Firm-level Networks

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June 2022

Motivation

- There exists an established macro literature on firm size and firm dynamics that look at
  - What explains the growth of firms (by employment/sales)?
  - Why is there a small number of very large firms? Figure
- Empirically, we also see that large firms are very well-connected in the production network
  - Large firms have large number of buyers and suppliers Figure
  - Yet, we do not know much about how the network connections of firms could affect growth/performance
- With advances in data collection, there are now detailed datasets that allow us to study firm-to-firm networks

Questions

- Is there a structural relationship between the firm-level network and firm performance? If there is, how can we explain it?
- What can we observe about the firm-level network? If we have a model, what are the potential issues that prevent identification?
- How do we address simultaneity bias? What are the identification/estimation issues that we encounter because the firm-level network is endogenous?
- What is the endogenous formation process of the firm-level network? Can we identify/explain it?

Objectives

- Develop a model where the production network could impact firms' choices due to spillovers in innovations
  - Adapt a linear social interactions model from Blume, Brock, Durlauf & Jayaraman (2015), h.f. 'BBDJ' who have established key identification results
  - Ask if identification conditions are met after the social interactions model is applied to firm-level networks
  - Discuss potential issues with endogeneity/simultaneity bias and how to resolve them

Literature

Shock propagation through networks: Acemoglu, Carvalho, Ozdaglar & Tahbaz-Salehi (2012); Bernard, Moxnes & Saito (2019); Carvalho, Nirei, Saito & Tahbaz-Salehi (2021)

Endogenous production networks: Lim (2018); Oberfield (2018); Demir, Fieler, Xu & Yang (2021)

Social interaction: Bramoulle, Djebbari & Fortin (2009); Calvo-Armengol, Patacchini & Zenou (2009); Blume, Brock, Durlauf & Jayaraman (2015)

Network formation: Jackson & Rogers (2007); Atalay, Hortacsu, Roberts & Syverson (2011); Badev (2021); Battaglini, Patacchini & Rainone (2021); Sheng & Sun (2022)

Definitions

Suppose that there are N firms in the economy, indexed by i, and let $M_{ij} \geq 0$ denote i's expenditure on intermediates from j

The supplier matrix $\mathcal{S}$ is a $N \times N$ adjacency matrix with ij^{th} element

$$s_{ij} = \frac{M_{ij}}{\sum_k M_{ik}} \quad (j\text{'s share of } i\text{'s total purchases from other firms})$$

The buyer matrix $\mathcal{B}$ is a $N \times N$ adjacency matrix with ij^{th} element

$$b_{ij} = \frac{M_{ji}}{\sum_k M_{ki}} \quad (j\text{'s share of } i\text{'s total sales to other firms})$$

Firm-level model of action and spillovers

- Firm i is described by a vector (x_i, ν_i)
  - $x_i \in \mathbb{R}$ is publicly observed firm size (e.g. employment or sales)
  - $\nu_i \in \mathbb{R}$ is privately known to the firm (e.g. productivity)
- Some innovation or investment action ω_i (e.g. R&D) is simultaneously made by all firms
- Firm's payoff depends on (ω_i, x_i, ν_i), as well as on other firms' actions and characteristics:

$$\pi_i = \left(\gamma x_i + \nu_i + \delta \sum_j b_{ij} x_j \right) \omega_i + \phi \sum_j s_{ij} \omega_i \omega_j - \frac{1}{2} \omega_i^2$$

- Interested in parameters (γ, δ, ϕ)
- Want to find a Bayes-Nash equilibrium and a reduced-form for estimation

Reduced-form

From the FOC of the firm's problem we get

$$\mathbb{E}(\omega_i|\boldsymbol{x}) = \gamma x_i + \underbrace{\delta \sum_j b_{ij} x_j}_{\text{downstream-effects}} + \underbrace{\phi \sum_j s_{ij}\, \mathbb{E}(\omega_j|\boldsymbol{x})}_{\text{upstream-effects}} + \mathbb{E}(\nu_i|\boldsymbol{x})$$

