Econ 272a Winter 2022
Problem Set 2

February 20, 2022

Due March 4, 2022 at 6pm PST

In this problem set, we will analyze an oligopolistic, infinite-horizon game with entry and exit.

Assume the following:

(a) Time is discrete and indexed by $t \in \{1, 2, \ldots, \infty\}$.

(b) All firms discount future periods at a common rate $\beta \in (0, 1)$.

(c) There are at most three active firms (incumbents), indexed by $i \in \{1, 2, 3\}$.

(d) Incumbent firms i differ by their productive efficiency,

$$\omega_i \in \Omega \equiv \{1, 2, 3, 4, 5, 6\}.$$

Specifically, firms produce units of output at marginal cost $c_i = \frac{\alpha}{\alpha+\omega_i}$, where $\alpha > 0$ is some constant.

(e) In each period, incumbents compete on quantities in a homogenous product market, with an inverse demand curve,

$$P(Q) = Q^{-1/\theta},$$

where Q is the total quantity supplied over all firms.

(f) A firm will only ever exit if it is the least-productive among the incumbents. Such firm(s) draw a (random) scrap value $\phi_{it} \sim F_\phi$ in each period and decide whether to remain.

(g) Entry occurs only when there is fewer than three firms. In such periods, a prospective entrant j draws fixed cost $\kappa_{jt} \sim G_\kappa$, then decides whether to enter or disappear.

(h) Incumbents play symmetric, pure Markov strategies. The potential entrant also plays a pure Markov strategy. All firms know one another's productivities. The timing of the game is as follows:

1. Each active firm simultaneously chooses a quantity $q_{it} \geq 0$ to sell in the market.
2. Given $Q_t = \sum_{i \in N_t} q_{it}$, the market clears at a price $P_t = P(Q_t)$.
3. Each incumbent firm obtains flow profits, $\pi_{it} = P_t q_{it} - c_i q_{it}$.

4. One incumbent with the lowest productivity privately draws a scrap value, $\phi_{it} \sim F_\phi$.

5. If there are fewer than three incumbents, a single prospective entrant, j, draws a private fixed cost, $\kappa_{jt} \sim G_\kappa$.

6. The least-productive incumbent and the prospective entrant simultaneously choose whether to exit and enter, obtaining ϕ_{it} and $-\kappa_{jt}$, respectively.

7. If the entrant enters, they are endowed with an efficiency level ω_j, drawn uniformly at random from the set Ω. Otherwise, they disappear and obtain an outside option of zero.

Our objective is to learn about the cost structure of this industry.

1.1 Conceptual questions

1. Argue that the state variable of this game can be written

$$s_t = (s_{1t}, s_{2t}, s_{3t}) \in \{0, \Omega\}^3 \equiv S$$

where $s_{jt} = \omega_j$ if there is a firm in position $j \in \{1, 2, 3\}$ with efficiency ω_j, and 0 otherwise. What is the dimension of S? Use symmetry to reduce its dimension to 196.

2. Derive a system of equations that characterizes the optimal production decisions by firms given the state s_t. Show that optimal decisions satisfy

$$\left(1 - \frac{1}{\theta}\frac{q^*_{it}}{Q^*_t}\right) P(Q^*_t) = \frac{\alpha}{\alpha - s_{it}} \tag{1}$$

for all active firms i (such that $s_{it} > 0$), which allows us to implicitly define flow profits as functions $\pi_i(s_t)$ for each $i \in \{1, 2, 3\}$.

3. Consider a known sequence of states, $(s_0, s_1, \dots)$, and assume that you know the probability that firm i exits, $p_i^{\text{exit}}(s)$, for each $s \in S$.

Define the value of an incumbent along such a sequence of states as

$$V_i(s_0) = \sum_{t=0}^{\infty} \beta^t \left[\left(1 - p_i^{\text{exit}}(s_t)\right) \cdot \pi_i(s_t) + p_i^{\text{exit}}(s_t) \cdot \tilde{\phi}_i(s_t)\right]. \tag{2}$$

Explain why $\tilde{\phi}_i(s_t)$ will not, in generally, equal the expected scrap value $\mathbb{E}[\phi_i]$.

4. Suppose you know $p_{\text{exit}}(s) = \{p_i^{\text{exit}}(s)\}_i$ and $p_{\text{enter}}(s)$. Describe the state transitions.

5. Let $H(s'|s)$ denote the transition probability from s to s' as obtained in the previous part. Define the expected value of an incumbent,

$$\mathbb{E}[V_i(s_0)] = \sum_{t=0}^{\infty} \beta^t \left[\sum_{s_t \in S} \left[\left(1 - p_i^{\text{exit}}(s_t)\right) \cdot \pi_i(s_t) + p_i^{\text{exit}}(s_t) \cdot \tilde{\phi}_i(s_t)\right] H^t(s_t|s_0)\right],$$

using the iterated application of H, denoted by $H^t(s_t|s_0)$.

How does $\tilde{\phi}_i(s_t)$ depend on the expected value function $\mathbb{E}[V_i(s_t)]$?

6. Write down the expected value of an entrant at s as a function of $\mathbb{E}[V_i(s)]$. Write down the prospective entrant's Markov perfect equilibrium strategy as a cutoff rule at each s involving this expected value and the entry cost draw.

1.2 Simulation and estimation

1. Static Cournot

(a) Download the dataframe `cournot.RData` from the course website. This dataset contains prices and firm-level quantities for a large number of markets. Tabulate some summary statistics.

(b) Estimate the elasticity of demand $\hat{\theta}$ using OLS (there are no omitted variables).

(c) Calculate marginal costs c_i (or equivalently, $\hat{\alpha}$) for each $\omega_i \in \Omega$ with $\hat{\theta}$ and equation (1).

