1.

$$A = LU = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 11/13 & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} 2 & -3 & -1 \\ 0 & 13/2 & -7/2 \\ 0 & 0 & 32/13 \end{bmatrix}}$$

$Ax = b$ $\quad b^T = [1 \ -1 \ 2]$

$$b = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$Ax = LUx = b$ $\quad Ly = b$

$y \equiv Ux$ $\quad Ux = y$

$$\overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & \frac{11}{13} & 1 \end{bmatrix}} \overset{y}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \overset{b}{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}}$$

$y_1 = 1$

$\frac{3}{2} y_1 + y_2 = -1$

$\frac{3}{2} + y_2 = -1$

$y_2 = -2.5$

$\frac{1}{2} y_1 + \frac{11}{13} y_2 + y_3 = 2$

$\frac{1}{2}(1) + \frac{11}{13}(-2.5) + y_3 = 2$

$\frac{1}{2} - \frac{55}{26} + y_3 = 2$

$y_3 = -\frac{8}{13}$

$y^T = [1, -2.5, \frac{47}{13}]$

$$\begin{bmatrix} 1 \\ -2.5 \\ \frac{47}{13} \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ 0 & 13/2 & -\frac{7}{2} \\ 0 & 0 & 32/13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$1 = 2x_1 - 3x_2 - x_3$

$-2.5 = \frac{13}{2} x_2 - \frac{7}{2} x_3$

$\frac{47}{13} = \frac{32}{13} x_3$

$x = \left(\frac{13}{32}\right)\left(\frac{47}{13}\right)$

$\boxed{x_3 = \frac{47}{32}}$

$-2.5 = \frac{13}{2} x_2 - \frac{7}{2}\left(\frac{47}{32}\right)$

$\boxed{x_2 = \frac{13}{32}}$

$1 = 2x_1 - 3\left(\frac{13}{32}\right) - \frac{47}{32}$

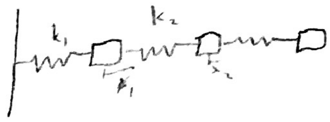
$x_1 = \frac{59}{32}$

$$x = \begin{bmatrix} \frac{59}{32} \\ \frac{13}{32} \\ \frac{47}{32} \end{bmatrix}$$

2. a)

$$\begin{bmatrix} m_1 & 1 & 0 & 0 \\ m_2 & -1 & 1 & 0 \\ m_3 & 0 & -1 & 1 \\ m_4 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} m_1 g(\sin\theta - \mu_1 \cos\theta) \\ m_2 g(\sin\theta - \mu_2 \cos\theta) \\ m_3 g(\sin\theta - \mu_3 \cos\theta) \\ -m_4 g \end{bmatrix}$$

3. a) $\sum F_i = m_i \ddot{x}_i$



Eqns of motion

$$\sum F_1 = -k_1 x_1 - k_2(x_1 - x_2) = m_1 \ddot{x}_1$$

$$\sum F_2 = -k_2(x_2 - x_1) - k_3(x_2 - x_3) = m_2 \ddot{x}_2$$

$$\sum F_3 = -k_3(x_3 - x_2) - k_4(x_3 - x_4) = m_3 \ddot{x}_3$$

$$\sum F_4 = -k_4(x_4 - x_3) - k_5(x_4 - x_5) = m_4 \ddot{x}_4$$

$$\vdots$$

$$\sum F_n = -k_n(x_n - x_{n-1}) - k_{n+1} x_n = m_n \ddot{x}_n$$

(1) $$-\left(\frac{k_1 + k_2}{m_1}\right)(x_1) + \left(\frac{k_2}{m_1}\right)(x_2) = \ddot{x}_1 = -\omega^2 x_1$$

(2) $$\left(\frac{k_2}{m_2}\right)(x_1) - \left(\frac{k_2 + k_3}{m_2}\right)x_2 + \left(\frac{k_3}{m_2}\right)x_3 = \ddot{x}_2 = -\omega^2 x_2$$

(3) $$\left(\frac{k_3}{m_3}\right)x_2 - \left(\frac{k_3 + k_4}{m_3}\right)x_3 + \left(\frac{k_4}{m_3}\right)x_4 = \ddot{x}_3 = -\omega^2 x_3$$

