Christopher Osell

1. a)

$$f(x) = x^5 - 3x^3 + 15x^2 + 29x + 9$$

$$-29x = x^5 - 3x^3 + 15x^2 + 9$$

$$g(x) = \frac{x^5 - 3x^3 + 15x^2 + 9}{-29}$$

$$g(x_2) = \sqrt{\frac{x^5 - 3x^3 + 29x + 9}{-15}}$$

$$0 = x^6 - 3x^4 + 15x^3 + 29x^2 + 9x$$

$$g(x_3) = \frac{x^6 - 3x^4 + 15x^3 + 29x^2}{-9}$$

$$g'(x_1) = \frac{5}{-29}x^4 + \frac{9}{29}x^2 - \frac{30}{29}x$$

$$g'(x_2) = \frac{-5x^4 + 9x^2 - 29}{2\sqrt{15}\sqrt{-x^5 + 3x^3 - 29x - 9}}$$

$$g'(x_3) = \frac{-2}{3}x^5 + \frac{4}{3}x^3 - 5x^2 - \frac{58}{9}x$$

b) for method 1, I think they should converge between $-2.2 < x < -1.8$ and $-1 < x < 1$

For method 2, $-1.9 < x < -.5$

Method 3, $-1.9 < x < -1$ & $-.3 < x < .3$

d) Summary of roots can be found in output code.
I didn't expect a 3rd convergence in the first method at $-.8$ & $-.3$, I only expected 1.

Continue →

d cont.) I figured there might have been a root at around $-.5$, in method 2, but I had low expectations because the function didn't look like it crossed x axis)

In method 3, I was surprised at how many roots there were because while I saw a lot of convergence, the actual function only reached the x axis at $x=0$.

I think N-R still superior.

2. $2z^3 - 1 + 3i \qquad z = x + iy$

$2(x+iy)^3 - 1 + 3i$

$2(x+iy)(x+iy)(x+iy) - 1 + 3i$

$2(x^2 + 2iyx - y^2)(x+iy) - 1 + 3i$

$2(x^3 + 2iyx^2 - y^2x + iyx^2 - 2y^2x - iy^3) - 1 + 3i$

$2x^3 + \underbrace{4iyx^2 - 2y^2x + 2iyx^2}_{} - 4y^2x - 2iy^3 - 1 + 3i$

$2x^3 + 6iyx^2 - 6y^2x - 2iy^3 - 1 + 3i$

Re: $2x^3 - 6y^2x - 1 \quad \leftarrow$ Eq. 1

Im: $6iyx^2 - 2iy^3 + 3i \quad \leftarrow$ Eq. 2

$i(6x^2y - 2y^3 + 3)$

b) inverse $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{\det}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \begin{bmatrix} 6x^2 - 6y^2 & -12yx \\ 12xy & 6x^2 - 6y^2 \end{bmatrix}$$

$$J^{-1} = \begin{bmatrix} (6x^2 - 6y^2) & (12xy) \\ -(12xy) & (6x^2 - 6y^2) \end{bmatrix} \frac{1}{(6x^2 - 6y^2)^2 + (12xy)^2}$$

c) There should be 3 unique roots,
I found 3 roots, which hopefully should be all. (0,0) was problematic and didn't converge because a 0 got in the denominator.

Problem 3

a. 1) $eq1 = 6870 = \frac{c}{1+e\sin(\frac{-\pi}{6}+\alpha)}$ solve for 0

2) $eq2 = 6728 = \frac{c}{1+e\sin(0+\alpha)}$

3) $eq3 = 6615 = \frac{c}{1+e\sin(\frac{\pi}{6}+\alpha)}$

1) $\frac{c}{1+e\sin(\frac{-\pi}{6}+\alpha)} - 6870 = 0$

2) $\frac{c}{1+e\sin(\alpha)} - 6728 = 0$

3) $\frac{c}{1+e\sin(\frac{\pi}{6}+\alpha)} - 6615 = 0$

$X_1 = \alpha,\ x_2 = c,\ x_3 = e$

$\alpha = .3408$

$c = 6819.2934$

$e = .0406$

$$R(\theta) = \frac{c}{1+e\sin(\theta+\alpha)}$$

$$R'(\theta) = \frac{-c \cdot (e\cos(\theta+\alpha))}{(1+e\sin(\theta+\alpha))^2}$$

$$R'(\theta) = \frac{-c \cdot (e\cos(\theta+\alpha))}{(1+e\sin(\theta+\alpha))^2} = 0$$

$\theta = \arccos(0) - \alpha$

$\theta_1 = \frac{\pi}{2} - \alpha = 1.23$

$\theta_2 = \frac{3\pi}{2} - \alpha = 4.3716$

$R(\theta_1) = 6553.2391$ ← minimum

$R(\theta_2) = 7107.8657$