$$\frac{K\theta}{\theta-1} = \frac{1}{n}\sum y$$

$$\frac{K\theta}{\theta-1} = \bar{y}$$

$$K\theta = \bar{y}\theta - \bar{y}$$

$$K\theta - \bar{y}\theta = -\bar{y}$$

$$\theta = \frac{-\bar{y}}{K-\bar{y}} = \frac{\bar{y}}{\bar{y}-K}$$

Using likelihood:

$$\prod \theta K^{\theta}\left(\frac{1}{y}\right)^{\theta+1} = \left(\theta K^{\theta}\left(\frac{1}{y_1}\right)^{\theta+1}\right)\cdots\left(\theta K^{\theta}\left(\frac{1}{y_n}\right)^{\theta+1}\right]$$

$$L\theta = \theta^{n} K^{\theta n}\left(\frac{1}{\prod y}\right)^{\theta+1}$$

$$\ln L(\theta) = n\ln\theta + \theta n\ln K - (\theta+1)\ln\prod y$$

$$= n\ln\theta + \theta n\ln K - \theta\ln\prod y - \ln\prod y$$

$$\partial/\partial\theta \ln L(\theta) = \frac{n}{\theta} + n\ln K - \ln\prod y$$

$$0 = \frac{n}{\theta} + n\ln K - \ln\prod y$$

$$\frac{n}{\theta} = \ln\prod y - n\ln K$$

$$\theta = \frac{n}{\ln\prod y - n\ln K}$$

These are dissimilar b/c $\bar{y}/\bar{y}-K \neq \frac{n}{\ln\prod y - n\ln K}$