$\frac{3\theta}{5} = \frac{1}{n}\sum y$

$\frac{3}{5}\theta = \bar{y}$

$\theta = \frac{5}{3}\bar{y}$

$E(\hat{\theta}) = E(\frac{5}{3}Y) = \frac{5}{3}E(\bar{Y}) = \frac{5}{3}(\frac{3}{5}\theta) = \theta$

$\bar{y} = 63/4$ given obs are 14, 10, 18, and 21

$\theta = 105/4$

5.4.20: $Var(\hat{\lambda}_1) = Var(X_1) = \lambda$

$Var(\hat{\lambda}_2) = Var(\bar{X}) = Var\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right) = Var\left(\frac{1}{n}\sum X\right)$

$Var(\frac{1}{n}\sum X) = \frac{1}{n^2}\sum Var(X) = \frac{1}{n^2} n\lambda = \lambda/n$

$Var(\hat{\lambda}_2)$ is more efficient b/c λ/n is less than λ.

$Var(\hat{\lambda}_2)$ is more efficient: $\frac{\lambda/n}{\lambda} = 1/n$

5.6.6 $f_Y(y,\theta) = \theta y^{\theta-1}$; $W = \prod_{i=1}^{n} Y_i$

$L(\theta) = \prod \theta y^{\theta-1} = (\theta y_1^{\theta-1})(\theta y_2^{\theta-1})\cdots(\theta y_n^{\theta-1})$

$= \theta^n [\prod Y_i]^{\theta-1}$

Let $\theta^n = 1$. Then, $L(\theta) = (\prod Y_i)^{\theta-1}$

and $L(\theta) = (W)^{\theta-1}$

Therefore, W is a sufficient statistic

$L(\theta) = \theta^n[\prod y_i]^{\theta-1}$

$\ln L(\theta) = n\ln\theta + \theta\ln\prod y_i - \ln\prod y_i$

$\partial/\partial\theta \ln L(\theta) = \frac{n}{\theta} + \ln\prod y_i$

$0 = \frac{n}{\theta} + \ln\prod y_i$

$-\ln\prod y_i = n/\theta$

$\theta = n/-\ln\prod y_i = -\frac{n}{\ln\prod Y} = \frac{-n}{\ln(W)}$ The max. likelihood estimator is a function of W