

# ECON281A - Problem Set #3

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## Question 1

CDP first replicate the procedure of ADH to identify the component of imports from China that is driven by China. They estimate the regression:

$$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j$$

where:

- $j$  indexes 12 manufacturing sectors,
- $\Delta M_{USA,j}$  is the change in US imports from China in sector  $j$ ,
- $\Delta M_{other,j}$  is the change in advanced economies' imports from China
- $u_j$  is an unobserved error term.

To calibrate the China shock, CDP use  $\Delta \hat{M}_{USA,j}$  predicted by the fitted regression above and back out the size of TFP changes for each manufacturing sector  $j$ :

$$A_{2007}^{CHINA,j} / A_{2000}^{CHINA,j}$$

such that the model-predicted imports match  $\Delta \hat{M}_{USA,j}$ .

In CDP's model, the counterfactuals are fed through the model using time-varying fundamentals denoted by  $\Theta_t$ :

$$\Theta_t = (\{A_t^{nj}\}_{n=1,j=1}^{N,J}, \{\kappa_t^{nj,ij}\}_{n=1,i=1,j=1}^{N,N,J})$$

Hence, the alternative way to calibrate the China shock would be to back out the size of trade cost changes that match  $\Delta \hat{M}_{USA,j}$ :

$$\kappa_{2007}^{CHINA,j;USA,j} / \kappa_{2000}^{CHINA,j;USA,j}$$

## Question 2

**Definition:** Given  $(L_t, \Theta_t, \bar{\Theta})$ , a **temporary equilibrium** is a vector of wages  $w(L_t, \Theta_t, \bar{\Theta})$  that satisfies:

$$x_t^{nj} = B^{nj} \left( (r_t^{nj})^{\xi^n} (w_t^{nj})^{1-\xi^n} \right)^{\gamma^{nj}} \prod_{k=1}^J (P_t^{nk})^{\gamma^{nj,nk}} \quad (1)$$

$$P_t^{nj} = \Gamma^{nj} \left( \sum_{i=1}^N (x_t^{ij} \kappa_t^{nj,ij})^{-\theta_j} (A_t^{ij})^{\theta_j \gamma_{ij}} \right)^{-\frac{1}{\theta_j}} \quad (2)$$

$$\pi_t^{nj,ij} = \frac{(x_t^{ij} \kappa_t^{nj,ij})^{-\theta_j} (A_t^{ij})^{\theta_j \gamma_{ij}}}{\sum_{m=1}^N (x_t^{mj} \kappa_t^{nj,mj})^{-\theta_j} (A_t^{mj})^{\theta_j \gamma_{mj}}} \quad (3)$$

$$X_t^{nj} = \sum_{k=1}^J \gamma^{nk,nj} \sum_{i=1}^N \pi_t^{ik,nk} X_t^{ik} + \alpha^j \left( \sum_{k=1}^J w_t^{nk} L_t^{nk} + \iota^n \chi_t \right) \quad (4)$$

$$w_t^{nj} L_t^{nj} = \gamma^{nj} (1 - \xi^n) \sum_{i=1}^N \pi_t^{ij,nj} X_t^{ij} \quad (5)$$

$$r_t^{nj} H^{nj} = \gamma^{nj} \xi^n \sum_{i=1}^N \pi_t^{ij,nj} X_t^{ij} \quad (6)$$

where  $\chi_t = \sum_{i=1}^N \sum_{k=1}^J r_t^{ik} H^{ik}$  in (4).

**Definition:** Given  $(L_t, \{\Theta_t\}_{t=0}^\infty, \bar{\Theta})$ , a **sequential competitive equilibrium** is a sequence  $\{L_t, \mu_t, V_t, w(L_t, \Theta_t, \bar{\Theta})\}_{t=0}^\infty$  that solves:

$$V_t^{nj} = U(C_t^{nj}) + \nu \log \left( \sum_{i=1}^N \sum_{k=0}^J \exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{\frac{1}{\nu}} \right) \quad (7)$$

$$\mu_t^{nj,ik} = \frac{\exp(\beta V_{t+1}^{ik} - \tau^{nj,ik})^{\frac{1}{\nu}}}{\sum_{m=1}^N \sum_{h=0}^J \exp(\beta V_{t+1}^{mh} - \tau^{nj,mh})^{\frac{1}{\nu}}} \quad (8)$$

$$L_{t+1}^{nj} = \sum_{i=1}^N \sum_{k=0}^J \mu_t^{ik,nj} L_t^{ik} \quad (9)$$

and the temporary equilibrium (1)-(6) at each  $t$ .