(d) Download the matrix enumerating elements of $\mathcal{S}$ from the course website (`states.RData`). Each row corresponds to an element of $\mathcal{S}$.

(e) Using $\hat{\theta}$ and $\hat{\alpha}$, code a function that calculates equilibrium flow profits $\pi_i(s)$ for $i = 1$ as a function of s. Tabulate your output over $s \in \mathcal{S}$. Inspect whether profit appears to fall with the number of rivals, fall with rivals' efficiency, and rise with one own's efficiency.

2. Policy functions

(a) Download the dataframe `bbl.RData` from the course website. This dataframe contains binary entry/exit outcomes and the efficiency levels of incumbents for a large number of markets.

(b) Estimate a probit model for entry, $\hat{p}_{\text{enter}}(s)$ over all states where entry is feasible (e.g., $s_{1t} = 0$). Use a second-degree polynomial in (s_{2t}, s_{3t}).

(c) Estimate a probit model for exit, $\hat{p}_{\text{exit}}(s)$, over every state except the zero vector. Use a second-degree polynomial in (s_{it}, s_{-it}), where $i = 1$ is the index of the least-productive firm.

3. Forward-simulation

(a) Use your results from part 2 to code a simple loop over $t \in \{1, 2, \ldots, T\}$ that generates sample paths of the state variable, $(s_0, \hat{s}_1, \hat{s}_2, \ldots, \hat{s}_T)$, from a given $s_0 \in \mathcal{S}$ and some T (e.g., $T = 50$).

(b) Fix s_0. Calculate the vector

$$\sum_{t=0}^{T} \beta^t \begin{pmatrix} (1 - \mathbf{1}_{\text{exit}}(\hat{s}_t))\pi_i(\hat{s}_t) \\ \mathbf{1}_{\text{exit}}(\hat{s}_t) \end{pmatrix} \tag{3}$$

for a large number (e.g., $K = 200$) of sample paths $\hat{s}$, each starting from s_0.

Observe that the average value over these simulations will approximate

$$\mathbb{E}[V_i(s_0)] = \sum_{t=0}^{T} \beta^t \begin{pmatrix} (1 - p_i^{\text{exit}}(s_t))\pi_i(s_t) \\ p_i^{\text{exit}}(s_0) \end{pmatrix}' \begin{pmatrix} 1 \\ \tilde{\phi}_i(s_0) \end{pmatrix},$$

and find the average value (3) over the K sample paths.

(c) For each sample path in ab), perturb the simulated exit strategy $T = \inf\{t \geq 0 : \mathbf{1}_{\text{exit}}(s_t) = 1\}$ by exiting randomly

$$\{-5, -4, -3, -2, -1, 1, 2, 3, 4, 5\}$$

periods after the true exit time. For each of some large number (e.g., $M = 200$) of perturbations, calculate (3) and its average over the K sample paths.

(d) Use your results to obtain bounds on the nuisance $\tilde{\phi}_i(s_0)$ via revealed preference.

(e) Loop steps (b)–(d) for each $s_0 \in \mathcal{S}$ and store the implied $\mathbb{E}[V_i(s_0)]$ for each such s_0, taking lower or upper bounds of $\tilde{\phi}_i(s_0)$ as their true values.

(f) Using your simulated $\mathbb{E}[V_i(s_0)]$, calculate the expected value of an entrant at s,

$$\mathbb{E}[V^{\text{entrant}}(s)] = \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \mathbb{E}[V(\omega, s_2, s_3)]$$

for each unique state $s = (0, s_2, s_3)$.

(g) Assume that G_κ is Gaussian with some mean μ_κ and standard deviation σ_κ. Estimate $(\mu_\kappa, \sigma_\kappa)$ by minimizing the distance between the vector of entry probabilities, $(\hat{p}_{\text{entry}}(s))_{s \in \mathcal{S}}$, and the probabilities constructed from an equilibrium cutoff strategy, the distributional parameters, and $\mathbb{E}[V^{\text{entrant}}(s)]$.

4. Counterfactuals

(a) Calculate a vector of equilibrium entry probabilities in $\mathcal{S}$ using your parameter estimates from 3(g) and the expected value of an entrant from 3(f). Note that this should be similar to $\hat{p}_{\text{entry}}(s)$.

(b) Calculate long-run expected discounted profits, entry rates, and markups from paths starting at $s_0 = (0, 3, 3)$

(c) Double your estimate $\hat{\mu}_\kappa$ and recalculate equilibrium entry probabilities holding $\mathbb{E}[V^{\text{entrant}}(s)]$ fixed. Denote this vector of equilibrium entry probabilities $\hat{p}_{\text{entry}}^{(1)} \in [0, 1]^{\mathcal{S}}$.

(d) Re-obtain a new $\mathbb{E}[V^{\text{entrant}}(s)]$ through forward-simulation with $\hat{p}_{\text{entry}}^{(1)} \in [0, 1]^{\mathcal{S}}$.

(e) Iterate (d) until your sequence of probabilities converges in the sense of $\|\hat{p}_{\text{entry}}^{(k+1)} - \hat{p}_{\text{entry}}^{(k)}\| < \varepsilon$ for some tolerance parameter $\varepsilon > 0$ that you find to be reasonable.

(f) Calculate the counterfactual sequence of long-run expected discounted profits and markups from paths starting at $s_0 = (0, 3, 3)$ using your approximation of $\lim_{k \to \infty} \hat{p}_{\text{entry}}^{(k)}$ from above. Contrast with (b) and interpret. What strategic feature from the benchmark model does this comparison hold fixed, and do you view such a simplification to be reasonable?