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A machine learning-based method for simulation of ship speed profile in a complex ice field

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ABSTRACT
Computational methods for predicting ship speed profile in a complex ice field have traditionally relied on mechanistic simulations. However, such methods have difficulties capturing the entire complexity of ship–ice interaction process due to the incomplete understanding of the underlying physical phenomena. Therefore, data-driven approaches have recently gained increased attention in this context. Hence, this paper proposes a concept of a first machine learning-based simulator of ship speed profile in a complex ice field. The developed approach suggests using supervised machine learning to trace a function mapping several ship and ice parameters to the ship acceleration/deceleration between the two adjacent points along the route. The simulator is trained and tested on a dataset obtained from the full-scale tests of an icebreaking ship. The results show high accuracy of the developed method, with an average error of the simulated ship speed against the measured one ranging from 2.6% to 9.4%.

Abbreviations: AI: artificial intelligence; AIS: automatic identification system; ANN: artificial neural network; CFD: computational fluid dynamics; CPP: controllable pitch propeller; EM: electro-magnetic; FEA: finite element analysis; ML: machine learning; PC: polar class; SAR: synthetic aperture radar

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Artificial neural network; machine learning; ship ice transit simulations; ship resistance in ice; ship speed profile in ice

List of symbols

d	Distance between the two adjacent points along the simulated track [m]
D	Propeller diameter [m]
$\boldsymbol{f}$	Function relating the ship speed at a point to a set of independent parameters [-]
h	Ice thickness [m]
$\bar{h}$	Average ice thickness between the two adjacent points [m]
i	Index enumerating the simulated point [-]
L_{data}	Length of the full dataset after the transformations are applied [m]
L_{test}	Length of the test dataset [m]
N_{test}	Number of samples in the test dataset [–]
N_{train}	Number of samples in the training dataset [–]
P	Propeller pitch [m]
$\bar{P}$	Average propeller pitch between points i and $i-1$ [m]
$\boldsymbol{P}_{hull}$	Vector of hull-ice interaction parameters
$\boldsymbol{P}_{ice}$	Vector of ice resistance parameters
$\boldsymbol{P}_{mom}$	Vector of ship momentum parameters
$\boldsymbol{P}_{prop}$	Vector of propulsion parameters
rpm	Revolutions per minute [min^{-1}]
$\overline{rpm}$	Average revolutions per minute between points i and $i-1$ [min^{-1}]
v_i	Ship speed at point i [$\mathrm{m\ s}^{-1}$]
v_{init}	Initial ship speed when entering the ice field [$\mathrm{m\ s}^{-1}$]
v_{meas}	Ship speed measured during full-scale tests [$\mathrm{m\ s}^{-1}$]
v_{sim}	Ship speed simulated using the machine learning-based simulator [$\mathrm{m\ s}^{-1}$]
ε	Average relative error of simulated ship speed against the measured one [%]

1. Introduction

In engineering fields dealing with computational modelling of interaction between physical objects, scientists have traditionally relied on mechanistic simulations. These are computer simulations of physical interactions based on Newtonian mechanics, where the equations governing the physical processes are based on simplified mathematical formulations and solved either analytically, for simple problems, or numerically, for the more complex ones (e.g. using CFD, FEA, etc.). The main drawbacks of the mechanistic simulations can be summarised into two points: first, the governing equations strongly depend on the user bias, i.e. on how he thinks the causal mechanisms should work and which parameters are important; and second, the simplified mathematical formulations are usually capable of handling only a handful of independent parameters, while the real-world physical processes are governed by many more. Therefore, in an attempt to overcome this intrinsic limitation of mechanistic simulations and provide an alternative approach, researchers in recent years have started developing data-driven methods for simulations of physical systems based on machine learning (ML) algorithms, so-called ML-based simulations (see Baker et al. 2018 for the discussion about differences between mechanistic and ML models). The advantages of such ML-based approaches compared to the

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mechanistic ones, in the context of the two main drawbacks mentioned above, are the following: first, it is not necessary to understand all the underlying physical mechanisms and explicitly model them, as the algorithms learn these implicitly from the data; and second, the number of independent parameters that the ML-based models can handle is practically unlimited.

In view of the above, the purpose of this paper is to bring the novel technology of ML-based simulations to the field of ship ice transit simulations. The developed methodology is based on analysing the real-world ship and ice data towards creating a ML algorithm which can be iteratively used to calculate the ship acceleration/deceleration between the two adjacent points along the route and thus estimate the expected ship speed profile in an ice field.

