## Q11       0.88      0.89    0.90      0.26 7.8   0.0033 0.016  0.26
## Q12       0.88      0.89    0.90      0.26 7.7   0.0033 0.016  0.26
## Q13       0.88      0.89    0.90      0.26 7.7   0.0033 0.016  0.26
## Q14       0.88      0.89    0.90      0.26 7.8   0.0033 0.016  0.26
## Q15       0.88      0.89    0.90      0.26 7.9   0.0033 0.017  0.27
## Q16       0.88      0.89    0.90      0.26 7.7   0.0033 0.016  0.26
## Q17       0.88      0.89    0.90      0.26 7.8   0.0033 0.016  0.26
## Q18       0.88      0.88    0.90      0.26 7.7   0.0034 0.016  0.26
## Q19-      0.89      0.89    0.90      0.27 8.2   0.0032 0.017  0.29
## Q20       0.89      0.89    0.90      0.27 8.2   0.0032 0.017  0.28
## Q21       0.88      0.89    0.90      0.26 7.7   0.0033 0.016  0.26
## Q22-      0.89      0.89    0.91      0.28 8.4   0.0031 0.016  0.29
## Q23-      0.89      0.90    0.91      0.28 8.7   0.0030 0.014  0.29
##
##  Item statistics
##         n raw.r std.r r.cor r.drop mean   sd
## Q01  2571  0.55  0.57  0.54   0.50  3.6 0.83
## Q02- 2571  0.36  0.36  0.31   0.30  1.6 0.85
## Q03- 2571  0.65  0.64  0.62   0.59  2.6 1.08
## Q04  2571  0.61  0.61  0.59   0.55  3.2 0.95
## Q05  2571  0.54  0.55  0.52   0.48  3.3 0.96
## Q06  2571  0.56  0.55  0.53   0.49  3.8 1.12
## Q07  2571  0.67  0.67  0.65   0.62  3.1 1.10
## Q08  2571  0.51  0.53  0.51   0.46  3.8 0.87
```

```
## Q09- 2571  0.37  0.35  0.30   0.28  2.8 1.26
## Q10  2571  0.44  0.45  0.40   0.38  3.7 0.88
## Q11  2571  0.63  0.64  0.63   0.58  3.7 0.88
## Q12  2571  0.65  0.65  0.64   0.60  2.8 0.92
## Q13  2571  0.65  0.65  0.64   0.60  3.6 0.95
## Q14  2571  0.64  0.64  0.62   0.59  3.1 1.00
## Q15  2571  0.59  0.59  0.56   0.53  3.2 1.01
## Q16  2571  0.66  0.67  0.65   0.61  3.1 0.92
## Q17  2571  0.61  0.62  0.61   0.56  3.5 0.88
## Q18  2571  0.68  0.68  0.67   0.63  3.4 1.05
## Q19- 2571  0.47  0.46  0.42   0.40  2.3 1.10
## Q20  2571  0.45  0.45  0.41   0.38  2.4 1.04
## Q21  2571  0.64  0.64  0.63   0.59  2.8 0.98
## Q22- 2571  0.37  0.36  0.31   0.30  2.9 1.04
## Q23- 2571  0.23  0.22  0.15   0.15  3.4 1.04
##
## Non missing response frequency for each item
##        1    2    3    4    5 miss
## Q01 0.02 0.07 0.29 0.52 0.11    0
## Q02 0.01 0.04 0.08 0.31 0.56    0
## Q03 0.03 0.17 0.34 0.26 0.19    0
## Q04 0.05 0.17 0.36 0.37 0.05    0
## Q05 0.04 0.18 0.29 0.43 0.06    0
## Q06 0.06 0.10 0.13 0.44 0.27    0
## Q07 0.09 0.24 0.26 0.34 0.07    0
## Q08 0.03 0.06 0.19 0.58 0.15    0
## Q09 0.08 0.28 0.23 0.20 0.20    0
## Q10 0.02 0.10 0.18 0.57 0.14    0
## Q11 0.02 0.06 0.22 0.53 0.16    0
## Q12 0.09 0.23 0.46 0.20 0.02    0
## Q13 0.03 0.12 0.25 0.48 0.12    0
## Q14 0.07 0.18 0.38 0.31 0.06    0
## Q15 0.06 0.18 0.30 0.39 0.07    0
## Q16 0.06 0.16 0.42 0.33 0.04    0
## Q17 0.03 0.10 0.27 0.52 0.08    0
## Q18 0.06 0.12 0.31 0.37 0.14    0
## Q19 0.02 0.15 0.22 0.33 0.29    0
## Q20 0.22 0.37 0.25 0.15 0.02    0
## Q21 0.09 0.29 0.34 0.26 0.02    0
## Q22 0.05 0.26 0.34 0.26 0.10    0
## Q23 0.12 0.42 0.27 0.12 0.06    0
```

Summary

- What is factor analysis
- CFA versus PCA
- Variance in factor analysis
- Considertations for factor analysis

- Identifying / extracting factors
- Rotation
- Cronbach’s alpha