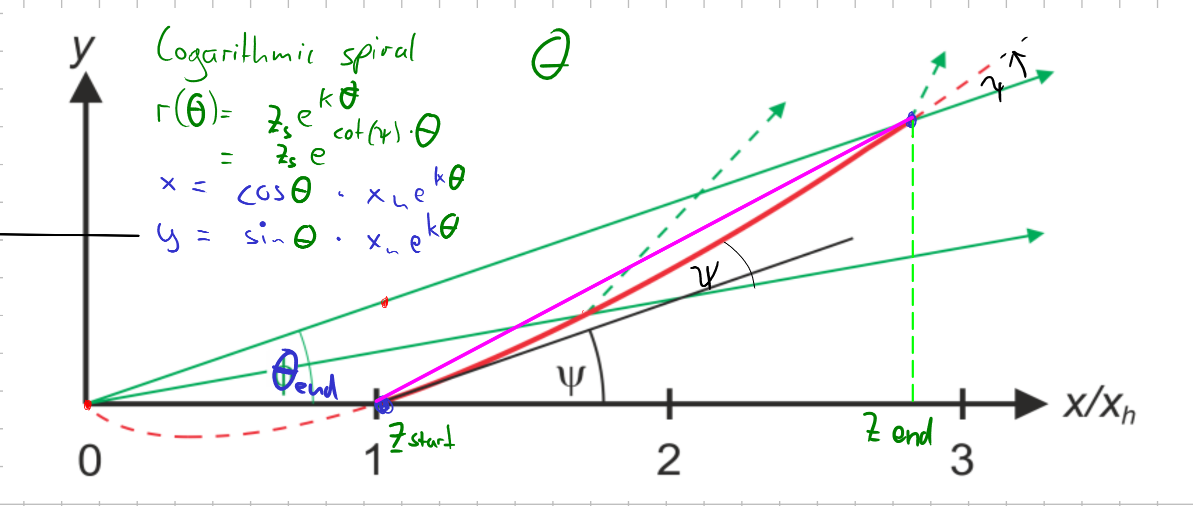
Short overview for log spiral



$k = \cot(\psi)$

Determination of θ_{end} $\Rightarrow$ $z_{end} = \cos\theta_{end} \cdot z_{start} \cdot e^{k\theta_{end}}$

Newton: $f(\theta) := 0 = -z_e + \cos\theta_e \cdot z_{start} \cdot e^{k\theta_e}$

$$\frac{\partial f(\theta)}{\partial\theta} = \cos\theta_e \cdot z_{start} \cdot k \cdot e^{k\theta_e} - \sin\theta_e \cdot z_{start} \cdot e^{k\theta_e}$$

$$= z_{start} \cdot e^{k\theta_e}\left(k\cos\theta_e - \sin\theta_e\right)$$

Initial value $\theta_0 = \ln\left(\frac{z_e}{z_{start}}\right) \cdot \frac{1}{k}$ Precise would be $\theta_e = \ln\left(\frac{r}{z_s}\right) \cdot \frac{1}{k}$

$z_e \neq r$

Intersection with line $y = mx + t$

(Geometry is has translat symmetry in 3rd Dimension)

$\rightarrow$ with x, y from above $\Rightarrow$ $\sin\theta\, r(\theta) = m\cos\theta \cdot r(\theta) + t$

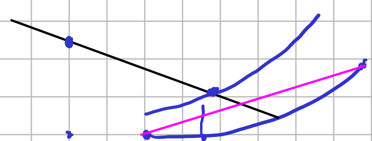
Newton to solve the well behaved equation $= f(\theta) = (m\cos\theta - \sin\theta)\, r(\theta) + t \overset{!}{=} 0$

$$\frac{d f(\theta)}{d\theta} = (-m\sin\theta - \cos\theta)\, z_s \cdot e^{k\theta} + (m\cos\theta - \sin\theta)\, k z_s e^{k\theta}$$

$$= z_s e^{k\theta}\left(\cos\theta(mk - 1) - \sin\theta(m + k)\right)$$

$\hookrightarrow$ Initial value for theta is obtained by approx mirror as a line and intersecting this with the neutron

