Christopher Gsell HW page 1 2/1/18
PHYS 331

1. a) $f(x) = x^3 - 750$

b) $$x_{n+1} = x_n - \frac{(x_n^3 - 750)}{(3x_n^2)}$$

$$x_{n+1} = x_n - \frac{x_n^3}{3x_n^2} + \frac{750}{3x_n^2}$$

$$x_{n+1} = x_n - \frac{x_n}{3} + \frac{250}{x_n^2}$$

$$x_{n+1} = \frac{3x_n}{3} - \frac{x_n}{3} + \frac{250}{x_n^2}$$

$$x_{n+1} = \frac{2}{3} \cdot x_n + \frac{250}{x_n \cdot x_n}$$

# of operations 5

c) $x_0 = 9$

$$x_{n+1} = \frac{2(9)}{3} + \frac{250}{(9)^2} = 9.086 = x_1$$

$$V(x_0) = 9^3 = 729 = V_0$$

$$V(x_1) = 9.086^3 = 750.2 = V_1$$

$$x_{n+1} = \frac{2(9.086)}{3} + \frac{250}{(9.086)^2} = 9.086 = x_2$$

$$V(x_2) = 9.086^3 = 750.00 = V_2$$

Two iterations were needed for stated accuracy of .1 cm^3

d)

To converge:

- $|x_n - x^*| \leq \varepsilon$
- $\frac{b-a}{2^n} \leq \varepsilon$

$10^3 - 9^3 = 271 = 1000 - 729$

$2^n \cdot (.1) \leq 271$

$2^n \leq \frac{271}{.1}$

$n \geq \frac{\log\left(\frac{271}{.1}\right)}{\log(2)}$

$n \geq 11.4$, therefore 12 iterations.

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e) For N-R
If we can do it in 2 iterations, and each iteration takes five operations total is 10. 5 operations are: $(2 \cdot x) \div 3 + 250 \div (x \cdot x)$

For bisection: Add 2 #'s then divide by 2, and check midpoint. Thus, 3 operations for 12 iterations. Total is 36.

$\downarrow$

$+, \div$, then checking which side midpoint is on.

N-R takes much fewer.

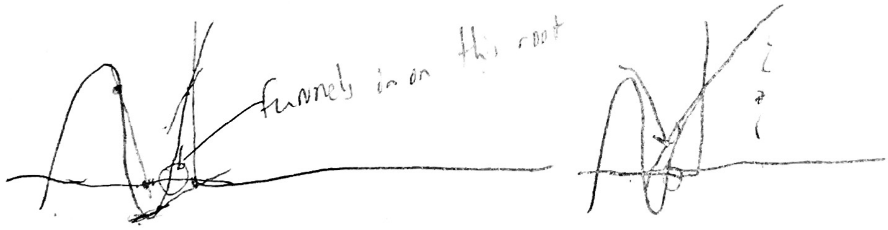
2. a) There are 3 real roots approximately at x: (-2.2, -1.4, -.5)

b) $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

c) Middle root = -.3984

d) It is satisfied in the region of all the roots. The only time when the absolute value is greater than 1 is between -2 and -1.7, -1.1 & -.5, and 1.5 & 3. This is what I expect because the N-R is not always effective if you're on the other side of a "hump."

e) It is reverting to x = -.39.



3. Root 1 = 1.875

Root 2 = 4.694

4 b) After 6 iterations

c) m ≈ 2. Yes because the N-R method is usually pretty darn fast.