Christopher Bell PHYS 331 HW 2/15/18

1. a) $f(x) = e^{-x}$

$f(x) = \frac{1}{e^{x}}$

Taylor Series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

$\sum_{n=0}^{\infty} \frac{(x-a)^n}{e^{x} n!}$ $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

$\sum_{n=0}^{\infty} \frac{x^n}{e^{x} n!}$

$\sum_{0}^{3} \frac{x^n}{e^{0} n!}$

$$\frac{x^0}{e^0(0!)} + \frac{x}{-c^0(1)} + \frac{x^2}{c^0(2)} + \frac{x^3}{-c^0(3\cdot 2)}$$

$$\frac{1}{e^0} - \frac{x}{-e^0} + \frac{x^2}{2e^0} - \frac{x^3}{6e^0}$$

$$e^{-x} = (1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3)$$

b) $f_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i$

Unknown parameters: a, b, c, d : $\boxed{\text{4 parameters}}$

Conditions: match the function $2(n-1)$ (conditions

At interior points: match first derivative of both sides $(n-2)$ conditions

· match second derivative from both sides $(n-2)$ conditions)

don't know if you wanted this.

Total: $2n-2 + n-2 + n-2$

$4n-6$

c) $(x_i, y_i),\ i = 1, \dots 4$

$f_1 = a_1 x_1^3 + b_1 x_1^2 + c_1 x_1 + d_1$

$f_2 = a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 + d_2$

$f_3 = a_3 x_3^3 + b_3 x_3^2 + c_3 x_3 + d_3$

Conditions

$f_1(x_1) = y_1 = a_1 x_1^3 + b_1 x_1^2 + c_1 x_1 + d_1$

$f_1(x_2) = y_2 = a_1 x_2^3 + b_1 x_2^2 + c_1 x_2 + d_1$

$f_2(x_2) = y_2 = a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 + d_2$

$f_2(x_3) = y_3 = a_2 x_3^3 + b_2 x_3^2 + c_2 x_3 + d_2$

$f_3(x_3) = y_3 = a_3 x_3^3 + b_3 x_3^2 + c_3 x_3 + d_3$

$f_3(x_4) = y_4 = a_4 x_4^3 + b_4 x_4^2 + c_4 x_4 + d_4$

$3a_1 x_2^2 + 2b_1 x_2 + c_1 = 3a_2 x_2^2 + 2b_2 x_2 + c_2$

$3a_2 x_3^2 + 2b_2 x_3 + c_2 = 3a_3 x_3^2 + 2b_3 x_3 + c_3$

$6a_1 x_2 + 2b_1 = 6a_2 x_2 + 2b_2$

$6a_2 x_3 + 2b_2 = 6a_3 x_3 + 2b_3$

2 more conditions are needed to completely specify the interpolating functions...

# of parameters: 12
# of equations: 10

d) $x_i = \left(-1, -\frac{1}{3}, \frac{1}{3}, 1\right)$

$y_i = f(x_i) = \left(e^{-1}, e^{-\frac{1}{3}}, e^{\frac{1}{3}}, e^{1}\right)$

$f_1(x_1) = c_1 x_1 + d_1 \rightarrow f_1(x_1) = c_1(-1) + d_1 = e$

$f_1(x_2) = c_1 x_2 + d_1 \rightarrow f_1(x_2) = c_1\left(\frac{-1}{3}\right) + d_1 = e^{\frac{1}{3}}$

$$\begin{array}{rl} \cancel{d_1} - c_1 x_1 & = e \\ - \quad \cancel{d_1} - \frac{1}{3} c_1 x_2 & = e^{\frac{1}{3}} \\ \hline -\frac{2}{3} c_1 & = e - e^{\frac{1}{3}} \end{array}$$

$c_1 = \frac{3}{2}\left(e^{\frac{1}{3}} - e\right) \rightarrow \boxed{c_1 = -1.9840}$

$d_1 - \left(\frac{3}{2}\left(e^{\frac{1}{3}} - e\right)\right) = e$

$d_1 = e + \left(\frac{3}{2}\left(e^{\frac{1}{3}} - e\right)\right)$

$= e + \left(\frac{3}{2} e^{\frac{1}{3}} - \frac{3}{2} e\right)$

$d_1 = \frac{3}{2} e^{\frac{1}{3}} - \frac{1}{2} e \rightarrow \boxed{d_1 = .9342}$

$$\boxed{\begin{array}{l} a_1 = 0 \\ b_1 = 0 \\ c_1 = -1.984 \\ d_1 = .7342 \end{array}}$$