$\hookrightarrow$ connecting start and endpoint

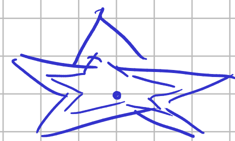


reflection

$$\vec r = \vec v - 2 \cdot \frac{(\vec n \cdot \vec v)}{|\vec n|^2} \cdot \vec n$$

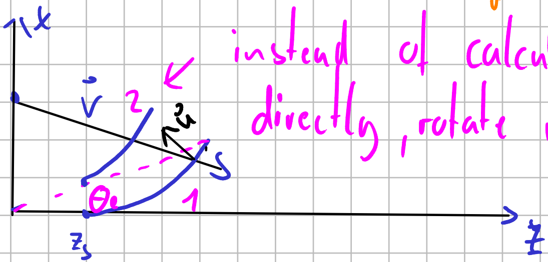
with $\vec n$ calculated from defining formula (see below for details)

$$\Rightarrow \quad \vec n = \begin{pmatrix} \cos\theta + k \sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix}$$



Rotation to simulate multiple mirrors

↳



instead of calculatin intersection of Neutron with branch 2 directly, rotate neutron by $-\theta_e$ and calc. intersection with branch 1 again

– As the interaction time will be correct (and is used to propagate neutron) only the normal vec $\vec n$ of the surface needs to be rotated to determine the state of the neutron after reflection

For each branch? Branches are rotated by θ_e with respect to the adjacent ones?

Complete Trace for each neutron

As long as collisions happen:

For each branch

i) rotate incoming neutron by θ_e such that in rot. frame, the intersection can be calc

ii) calculate the intersection with mirror as above (time of intersection)

iii) append time at which collision happens, if any to list

sort collisions by time and execute the one with the lowest time

↳ Executing coll: i) propagate onto the mirror by calculated time

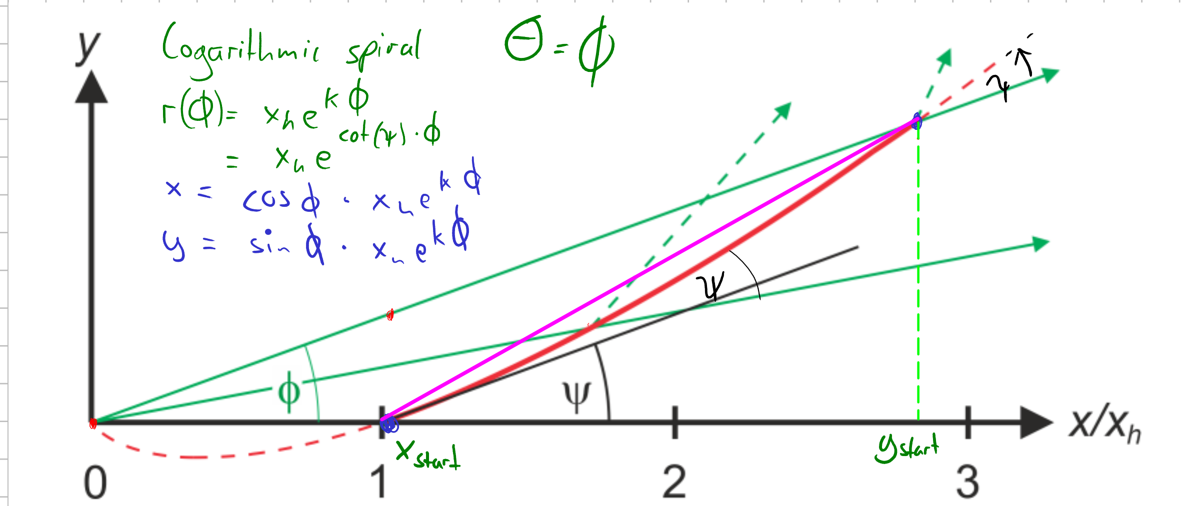
$$\text{ii)} \quad \vec{r} = \vec{v} - 2\frac{(\vec{n}\cdot\vec{v})}{|\vec{n}|^2}\vec{n}$$

reflected velocity — incoming velocity — normal vector in the non rotated frame

→ return neutron after all collisions

hyperbolic Spiral ?

how to determine the intersection between the Spiral and the neutron!



