STA9690 - Midterm

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Question 1

Part A

Prove that $E[y] = X\beta^*$ and $Cov[y] = \sigma^2 I$

We know the following:

- $E[\epsilon] = 0$
- $Cov[\epsilon] = \sigma^2 I$

Using the distributive property of the **expectation operator** we define the following:

$$y = X\beta^* + \epsilon$$

$$E[y] = E[X\beta^*] + E[\epsilon]$$
$$E[y] = E[X\beta^*] + 0$$
$$E[y] = E[X\beta^*]$$

X is a fixed quantity, while β^* is a known quantity **that does not vary** (though we don't know what the value is, we know it is a population parameter). If so, they are both constant values (constant matrix, constant vector, respectively), and by the expectation property of $E[a] = a$ we have the following:

$$E[y] = X\beta^*$$

Similarly, to find $Cov[y]$ we note that the covariance of a single random variable is the same as the variance of that random variable by the property of covariance:

$$Cov[Y] = \sigma_Y^2$$

Since we are only looking at a single random variable y, the **covariance operator** is equivalent to the **variance operator**. Therefore, all of the variance operator is applicable:

$$Cov[Y] = Cov[X\beta^* + \epsilon]$$

Since both X and β^* are constants, they are dropped by the variance operator:

$$Cov[Y] = Cov[\epsilon]$$

And by the known defintion of $Cov[\epsilon] = \sigma^2 I$

$$Cov[Y] = \sigma^2 I$$

Part B

We know the following:

- $v \in \mathbb{R}^p$ is a vector such that $Xv = 0$
- $X \in \mathbb{R}^{n \times p}$ is a $n \times p$ real matrix of predictor variables

If $Xv = 0$ is true than the vector $v = 0$ is guaranteed to be a solution out of many possible solutions

If so, then $\hat{\beta} + c.v$ is also a minimizer for any $c \in \mathbb{R}$ as:

$\hat{\beta} + c.v = \hat{\beta}$ if $v = 0$ for **any** $c \in \mathbb{R}$

Part C

If the matrix X has completely linear independent columns, than the matrix is full column rank and has a unique solution for $Xv = 0$, and the solution for v is $v = 0$