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Solving Recurrences

1. T(n)=2 * T(n/2) + 1000n ∀n≥2

$T(k) = 2T(\frac{k}{2}) + 1000k$

$T(n) = 2T(\frac{n}{2}) + 1000n$

$= 2^1[\,2T(\frac{n}{2^2}) + \frac{1000n}{2}] + 1000n = 2^2\,T(\frac{n}{2^2}) + 2(\frac{1000n}{2}) + 1000n$

$= 2^2[\,2T(\frac{n}{2^3}) + \frac{1000n}{2^2}] + 1000n = 2^3\,T(\frac{n}{2^3}) + 2^2(\frac{1000n}{2^2}) + 2^1(\frac{1000n}{2}) + 1000n$

$= 2^3[\,2T(\frac{n}{2^4}) + \frac{1000n}{2^3}] + 1000n = 2^4\,T(\frac{n}{2^4}) + 2^3(\frac{1000n}{2^3}) + 2^2(\frac{n}{2^2}) + 2^1(\frac{1000n}{2}) + 1000n$

This is a pattern that is occurring that can be represented below:

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + 1000n\sum_{i=0}^{k-1}\frac{2^i}{2^i}$$

$\frac{n}{2^k} = 1 \rightarrow n = 2^k \rightarrow k = \log_2 n$

$$2^k(1) + 1000n\sum_{i=0}^{k-1} 1$$

$1000n\sum_{i=0}^{k-1} 1 \rightarrow$ translates to $(\log_2 n - 1)$

$n + 1000n(\log_2 n - 1)$

$n + 1000n \log_2 n - 1000n$

T(n) ∈ O(n log n)

This function has a Big O that is: **O(n log n)**

2. T(n)=7*T(n/2)+18n^2 ∀n≥2 T(1) = 1

T(k) = $7T(\frac{k}{2}) + 18k^2$

$T(n) = 7T(\frac{n}{2}) + 18n^2$

$= 7\,[7\,T(\frac{n}{2^2}) + \frac{18n^2}{2}] + 18n^2 = 7^2T(\frac{n}{2^2}) + 7(\frac{18n^2}{2}) + 18n^2$

$= 7^2[7\ \mathrm{T}(\frac{\mathrm{n}}{2^3}) + \frac{18\mathrm{n}^2}{2^2}] + 7(\frac{18\mathrm{n}^2}{2}) + 18\mathrm{n}^2 = 7^3\mathrm{T}(\frac{\mathrm{n}}{2^3}) + 7^2\mathrm{T}(\frac{\mathrm{n}}{2^2}) + 7(\frac{18\mathrm{n}^2}{2}) + 7^0\mathrm{T}(\frac{\mathrm{n}}{2^0}) + 18\mathrm{n}^2$

$\mathrm{T(n)} = 7^3\mathrm{T}(\frac{\mathrm{n}}{2^3}) + 7^2\mathrm{T}(\frac{\mathrm{n}}{2^2}) + 7(\frac{18\mathrm{n}^2}{2}) + 7^0\mathrm{T}(\frac{\mathrm{n}}{2^0}) + 18\mathrm{n}^2$

$\mathbf{T(n)} = \boldsymbol{7^k T(\frac{n}{2^k}) + 18n^2} \sum_{\mathrm{i=0}}^{\mathrm{k-1}} \boldsymbol{\frac{7^i}{2^i}}$

$\frac{\mathrm{n}}{2^{\mathrm{k}}} = 1 \rightarrow \mathrm{n} = 2^{\mathrm{k}} \rightarrow \mathrm{k} = \log_2 \mathrm{n}$

$\sum_{\mathrm{i=0}}^{\mathrm{k-1}} \boldsymbol{\frac{7^i}{2^i}} \rightarrow \log_2 \mathrm{n} - 1$

$\mathrm{n} + 18\mathrm{n}^2(\log_2 \mathrm{n} - 1)$

$\mathrm{n} + 18\mathrm{n}^2\log_2 \mathrm{n} - 18\mathrm{n}^2$

T(n) ∈ O($\boldsymbol{n^2 \log n}$)

3. $\mathrm{T(n)} = 2\mathrm{T}(\frac{n}{2}) + n\ \ \forall n \geq 2$ where T(1) = 1

$\mathrm{T(k)} = 2\mathrm{T}(\frac{\mathrm{k}}{2}) + \mathrm{k}$

$\mathrm{T(n)} = 2\mathrm{T}(\frac{n}{2}) + n$

$= 2^1[\ 2T(\frac{n}{2^2}) + \frac{\mathrm{n}}{2}] + \mathrm{n} = 2^2\ T(\frac{n}{2^2}) + 2(\frac{\mathrm{n}}{2}) + \mathrm{n}$
$= 2^2[\ 2T(\frac{n}{2^3}) + \frac{\mathrm{n}}{2^2}] + \mathrm{n} = 2^3\ T(\frac{n}{2^3}) + 2^2(\frac{\mathrm{n}}{2^2}) + 2^1(\frac{\mathrm{n}}{2}) + \mathrm{n}$
$= 2^3[\ 2T(\frac{n}{2^4}) + \frac{\mathrm{n}}{2^3}] + \mathrm{n} = 2^4\ T(\frac{n}{2^4}) + 2^3(\frac{\mathrm{n}}{2^3}) + 2^2(\frac{n}{2^2}) + 2^1(\frac{\mathrm{n}}{2}) + \mathrm{n}$

$\frac{\mathrm{n}}{2^{\mathrm{k}}} = 1 \rightarrow \mathrm{n} = 2^{\mathrm{k}} \rightarrow \mathrm{k} = \log_2 \mathrm{n}$

$$\mathrm{T(n)} = 2^{\mathrm{k}}T\left(\frac{\mathrm{n}}{2^{\mathrm{k}}}\right) + n\sum_{\mathrm{i=0}}^{\mathrm{k-1}}\frac{2^{\mathrm{i}}}{2^{\mathrm{i}}}$$

$$2^{\mathrm{k}}(1) + n\sum_{\mathrm{i=0}}^{\mathrm{k-1}} 1$$

$n \sum_{i=0}^{k-1} 1 \rightarrow$ translates to $(\log_2 n - 1)$

$n + n(\log_2 n - 1)$
$n + n \log_2 n - n$

$= n \log_2 n$

$T(n) \in O(n \log n)$

This function has a Big O that is: **$O(n \log n)$**