(n) $$\left(\frac{k_n}{m_n}\right)x_{n-1} - \left(\frac{k_n + k_{n+1}}{m_n}\right)x_n = \ddot{x}_n$$

3 a) cont...

$$\begin{pmatrix} -\left(\frac{k_1+k_2}{m_1}\right) & \left(\frac{k_2}{m_1}\right) & 0 & 0 & 0 \cdots 0 \\ \left(\frac{k_2}{m_2}\right) & -\left(\frac{k_2+k_3}{m_2}\right) & \left(\frac{k_3}{m_2}\right) & 0 & 0 \cdots 0 \\ 0 & \left(\frac{k_3}{m_3}\right) & -\left(\frac{k_3+k_4}{m_3}\right) & \left(\frac{k_4}{m_3}\right) & 0 \cdots 0 \\ \vdots & & & & \vdots \\ 0 \cdots & 0 & 0 & \left(\frac{k_n}{m_n}\right) & -\left(\frac{k_n+k_{n+1}}{m_n}\right) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = -\omega^2 \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}$$

3 c) Are the eigenvalues and eigenvectors what you expected?

Eigenvalues: $\lambda_1 = -1$

$\lambda_2 = -3$

Eigenvectors: $w_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ swing equal and opposite

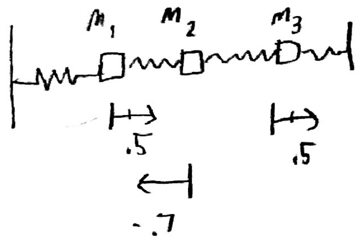
$w_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ swing together in phase.

Eigenvectors are what we expected because they explain the different directions of motion of each block. And because both blocks are equal masses, and the k's are equal magnitudes, the direction and magnitude of the direction can either be equal and opposite, or equal and the same.

The eigenvalues are the frequencies because $\lambda = -\omega^2$

Thus the natural frequencies are $\sqrt{1}$ & $\sqrt{3}$.

3 d) $\lambda_1 \approx -3.4$ $\quad w_1 \approx \sqrt{3.4}$



Eigenvalues: $\lambda_1 = -3.4142$

$\lambda_2 = -2$

$\lambda_3 = -0.585786$

Eigenvectors:

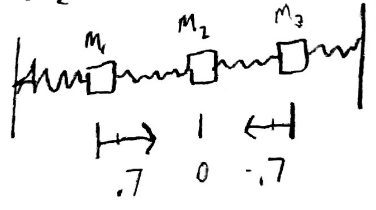
$$w_1 = \begin{pmatrix} .5 \\ -.707106781 \\ .5 \end{pmatrix}$$

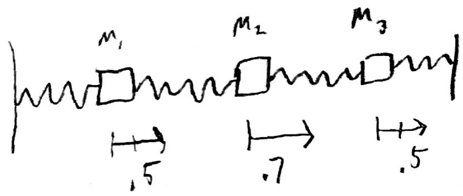
$$w_2 = \begin{pmatrix} .707106781 \\ 0 \\ -.70710678 \end{pmatrix}$$

$$w_3 = \begin{pmatrix} .5 \\ .707106781 \\ .5 \end{pmatrix}$$

these can be multiplied by scalars

$\lambda_2 = -2$ $\quad w_2 \approx \sqrt{2}$





3 e) All of the eigenvalues (ie frequencies) are between 0 and -1.3 & -2 and -3.4 and specifically at $\lambda \approx -3.3, -2.0, -1.3, 0$. What does this mean, There is a band gap. most of the vibrational frequencies are: 1.8, 1.4, 1.1, 0. ← natural.

With the mass at 1.2×, the eigenvalues: -2, -1.3 become less frequent and now the most common eigenvalues are clearly 0 & -3.8 } -3.8 is also different from the other maximum of -3.3 found when mass was 1.5. this means frequency increases to 1.9. (rather than 1.8)

Extra Credit: The eigenvectors that have higher energy (left of band gap) (higher frequency) the values of the eigenvectors alternate between positive and negative. There is more chaos. But at lower energy (eigvals between -1.3 & 0), the eigenvectors and displacements are similar, I think this means the atoms might be going in same direction forming a wave-type behavior.