1a)

$y' + 2xy = x \qquad y(-1) = 4$

$y(x) = c_1 e^{(-c_2 x^2)} + c_3 \qquad y'(x) = -2c_2 x \cdot c_1 e^{-c_2 x^2}$

$4 = c_1 e^{-c_2} + c_3$

$-2c_2 x c_1 e^{-c_2 x^2} + 2x\left(c_1 e^{(-c_2 x^2)} + c_3\right) = x$

$-2c_2 x c_1 e^{-c_2 x^2} + 2x c_1 e^{(-c_2 x^2)} + 2x c_3 = x$

$2x c_1 e^{-c_2 x^2}(-c_2 + 1) + 2x c_3 = x$

$-c_2 + 1 = 0 \qquad 2x c_3 = x$

$\boxed{c_2 = 1} \qquad 2c_3 = 1$

$\boxed{c_3 = \frac{1}{2}}$

$4 = c_1 e^{-1} + \frac{1}{2}$

$3.5 = c_1 e^{-1}$

$3.5e = c_1$

$\boxed{c_1 = 9.514}$

1d) The Euler integration overshoots the analytic solution. This error is a result of the second derivative $O(h)$. The mesh size plays a large role in the size of error. The smaller the mesh, the more accurate. When our mesh was 0.01, almost exactly on the analytical plot. Additionally, the bends in the plot, or when the derivative changes rapidly, is when the Euler integration is most vulnerable. This can be seen in my two zoomed in graphs.

2 a) $y''(x) + (2x+3)\,y'(x) + 6xy = x$

$y(0) = 1$

$y'(0) = 1$

$y''(x) = x - (2x+3)\,y'(x) - 6xy$

$y_1 = y$

$y_2 = y'$

$$\begin{cases} y_1' = y_2 \\ y_2' = x - (2x+3)\,y_2 - 6xy_1 \end{cases}$$

$y_1(0) = 1$

$y_2(0) = 1$

2c) The Runge Kutta is accurate for $h = .001, .01, .1$; however, for $h=1$ it explodes and becomes really inaccurate. The step size is way to big for it to get enough points to model the curve & doesn't approximate well.

3)

7. $$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

$$\tau = t\sqrt{\frac{g}{L}}$$

$$\frac{d^2\theta}{d\tau^2} = -\sin\theta$$

$$\theta_1 = \theta \qquad \theta_2 = \theta' \qquad \begin{cases} \theta_1' = \theta_2 \\ \theta_2' = -\sin\theta_1 \end{cases} \qquad \begin{cases} \theta_1(0) = 1\text{rad} \\ \theta_2(0) = 0 \end{cases}$$

$\leftarrow$ it is not moving at $t=0$

$$\theta_0 = 1 \text{ rad}$$

$$\tau = t\sqrt{g}$$

$$\frac{\tau}{\sqrt{g}} = t$$

1