5.7.2 $S_n^2 = \frac{1}{n}\sum_{i=1}^{n} y_i^2$; $Var(Y) = \sigma^2$

Consistent if $P(|S_n^2 - S^2| < \varepsilon) > 1 - \frac{Var(S_n^2)}{\varepsilon^2}$

$$Var(Y^2) = E(Y^4) - [E(Y^2)]^2$$

$$M_Z(t) = e^{\sigma^2 t^2/2}$$

$$M_Z'(t) = t\sigma^2 e^{\sigma^2 t^2/2} \quad ; \quad t=0: 0$$

$$M_Z^2(t) = \sigma^2 e^{\sigma^2 t^2/2} + t^2\sigma^4 e^{\sigma^2 t^2/2} \quad ; \quad t=0: \sigma^2$$

$$M_Z^3(t) = t\sigma^4 e^{\sigma^2 t^2/2} + 2t\sigma^4 e^{\sigma^2 t^2/2} + t^3\sigma^6 e^{\sigma^2 t^2/2} \quad ; \quad t=0: 0$$

$$M_Z^4(t) = 3\sigma^4 e^{\sigma^2 t^2/2} + 3t^2\sigma^6 e^{\sigma^2 t^2/2} + t^4\sigma^8 e^{\sigma^2 t^2/2} \quad ; \quad t=0: 3\sigma^4$$

$$Var(Y^2) = E(Y^4) - E(Y^2)^2$$

$$3\sigma^4 - (\sigma^2)^2 = 2\sigma^4$$

$$S_n^2 = \frac{1}{n} Var(Y^2) = \frac{2\sigma^4}{n}$$

For ε, δ, and σ^2, an n can be found that makes $2\sigma^4/n < \delta$. Therefore, S_n^2 is consistent estimator for $\sigma^2 = Var(Y)$