e) • $f_2 = a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 + d_2$

• $3a_1 x_2^2 + 2b_1 x_2 + c_1 = 3a_2 x_2^2 + 2b_2 x_2 + c_2$

$3(0)\ldots + 2(0)\ldots + c_1 = 3a_2 x_2^2 + 2b_2 x_2 + c_2$

$-1.984 = 3a_2 x_2^2 + 2b_2 x_2 + c_2$

• $6a_1 x_2 + 2b_1 = 6a_2 x_2 + 2b_2$

$6(0) + 0 = 6a_2 x_2 + 2b_2 = 0$

$3a_2 x_2 + b_2 = 0$

$b_2 = -3a_2 x_2$

$-1.984 = 3a_2 x_2^2 - 6a_2 x_2 \cdot x_2 + c_2$

$-1.984 = 3a_2 x_2 (x_2 - 2x_2) + c_2$

$-1.984 = 3a_2 x_2 (-x_2) + c_2$

$-1.984 = -3a_2 x_2^2 + c_2$

$-1.984 = x_2 b_2 + c_2$

e) $f_1(x) = a_1 x_2^3 + b_1 x_2^2 + c_1 x_2 + d_1$

$f_2(x) = a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 + d_2$

$f_1'(x_2) = 3a_1 x_2 + 2b_1 x_2 + c_1 = f_2'(x_2) = 3a_2 x_2^2 + 2b_2 x_2 + c_2$

$f_1''(x_2) = 6a_1 x_2 + 2b_1 = f_2''(x_2) = 6a_2 x_2 + 2b_2$

$$0 = 6a_2\left(-\frac{1}{3}\right) + 2b_2$$

$$\cancel{2}a_2 = \cancel{2}b_2$$

$$a_2 = b_2 \leftarrow \text{Eq } 1$$

$$-1.984 = 3a_2\left(-\frac{1}{3}\right)^2 + 2a_2\left(-\frac{1}{3}\right) + c_2$$

$$-1.9840 = \frac{a_2}{3} - \frac{2}{3}a_2 + c_2$$

$$-1.984 = \frac{-a_2}{3} + c_2$$

$$c_2 = -1.984 + \frac{a_2}{3} \leftarrow \text{Eq } 2$$

$$a_2 x_2^3 + a_2 x_2^2 + \left(c_1 + \frac{a_2}{3}\right)x_2 + d_2 = y_2$$

$$a_2 x_3^3 + a_2 x_3^2 + \left(c_1 + \frac{a_2}{3}\right)x_3 + d_2 = y_3$$

$$\frac{-a_2}{27} + \frac{a_2}{9} + \left(\frac{-c_1}{3} - \frac{a_2}{9}\right) + d_2 = e^{-\frac{1}{3}}$$

$$\frac{a_2}{27} + \frac{a_2}{9} + \left(\frac{c_1}{3} + \frac{a_2}{9}\right) + d_2 = e^{\frac{1}{3}}$$

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$$\frac{2a_2}{9} + 2d_2 = e^{\frac{1}{3}} + e^{-\frac{1}{3}} \rightarrow \text{Eq} \rightarrow 3$$

$$d_2 = \frac{e^{\frac{1}{3}}}{2} + \frac{e^{-\frac{1}{3}}}{2} - \frac{a^2}{9}$$

$$d_2 = \frac{e^{\frac{1}{3}}}{2} + \frac{e^{-\frac{1}{3}}}{2} - \frac{a_2}{9}$$

$$f(x_2) = a_2 x_2^3 + b_2 x_2^2 + c_2 x_2 + d_2 = y_2$$

$$= \frac{-a_2}{27} + \frac{a_2}{9} - \frac{c_1}{3} - \frac{a_2}{9} + \left(\frac{e^{\frac{1}{3}}}{2} + \frac{e^{-\frac{1}{3}}}{2} - \frac{a_2}{9}\right) = e^{\frac{1}{3}}$$

$$-\frac{c_1}{3} + \frac{e^{\frac{1}{3}}}{2} + \frac{e^{-\frac{1}{3}}}{2} - e^{\frac{1}{3}} = \frac{a_2}{27} + \frac{a_2}{9}$$

$\downarrow$

e) continued )

$$\frac{27}{4}\left(\frac{-c_1}{3} + \frac{e^{-\frac{1}{3}}}{2} - \frac{1}{2}e^{\frac{1}{3}}\right) = a_2$$

$$a_2 = 2.1721 = b_2$$

$$c_2 = c_1 + \frac{a_2}{3} = -1.984 + \frac{2.1721}{3}$$

$$c_2 = -1.2599$$

$$d_2 = \frac{e^{\frac{1}{3}} + e^{-\frac{1}{3}}}{2} - \frac{2.1721}{9} = .8147$$

$$\boxed{f_2(x) = 2.1721x^3 + 2.1721x^2 - 1.2599x + .8147}$$

Discussion: the coefficients on the $x^3$ & $x^2$ term were very different and caused the biggest disparity between functions. However, the parameters on the $x$ & constant term were similar.

2. a) $f(x) = |\sin(x)|$

i) Difficult at $x = n\pi$ for $n \in \mathbb{Z}$ because these points, while continuous, are not differentiable.

b) For almost all of the periods, the sin function does not reach $y=0$. This occurs because the mesh size is too big.

c) The nearest creates a stepwise function, that when averaged could replicate the given span, but in its given state does not. Linear fits it perfectly, and cubic and quadratic almost do too w/ the exception of the bottom points.
From least to greatest error for given span: linear < cubic = quadratic < nearest

d) Near the peak of the function because it is most "round" there. This will be at $n\frac{\pi}{2}$ for $n \in \mathbb{Z}$.

3c) The part b figure is much much smoother because of the smaller mesh size.
Also, the scale got much bigger, extending to 200 on both axis in part b