I) $y(x) = mx + t$ / Log spiral = Neutron line

$b = \cot(\psi)$, $x_h e^{k\theta} \sin(\theta) = m x_h e^{k\theta} \cos(\theta) + t$

$$e^{k\theta}\left(\sin(\theta) - m\cos(\theta)\right) = \frac{t}{x_h}$$

$$e^{k\theta}\left(\sin(\theta) - m\cos(\theta)\right) - \frac{t}{x_h} = 0$$

$$f(\theta) = e^{k\theta}\left(\sin\theta - m\cos\theta\right) - \frac{t}{x_n}$$

$$f'(\theta) = e^{k\theta}\left(k(\sin\theta - m\cos\theta) + \cos\theta + m\sin\theta\right)$$

$$= e^{k\theta}\left(\cos\theta\,(1 - k\cdot m) + \sin\theta\,(k + m)\right)$$

Numerical solution

1. calculate the intersection with line approximation of line segment
2. check if neutrons hit this
3. use the resulting data as intial guess to Newton the below equation

$$e^{bt}\left(\sin(t) - m\cos(t)\right) - \frac{b}{a} = 0$$

4. Repeat for all branches by turning the neutron

Question where should the mirrors sit?

$$x(\phi) = \cos\phi \cdot x_h e^{k\phi}$$
$$y(\phi) = \sin\phi \cdot x_h e^{k\phi}$$

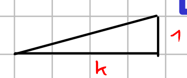
now for the derivative

$$\dot{x}(\phi) = x_h k \cos\phi\, e^{k\phi} - \sin\phi\, x_h e^{k\phi}$$
$$\dot{y}(\phi) = x_h k \sin\phi\, e^{k\phi} + \cos\phi\, x_h e^{k\phi}$$

$$\begin{pmatrix} x(\phi) \\ y(\phi) \end{pmatrix} = x_h e^{k\phi} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}(\phi) \\ \dot{y}(\phi) \end{pmatrix} = x_h e^{k\phi} \begin{pmatrix} k\cos\phi - \sin\phi \\ k\sin\phi + \cos\phi \end{pmatrix}$$