- **Downstream-effects**: buyers' size affect the marginal payoff from firm action ω_i
  - If $\delta > 0$, firm i has larger payoffs by selling to larger buyers
- **Upstream-effects**: suppliers' actions affect firm i's action
  - If $\phi > 0$, upstream action has positive spillovers on downstream action

Existence of equilibrium

BBDJ show the existence of a pure-strategy Bayes-Nash equilibrium under some conditions

1. Matrices $\mathcal{B}$ and $\mathcal{S}$ are nonnegative. $\forall\ i \in N, \sum_j b_{ij} = 1$ and $\sum_j s_{ij} = 1$ and $b_{ii} = s_{ii} = 0$.
   - These are satisfied by defining $\mathcal{B}$ and $\mathcal{S}$ accordingly

2. $[\mathcal{I} - \phi\mathcal{S}]$ is invertible Rank condition

3. Firm types $(x,z) \in \mathcal{T} = \mathbb{R}^{2N}$ are given by an exogenous probability distribution ρ on $\mathcal{T}$ and second moments of ρ exist
   - Ensures that expected payoffs are well-defined for a large number of strategy profiles
   - This will be an assumption required by the model

Identification

1. (x_i, ν_i) are i.i.d. across $i = 1, \ldots, N$ and $\mathbb{E}(\nu_i | x_i) = 0$

2. $\mathcal{B}, \mathcal{S}$ are exogenous and a priori known to the econometrician

3. There exists at least one pair of firms who exert downstream-effects on each other and at least one pair of firms that exert upstream-effects on each other

4. (ω_i, x_i) is observed $\forall$ i and the support of the marginal distribution of x has dimension N

With firm-level networks, most concerned about (1) and (2).

Key concerns

1. $\mathbb{E}(\nu_i|x_i) \neq 0$ because productivity depends on firm's characteristics $\implies$ use a control function for ν_i
   - Olley & Pakes (1996), Ackerberg, Caves & Frazer (2015) provide methods to recover Hicks-neutral productivity

2. Strong assumption that $\mathcal{B}, \mathcal{S}$ are exogenous
   - Similar concerns regarding network exogeneity are abound in the education/peer-effects literature
   - "no general theoretical model of network formation"
   - BBDJ suggest extension towards a 2-stage game where networks are formed in the first stage and actions are determined in the second
   - Alternatively, develop a model for the network bias and use a bias-correction in the reduced-form

To be continued

Outstanding questions

1. What are the identification/estimation issues that we encounter because the firm-level network is endogenous?

2. What is the endogenous formation process of the firm-level network? Can we identify/explain it?

Next steps

1. Gather data, begin descriptive analysis and work on estimation of structural parameters (start with sector-level data if firm-level data is not possible), or simulate a completely random network and test for zero effects

2. Extend this project towards a 2-stage game to include a network formation model or find a bias-correction for network endogeneity

