# ECON Brainstorming

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We will begin by restating some useful theorems: **fill me in**

## 1

### 1.a

Yes, $A$ and $B$ can be mutually exclusive. Recall that:

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B] \implies \mathbb{P}[A \cap B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cup B] \tag{1}$$

We also know that: **need to resolve this still**

$$\max(\mathbb{P}[A], \mathbb{P}[B]) \leq \mathbb{P}[A \cup B] \leq \tag{2}$$

**I need to explain this lol**

### 1.b

### 1.c

### 1.d

### 1.e

No. Suppose, by contradiction, that $A, B$ mutually exclusive and $A \perp\!\!\!\perp B$. If this were the case, then:

$$\mathbb{P}[A \cap B] = 0 = \mathbb{P}[A]\mathbb{P}[B] \tag{3}$$

However, we know that $\mathbb{P}[A]\mathbb{P}[B] = 0.12$, a contradiction. Thus, $A, B$ cannot be both.

### 1.f

Recall our equation from (e):

$$\mathbb{P}[A \cap B] = 0 = \mathbb{P}[A]\mathbb{P}[B] \tag{4}$$

This is true if and only if one of the probabilities take on the value 0. Thus, because all probabilities are nonnegative,

$$\min\{\mathbb{P}[C], \mathbb{P}[D]\} = 0 \tag{5}$$

## 2

Call the event $W \leq 0$ to be $\mathcal{E}_{\leq}$ and $W \geq 0$ to be $\mathcal{E}_{\geq}$. Then, note that $\mathcal{E}_{\leq} \cap \mathcal{E}_{\geq}$ is precisely the event where $W = 0$. Thus, by provided theorems, we know that:

$$\mathbb{P}[\mathcal{E}_{\leq} \cup \mathcal{E}_{\geq}] = \mathbb{P}[\mathcal{E}_{\leq}] + \mathbb{P}[\mathcal{E}_{\geq}] - \mathbb{P}[\mathcal{E}_{\leq} \cap \mathcal{E}_{\geq}] \tag{6}$$

$$1 = c + d - \mathbb{P}[W = 0] \tag{7}$$

$$\mathbb{P}[W = 0] = c + d - 1 \tag{8}$$

## 3

### 3.a

No - because $\mathbb{P}[A \cap B] \neq 0$, $A, B$ are not mutually exclusive.

### 3.b

From prior theorems, note that:

$$\mathbb{P}[A \cap B] \leq \mathbb{P}[B] \leq \mathbb{P}[A \cup B] \implies \mathbb{P}[B] \in [1/8, 1/2] \tag{9}$$

### 3.c

We will employ a proof by contradiction. Suppose that $A \perp\!\!\!\perp B$, so that:

$$\frac{1}{8} = \mathbb{P}[A]\mathbb{P}[B] \tag{10}$$

Consider that:

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B] \implies \frac{5}{8} - \mathbb{P}[B] = \mathbb{P}[A] \tag{11}$$

This requires:

$$\frac{1}{8} = \left(\frac{5}{8} - \mathbb{P}[B]\right)\mathbb{P}[B] \implies \mathbb{P}[B]^2 - \frac{5}{8}\mathbb{P}[B] + \frac{1}{8} = 0 \tag{12}$$

We can solve this via the quadratic equation, but it's unnecessary; a simple test of $b^2 - 4ac$ demonstrates that there are no real solutions for $\mathbb{P}[B]$:

$$\frac{25}{64} - \frac{1}{2} < 0 \tag{13}$$

Thus, our assumption was false and we cannot have that $A \perp\!\!\!\perp B$.

# 4

## 4.a

Recall that $\mathbb{P}[A \cup B] \geq \max(\mathbb{P}[A], \mathbb{P}[B])$, so $r \geq \max(\mathbb{P}[A], \mathbb{P}[B]), s \geq \max(\mathbb{P}[A], \mathbb{P}[C])$. Thus, it's conceivable that, when $\mathbb{P}[B] = r, \mathbb{P}[C] = s$, that we can allow $\mathbb{P}[A] = 0$.

On the other hand, let's consider the maximal value $\mathbb{P}[A]$ could be. Note that $\mathbb{P}[A] \leq \mathbb{P}[A \cup B], \mathbb{P}[A] \leq \mathbb{P}[A \cup C]$ by the above statements. Thus, $\mathbb{P}[A] \leq \min(\mathbb{P}[A \cup B], \mathbb{P}[A \cup C]) = \min(r, s)$. So, we conclude:

$$0 \leq \mathbb{P}[A] \leq \min(r, s) \tag{14}$$

## 4.b

**um who knows lmao**

# 5

## 5.a

## 5.b

## 5.c

## 5.d

## 5.e

## 5.f

## 5.g

# 6

## 6.a

## 6.b

## 6.c

## 6.d

# 7

## 7.a

*Proof.* We seek to show that $A^C \perp\!\!\!\perp B^C$ by demonstrating that $\mathbb{P}[A^C \cap B^C] = \mathbb{P}[A^C]\mathbb{P}[B^C]$. Recall that, if we partition the sample space, it's probability must sum to 1. Thus,

$$\mathbb{P}[A \cup B] + \mathbb{P}[\widetilde{A \cup B}] = 1 \tag{15}$$

$$\mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B] + \mathbb{P}[A^C \cap B^C] = 1 \tag{16}$$

$$\mathbb{P}[A^C \cap B^C] = 1 - \mathbb{P}[A] - \mathbb{P}[B] + \mathbb{P}[A]\mathbb{P}[B] \tag{17}$$

$$= 1 - \mathbb{P}[A] - \mathbb{P}[B](1 - \mathbb{P}[A]) \tag{18}$$

$$= (1 - \mathbb{P}[B])(1 - \mathbb{P}[A]) \tag{19}$$

$$\mathbb{P}[A^C \cap B^C] = \mathbb{P}[A^C]\mathbb{P}[B^C] \tag{20}$$

As desired. □

## 7.b

## 7.c

$A \perp\!\!\!\perp B$:

*Proof.* We seek to show $\mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$. Note that $\mathbb{P}[A]$ is $\frac{1}{6}$, as is $\mathbb{P}[B]$. We also know that $\mathbb{P}[A \cap B] = \frac{1}{36}$, because there is only one possibility where both die are 6 $(6, 6)$, while there are 36, equally likely possibilities for both die roles. Thus,

$$\mathbb{P}[A \cap B] = \frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6} = \mathbb{P}[A]\mathbb{P}[B] \tag{21}$$

□

$A \not\!\perp\!\!\!\perp B | D$ **?**

*Proof.* □