$$\boxed{\tan\alpha = \frac{1}{k}}$$

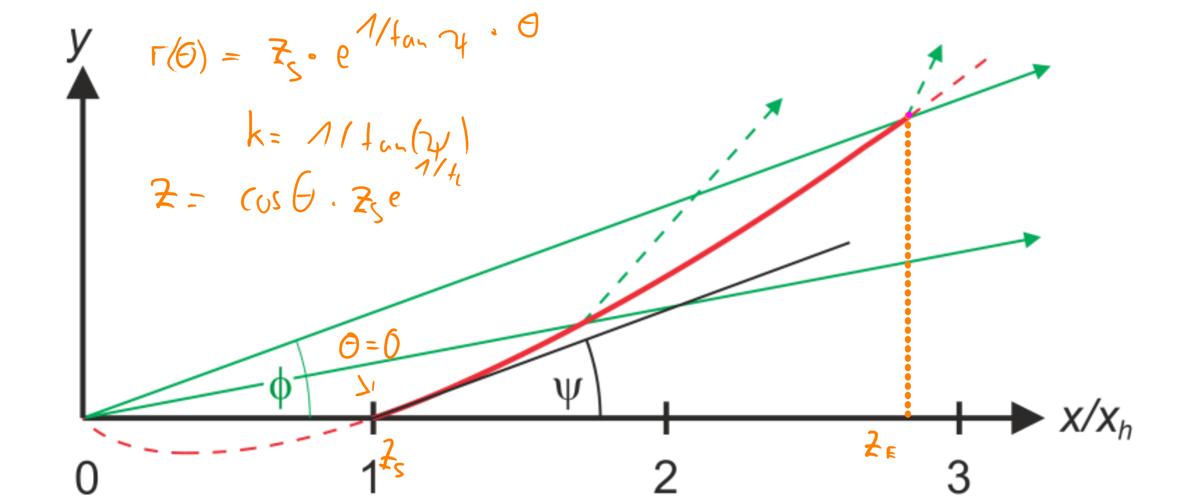


$$\dot{\vec{x}}(0) = x_h \cdot \begin{pmatrix} k \\ 1 \end{pmatrix}$$

$$\lim_{\phi \to 0} \dot{\vec{x}}(0) = x\left(1 + k\phi + \frac{k^2\phi^2}{2}\right)\begin{pmatrix} k(1-\phi^2) - \phi \\ k\phi + (1-\phi^2) \end{pmatrix}$$

What kind of parameters?

x_h the start in x



from $\theta_s = 0$

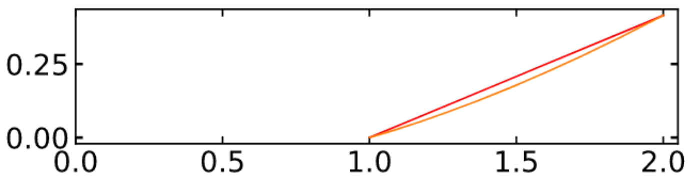
to θ_E | $\cos\theta_E \cdot x_s e^{k\theta_E} = x_E$

start at $r(\theta_E) = x_s e^{k\theta_E} \Rightarrow \theta_E \approx \ln\left(\frac{x_E}{x_s}\right)\frac{1}{k}$

newton $\cos\theta_E \cdot x_s e^{k\theta_E} - x_E = 0$

$f' = x_s e^{k\theta_E}(\cos\theta_E \cdot k - \sin\theta_E)$

$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$



return a linear equation to get prelimin

$m_t \cdot x + y_{line} = m_{spir} x + y_{spir}$

$x = \frac{y_{spir} - y_{line}}{m_l - m_{spir}}$

Calculation of normal vector

$\vec{n} = \vec{\nabla} f$ $\quad f \Rightarrow r(\theta) = z_h e^{k\theta} \Rightarrow \begin{pmatrix} \partial r \\ \partial\theta \end{pmatrix} = \begin{pmatrix} 1 \\ x_h k e^{k\theta} \end{pmatrix} \Rightarrow \begin{matrix} \cos\theta\, x_h k e^{k\theta} \\ \sin\theta\, x_h k e^{k\theta} \end{matrix}$

transform to cartesian coords

$\tan\theta = \frac{y}{x}$

$\sqrt{x^2 + y^2} = z_h e^{k \arctan\frac{y}{x}}$

$0 = \sqrt{x^2 + y^2} - z_h e^{k \arctan\frac{y}{x}}$

In[4]:= $\partial_x\left(\sqrt{x^2+y^2} - xh\, e^{k \operatorname{ArcTan}[(\frac{y}{x})]}\right)$

Out[4]= $\dfrac{x}{\sqrt{x^2+y^2}} + \dfrac{e^{k \operatorname{ArcTan}[\frac{y}{x}]}\, k\, xh\, y}{x^2\left(1+\frac{y^2}{x^2}\right)} = \cos\theta + \dfrac{\overbrace{e^{k\theta}\cdot x_h}^{r}\cdot k\cdot y}{r^2} = \cos\theta + k\sin\theta$

In[5]:= $\partial_y\left(\sqrt{x^2+y^2} - xh\, e^{k \operatorname{ArcTan}[(\frac{y}{x})]}\right)$