Distribution (tail CDF) of US firm size by revenue

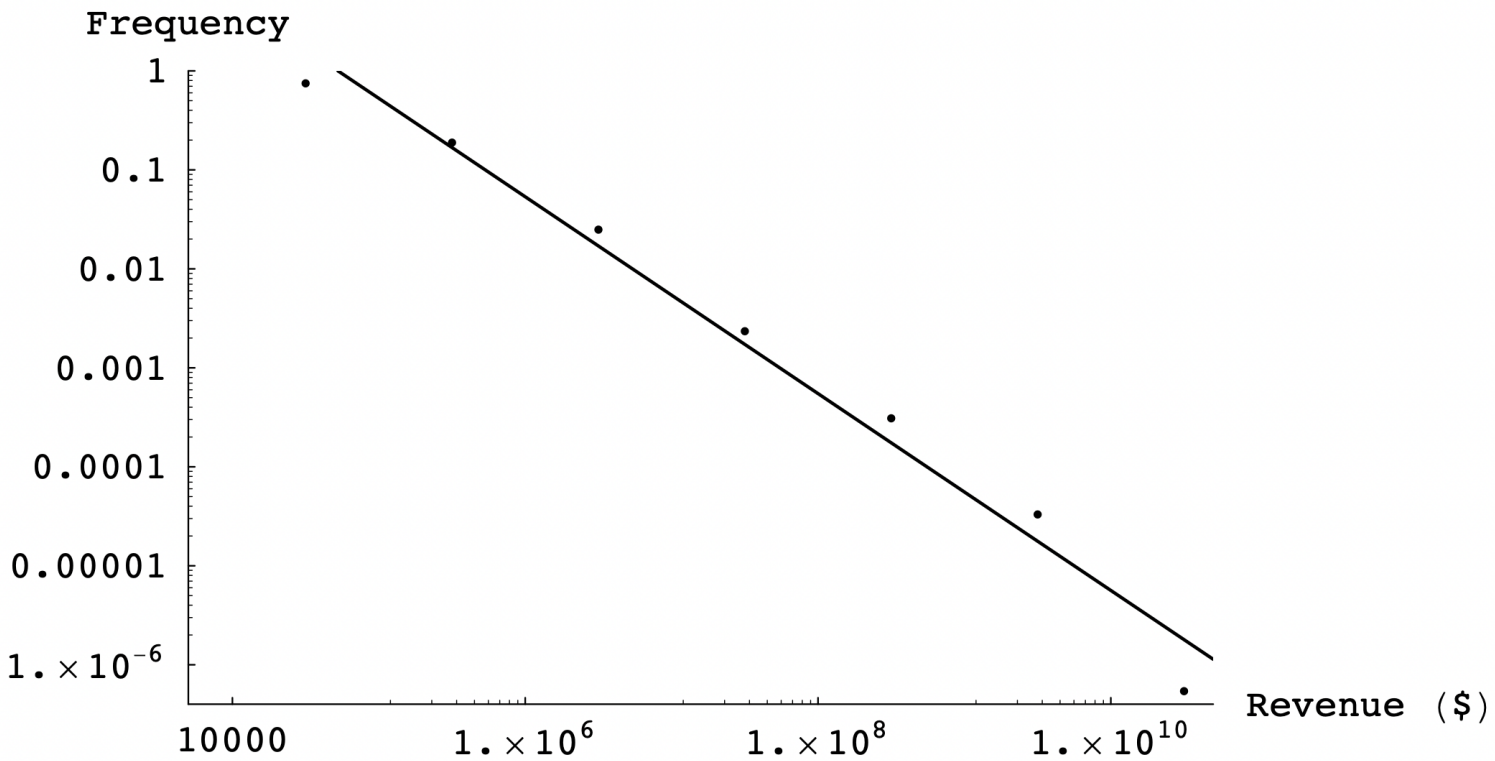

Figure 1: 1997 US Census data (Axtell 2001)

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Supplier/customer degree and firm size