## Question 2 (continued)

The system of equations for the numerical solution are:

For the **sequential competitive equilibrium**:

$$\mu_{t+1}^{nj,ik} = \frac{\mu_t^{nj,ik} (\dot{u}_{t+2}^{ik})^{\beta/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \mu_t^{nj,mh} (\dot{u}_{t+2}^{mh})^{\beta/\nu}} \quad (10)$$

$$\dot{u}_{t+1}^{nj} = \dot{\omega}^{nj}(\dot{L}_{t+1}, \dot{\Theta}_{t+1}) \left( \sum_{i=1}^N \sum_{k=0}^J \mu_t^{nj,ik} (\dot{u}_{t+2}^{ik})^{\beta/\nu} \right)^\nu \quad (11)$$

$$L_{t+1}^{nj} = \sum_{i=1}^N \sum_{k=0}^J \mu_t^{ik,nj} L_t^{ik} \quad (12)$$

For the **temporary equilibrium**:

$$\dot{x}_{t+1}^{nj} = (\dot{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n} (\dot{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_{k=1}^J (\dot{P}_t^{nk})^{\gamma^{nj,nk}} \quad (13)$$

$$\dot{P}_{t+1}^{nj} = \left( \sum_{i=1}^N \pi_t^{nj,ij} (\dot{x}_{t+1}^{ij} \dot{\kappa}_{t+1}^{nj,ij})^{-\theta_j} (\dot{A}_{t+1}^{ij})^{\theta_j \gamma_{ij}} \right)^{-\frac{1}{\theta_j}} \quad (14)$$

$$\pi_{t+1}^{nj,ij} = \pi_t^{nj,ij} \left( \frac{\dot{x}_{t+1}^{ij} \dot{\kappa}_{t+1}^{nj,ij}}{\dot{P}_{t+1}^{nj}} \right) (\dot{A}_{t+1}^{ij})^{\theta_j \gamma_{ij}} \quad (15)$$

$$X_{t+1}^{nj} = \sum_{k=1}^J \gamma^{nk,nj} \sum_{i=1}^N \pi_{t+1}^{ik,nk} X_{t+1}^{ik} + \alpha^j \left( \sum_{k=1}^J \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk} w_t^{nk} L_t^{nk} + \iota^n \chi_{t+1} \right) \quad (16)$$

$$\dot{w}_{t+1}^{nj} \dot{L}_{t+1}^{nj} w_t^{nj} L_t^{nj} = \gamma^{nj} (1 - \xi^n) \sum_{i=1}^N \pi_{t+1}^{ij,nj} X_{t+1}^{ij} \quad (17)$$

where  $\chi_{t+1} = \sum_{i=1}^N \sum_{k=1}^J \frac{\xi^n}{1-\xi^n} \dot{w}_{t+1}^{ik} \dot{L}_{t+1}^{ik} w_t^{ik} L_t^{ik}$  in (16)

## Question 2 (continued)

We want to calibrate the following shock over 32 quarters between 1Q2000 and 4Q2007:

$$\frac{A_{2007}^{China,mfg}}{A_{2000}^{China,mfg}} = 44$$

Assume that  $A_{2007}^{China,mfg} = 44$  and  $A_{2000}^{China,mfg} = 1$  and interpolate between  $t = 1$  and  $t = 32$  to derive the growth rate  $\dot{A}_{t+1} = \frac{A_{t+1}}{A_t} = 1.13$  (approximately 13% growth quarter-on-quarter for 31 quarters after quarter 1). Hence, consider a sequence of changes in fundamentals  $\{\dot{\Theta}_t\}_{t=1}^\infty = \{\dot{A}_t, \dot{\kappa}_t\}_{t=1}^\infty$  and set  $\dot{A}_t^{China,mfg} = 1.13$  for  $t = 1, \dots, 31$  and all other elements to 1.

## Questions 3 and 4

See attached MATLAB code ("CDP.CODE.m"). I followed the code of the authors and calculated two versions of the baseline economy to perform the counterfactual analysis. I attempted to write my own code following the algorithm in Appendix D, but could not do the inversion for  $X_{t+1}^{nj}$  in Step 4c. My incomplete code is also attached ("CDP.INCOMPLETE.m").

## Question 5

The results are summarized as follows:

Model	83%
Autor, Dorn & Hanson (2013)	5.0%
Adao, Arkolakis & Esposito (2019)	0.5%

## Question 6

The alternative method to measure the impact of the China shock using the CDP method would be to consider a baseline economy with changing (rather than constant) fundamentals. To do so,

1. Calculate a baseline economy with additional data on actual changes in fundamentals over time that will comprise: (i) Gross migration flows between regions; (ii) Trade flows between regions; and (iii) Expenditures for each region.
2. Calculate the counterfactual economy where agents expected the the actual changes in fundamentals but the China shock **did not** happen, using the same calibration of the China shock as described above.