Out[5]= $\dfrac{y}{\sqrt{x^2+y^2}} - \dfrac{e^{k \operatorname{ArcTan}[\frac{y}{x}]}\, k\, xh\, x}{x^2\left(1+\frac{y^2}{x^2}\right)} = \sin\theta - \dfrac{\overbrace{e^{k\theta}\, x_h}^{r}\cdot x}{r^2}\, k = \sin\theta - k\cdot\cos\theta$

$$\vec{n} = \begin{pmatrix} \cos\theta + k\sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix}$$

Other way to calculate this

$$0 = r(\theta) - x_h e^{k\theta}$$

$$\partial_x f = \frac{f(r(x+h,y),\theta(x+h,y)) - f(r,\theta)}{h}$$

$$= \frac{f(r(x+h),\theta(x+h))\,(r(x+h)-r(x))}{(r(x+h)-r(x))\, h} - \frac{f(r,\theta(x+h))}{h}\frac{r(x+h)-r(x)}{r(x+h)-r(x)} - \frac{f(r,\theta)}{h}$$

$$+ \frac{f(r,\theta(x+h))}{h}\cancel{\frac{r(x+h)-r(x)}{r(x+h)-r(x)}}$$

$$= \frac{\left(f(r(x+h),\theta(x+h)) - f(r,\theta(x+h))\right)(r(x+h)-r(x))}{(r(x+h)-r(x))\, h} + \frac{\left(f(r,\theta(x+h)) - f(r,\theta)\right)\cdot(\theta(x+h)-\theta(x))}{(\theta(x+h)-\theta(x))\, h}$$

$$= \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial x}$$

$$f = r - x e^{k\theta} \qquad \theta = \arctan\left(\frac{y}{x}\right)$$

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}} = \cos\theta;\ \frac{\partial r}{\partial y} = \sin\theta$$

$$\frac{\partial\theta}{\partial x} = -\frac{y}{(x^2+y^2)} \qquad \frac{\partial\theta}{\partial y} = \frac{x}{(x^2+y^2)}$$

$$\begin{matrix}\partial_x \\ \partial_y\end{matrix} = \begin{pmatrix} 1\cdot\frac{\partial r}{\partial x} - kxe^{k\theta}\frac{\partial\theta}{\partial x} \\ 1\cdot\frac{\partial r}{\partial y} - kxe^{k\theta}\frac{\partial\theta}{\partial y}\end{pmatrix}$$

$$= \begin{matrix}\cos\theta - \left(-\frac{k\cancel{r}y}{r^{\cancel{2}}}\right) \\ \sin\theta - \left(\frac{k\cancel{r}x}{r^{\cancel{2}}}\right)\end{matrix}$$

$$\vec{n}(\theta) = \boxed{\begin{pmatrix} \cos\theta + k\sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix}} \Rightarrow$$

Same result which is good
So now we can get the reflection

$$\vec n = \begin{pmatrix} \cos\theta + k\sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix} \Rightarrow \hat n = \begin{pmatrix} \cos\theta + k\sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix} \cdot \frac{1}{\left(\cos^2\theta + k^2\sin^2\theta + \sin^2\theta + k^2\cos^2\theta\right)^{1/2}}$$

normalized normal vector

$$\hat n = \frac{1}{\sqrt{1+k^2}} \begin{pmatrix} \cos\theta + k\sin\theta \\ \sin\theta - k\cos\theta \end{pmatrix} = \frac{1}{\left((x^2+y^2)(1+k^2)\right)^{1/2}} \begin{pmatrix} x + ky \\ y - kx \end{pmatrix}$$

reflected ray has new direction

$$\vec r = \vec v - 2(\vec v \hat n)\cdot \hat n$$

$$\vec r^{\,2} = \vec v^{\,2} - 4(\vec v\hat n)(\vec v\hat n) + 4(\vec v\hat n)^2$$

If $\vec v \vec n > 0$ bad juju

Steps to trace

1. Point of intersection $\vec p$
2. calc $\vec n$
3. calc $\vec r$
4. reflected $\vec p + \vec r t \quad t \in]0, \infty[$

