1a) The function is undefined at $x = 0$.

b) It is harder to compute the integral near the undefined point. It's easier farther away.

d) Error decreases exponentially as n increases. The error of the trapezoidal method is much higher than that of Simpson's. Both of Simpson's methods have comparable error rates. Simpson's 1/3 has lower error for lower n, but higher error for larger n relative to Simpson's 3/8. Overall, the lowest error occurs at log of n, when $n = 9$ by Simpson's 3/8, because after this point, both of Simpson's methods go up in error. So no benefit to increasing n beyond this point. This occurs because eventually the mesh size gets so small that the machine precision can't handle it and becomes imprecise.

2a) $y = (b-x)^{1-\gamma}$ where $0 < \gamma < 1$

$$\int_a^b f(x)\,dx$$

$$y^{\frac{1}{1-\gamma}} = b - x$$

$$x = b - y^{\frac{1}{1-\gamma}}$$

$$dx = -\left(\frac{1}{1-\gamma}\right) y^{\left(\frac{1}{1-\gamma}\right)-1} dy$$

$$dx = -\left(\frac{1}{1-\gamma}\right) y^{\left(\frac{\gamma}{1-\gamma}\right)} dy$$

$$\int_{x=a}^{x=b} f(x)\,dx = -\int_{y=(b-a)^{(1-\gamma)}}^{y=0} f\left(b-y^{\frac{1}{1-\gamma}}\right)\left(\frac{1}{1-\gamma}\right) y^{\left(\frac{\gamma}{1-\gamma}\right)} dy$$

2b) $\int_0^3 \frac{1}{(3-x)^p}\,dx$ $\quad dx = -\left(\frac{1}{1-\gamma}\right) y$

$a = 0$, $b = 3$

$$\frac{-1}{1-\gamma}\int_{3^{1-\gamma}}^{0} y^{\frac{\gamma}{1-\gamma}} \left(y^{\frac{-p}{1-\gamma}}\right) dy$$

$$= \frac{-1}{1-\gamma}\int_{3^{1-\gamma}}^{0} y^{-(p-\gamma)/(1-\gamma)}\, dy$$

$$= \frac{1}{1-\gamma}\int_{0}^{3^{1-\gamma}} y^{\frac{-p+\gamma}{1-\gamma}}\, dy$$

Integral converges if $\frac{-p+\gamma}{1-\gamma} \geq 0 \Rightarrow -p+\gamma \geq 0$

$\gamma \geq p$

2c) $I = \int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$

$$-\int_{y=(b-a)^{(1-\gamma)}}^{y=0} f\left(b-y^{\frac{1}{1-\gamma}}\right)\left(\frac{1}{1-\gamma}\right) y^{\left(\frac{\gamma}{1-\gamma}\right)} dy$$

$y=(1-x)^{1-\frac{1}{2}}$ $\quad b=1 \quad a=0 \quad \gamma=\frac{1}{2}$

$$-2\int_1^0 \sqrt{1-\left(1-y^{\frac{1}{1-.5}}\right)^2}\; y\; dy$$

$$= 2\int_0^1 y\sqrt{1-(1+y^4-2y^2)}\,dy$$

$$= 2\int_0^1 y\sqrt{-y^4+2y^2}\,dy$$

$$= 2\int_0^1 y\sqrt{y^2(2-y^2)}\,dy$$

$$= 2\int_0^1 y^2\sqrt{2-y^2}\,dy$$

- I suspect func 2 is harder to compute the integral for than func 3 because func 2 appears to approach an asymptote - complicating calculations. Func 3, on the other hand, just looks like a simple section of a function that's easy to integrate.

- The Simpson 3/8 method & Simpson 1/3 method were very accurate in the substituted function. All 3 methods were on the same order of magnitude for the unmodified function. The trapezoid error got a little better after substitution

3a) $I = \int_0^1 \sqrt{1-x^2}\, dx = \frac{1}{2}\int_{-1}^{1} (1-x)^{-\frac{1}{2}} (1-x^2)\, dx = \frac{\pi}{6} \sum_{i=0}^{2} (1-x_i^2)$

$\int_0^1 F(x) = \int_0^1 \sqrt{1-x^2}\, dx = \int_0^1 \frac{\pi}{3}$

$A_0 \sum f(x_i)$

$f(x) = 1 - x^2$

$x = [0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}]$

$w = [\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}]$

$= \frac{\pi}{6}\left[(1-0) + \left(1 - \left(\frac{\sqrt{3}}{2}\right)^2\right) + \left(1 - \left(-\frac{\sqrt{3}}{2}\right)^2\right)\right]$

$= \frac{\pi}{6}\left[1 + \left(1 - \frac{3}{4}\right) + \left(1 - \frac{3}{4}\right)\right]$

$= \frac{\pi}{6}\left[1 + \left(\frac{4-3}{4}\right) + \left(\frac{4-3}{4}\right)\right]$

$= \frac{\pi}{6}\left[1 + \frac{1}{4} + \frac{1}{4}\right] = \frac{\pi}{6}\left[1 + \frac{1}{2}\right] = \frac{\pi}{6} \cdot \frac{3}{2} = \frac{\pi}{4}$

The approximation is exact.

Only 3 function calls are required in this method.

This method is much more accurate than Simpson's methods, in fact, because it's exact, it will be better no matter how many n's you use in this case. It's also much faster, only requiring 3 function calls compared to thousands for Simpson's methods.

3b) $C = \int_0^1 \left[(\sqrt{2}-1)^2 - (\sqrt{1+z^2}-1)^2\right]^{-\frac{1}{2}} dz$

$$\left[(\sqrt{2}-1)^2 - (\sqrt{1+z^2}-1)^2\right]^{-\frac{1}{2}} = (1-z^2)^{-\frac{1}{2}} f(z)$$

$$(1-z^2)^{\frac{1}{2}} \cdot \left[(\sqrt{2}-1)^2 - (\sqrt{1+z^2}-1)^2\right]^{-\frac{1}{2}} = f(z)$$

It took 10 function calls because peak n was 9.