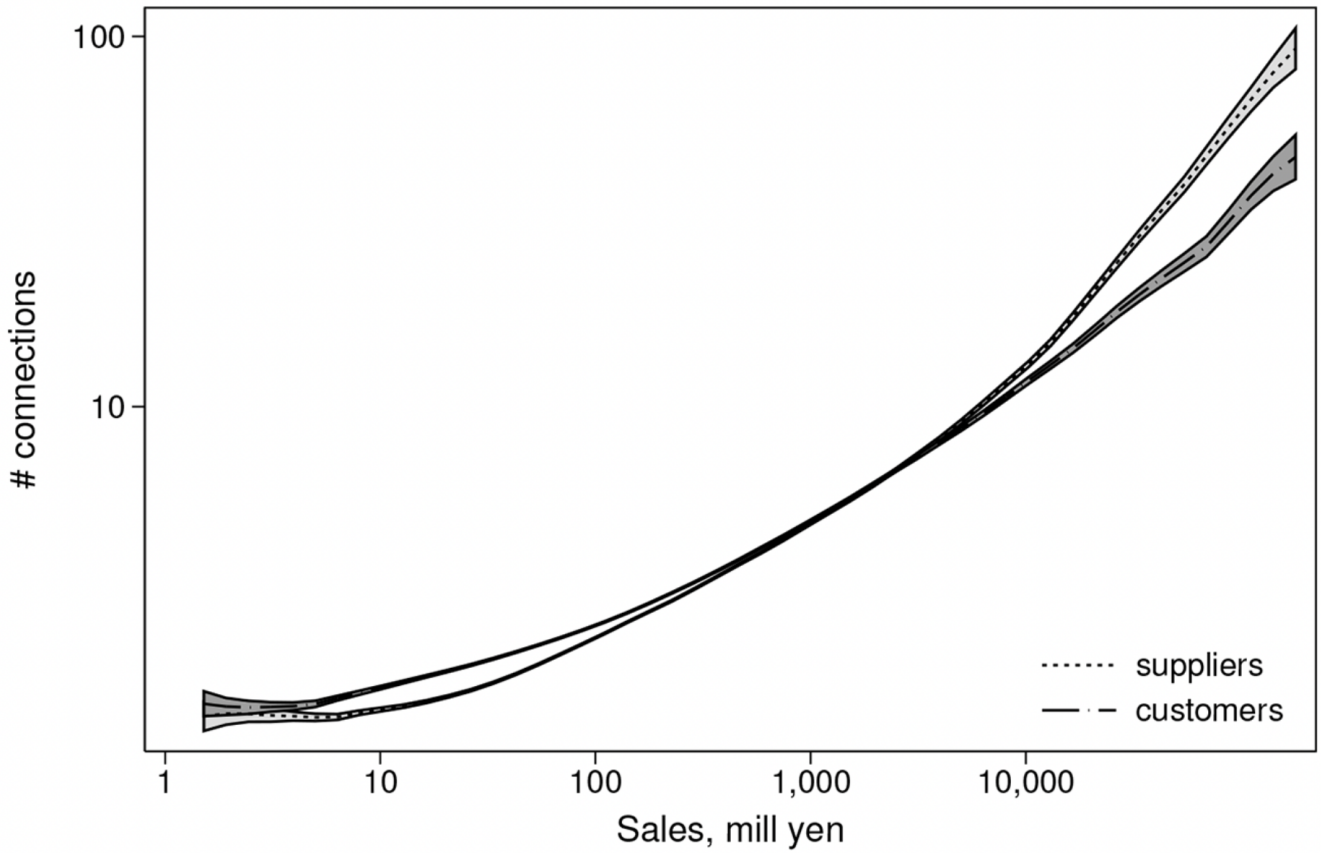

Figure 2: 2005 data on Japanese firms (Bernard et al. 2019)

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Rank condition

Recall that from the FOC of the firm's problem we get the strategy profile

$$\mathbb{E}(\omega_i|\boldsymbol{x}) = \gamma x_i + \delta \sum_j b_{ij} x_j + \phi \sum_j s_{ij}\, \mathbb{E}(\omega_j|\boldsymbol{x}) + \mathbb{E}(\nu_i|\boldsymbol{x})$$

In matrix form, the Bayes-Nash equilibrium can be expressed as

$$\mathbb{E}(\boldsymbol{\omega}|\boldsymbol{x}) = [\mathcal{I} - \phi\mathcal{S}]^{-1}[(\gamma + \delta\mathcal{B})\boldsymbol{x} + \mathbb{E}(\boldsymbol{\nu}|\boldsymbol{x})]$$

where $\mathcal{I}$ denotes a $N \times N$ identity matrix Back