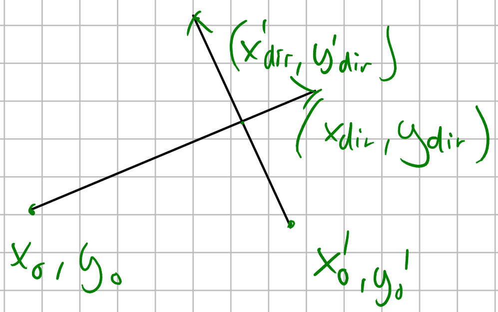
Rotation of beam to det. the intersection with multiple mirrors

matrix of rotation in 2D $R \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$\vec v = \begin{pmatrix} 1 \\ m \end{pmatrix} \Rightarrow \vec v_r = \begin{pmatrix} \cos\theta \cdot 1 - \sin\theta\, m \\ \sin\theta\, 1 + \cos\theta\, m \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{\sin\theta + \cos\theta\, m}{\cos\theta - \sin\theta\, m} \end{pmatrix}$$

$$\vec p = \begin{pmatrix} 0 \\ y_0 \end{pmatrix} \Rightarrow \vec p_r = \begin{pmatrix} -\sin\theta\, y_0 \\ \cos\theta\, y_0 \end{pmatrix} \Rightarrow \text{the new } y_0?$$

$$\Rightarrow \qquad \Rightarrow \vec p_r = \begin{pmatrix} 0 \\ y_0\left[\cos\theta + \sin\theta\left(\frac{\sin\theta + \cos\theta\, m}{\cos\theta - \sin\theta\, m}\right)\right] \end{pmatrix}$$



calculate intersection

$$\Rightarrow \quad x_0 + t\, x_{dir} \overset{!}{=} x_0' + \lambda x'_{dir}$$

1. $(x_0 - x_0') + t\, x_{dir} - \lambda x'_{dir} = 0$

2. $(y_0 - y_0') + t\, y_{dir} - \lambda y'_{dir} = 0$

1. $t = \dfrac{\lambda x'_{dir} + (x_0' - x_0)}{x_{dir}}$

2. $\lambda = \dfrac{(y_0 - y_0') + t\, y_{dir}}{y'_{dir}}$

$$\lambda = \frac{(y_0 - y_0') + \frac{\lambda x'_{dir} + (x_0' - x_0)}{x_{dir}}\, y_{dir}}{y'_{dir}}$$

$$\lambda \left(y'_{dir} - \frac{x'_{dir}\, y_{dir}}{x_{dir}} \right) = (y_0 - y_0') + \frac{y_{dir}}{x_{dir}} (x_0' - x_0)$$

$$\boxed{\lambda = \frac{x_{dir}(y_0 - y_0') + y_{dir}(x_0' - x_0)}{y'_{dir} \cdot x_{dir} - x'_{dir}\, y_{dir}}}$$

oder

1. $\lambda = \dfrac{(x_0 - x_0') + t\, x_{dir}}{x'_{dir}}$

2. $t\, y_{dir} = \lambda y'_{dir} + (y_0' - y_0)$

$$y_{dir}\, t = \frac{y'_{dir}}{x'_{dir}} \left[(x_0 - x_0') + t\, x_{dir} \right] + (y_0' - y_0)$$

$$t \left(y_{dir} - \frac{x_{dir}}{x'_{dir}}\, y'_{dir} \right) = \frac{y'_{dir}}{x'_{dir}} (x_0 - x_0') + (y_0' - y_0)$$

$$\boxed{t = \frac{y'_{dir}(x_0 - x_0') + x'_{dir}(y_0' - y_0)}{\left(x'_{dir}\, y_{dir} - x_{dir}\, y'_{dir} \right)}}$$

Point intersection ?

$$\vec{x} + t\, \vec{x}_{dir} = \vec{x}' + \lambda \vec{x}'_{dir}$$

Rotation of Beam and back to avoid multiple Branches