The paper is structured as follows: Section 2 analyses the current state of the art and positions the present study within it. Section 3 develops the generalised concept of the ML-based simulator. Section 4 tests the developed simulator on a case study. Section 5 discusses the results, and Section 6 concludes the paper with recommendations for future work.

2. State of the art

Complex ice fields may consist of small ice floes, large ice floes comprising of level ice and ice ridges, ice channels, and open water (MANICE 2005). Computer simulations of ship performance in such ice fields – also known as ship ice transit simulations – present an important tool for designers of ice-capable ships streaming for the most efficient design (von Bock und Polach et al. 2015), as well as for the ship operators who need to accurately estimate the ship performance in ice in order to reduce the risk of accidents (Goerlandt et al. 2017). The majority of the existing tools for the simulation of ship resistance/speed profile in ice are based on mechanistic approaches, as summarised in Li et al. (2018). These can be roughly divided in two groups: (1) simulations using semi-empirical methods (e.g. Lindqvist 1989; Riska et al. 1997; Jeong et al. 2017; which are summarised and compared in Erceg and Ehlers 2017) for ice resistance estimation (Frederking 2003; Kotovirta et al. 2009; Valkonen et al. 2013; Bergström et al. 2016; Kuuliala et al. 2017); (2) numerical simulations of ship performance in ice (Valanto 2001; Wang 2001; Lau 2006; Sawamura et al. 2008; Su et al. 2010; Lubbad and Løset 2011; Erceg et al. 2014; Zhou et al. 2016; Li et al. 2019). Both of these approaches come with their advantages and shortcomings. Methods in group (1) are computationally relatively economical, which comes at the expense of the accuracy. Nevertheless, they can still simulate ship speed profile in a complex ice field fairly accurately, as shown in e.g. Fig. 9 in Kuuliala et al. (2017). Methods in the group (2) are inherently more accurate as they are based on first-principle approaches, but can become computationally very expensive.

However, even the most sophisticated of these methods fail to capture the full complexity of ship–ice interactions, since the ice cover is a highly complex and heterogeneous natural phenomenon, with many of the underlying physical mechanisms still being unknown. Therefore, data-driven approaches based on ML have in the recent years been increasingly used for the prediction of ship performance in ice. Montewka et al. (2013) and Montewka et al. (2015) developed data-driven models that are capable of predicting ship speed and probability of ship besetting in ice using Bayesian networks. Fu et al. (2016) combined data with expert knowledge using Bayesian belief networks and developed a probabilistic model for estimating the probability of ship besetting in ice along the Northeast Passage. Li et al. (2017) used Bayesian networks on full-scale data to develop a model which yields the probability of certain ship speed under given ice conditions. Similä and Lensu (2018) used regression to estimate ship speed from SAR imagery and ship data. Finally, Montewka et al. (2019) developed a hybrid model for estimating ship performance in ice combining traditional engineering and data-driven approaches.

However, none of the existing data-driven methods for assessment of ship performance in ice addresses the stepwise simulation of ship speed profile in a complex ice field. Therefore, the goal of this paper is to present a concept of a first ML-based simulator for the simulation of ship speed profile in a complex ice field and to test it on the full-scale data. The goal of the developed simulator is to provide a computationally non-expensive method which based on a limited amount of available information about the ice cover can predict the expected ship speed profile in a sufficiently accurate way.

3. Methodology

The hypothesis is that the ship speed profile in a complex ice field can be simulated by iteratively applying the following function: ship speed at a point i (v_i) along the track is a function of ship speed at a point $i-1$ (v_{i-1}) together with known ship and ice parameters between arbitrarily distant (d) points i and $i-1$, which is given by

$$v_i = \boldsymbol{f}(\boldsymbol{P}_{hull},\ \boldsymbol{P}_{ice},\ \boldsymbol{P}_{mom},\ \boldsymbol{P}_{prop}) \quad (1)$$

Physically, $\boldsymbol{P}_{hull}$ is a hull-ice interaction term described as a vector of ship hull parameters important for ship–ice interaction such as main particulars and hull angles. $\boldsymbol{P}_{ice}$ is an ice resistance term described as a vector of ice parameters consisting of mechanical ice properties such as flexural/compressive strength, porosity, and ice-water density ratio. It also contains a measure of ice volume between the points i and $i-1$, as well as the ice compression and any other required ice parameter. In case other environmental parameters need to be accounted for, such as wind or water current, this vector can be extended. $\boldsymbol{P}_{mom}$ is a momentum term consisting of a ship speed at a previous point (v_{i-1}), ship mass, and added mass. $\boldsymbol{P}_{prop}$ is a propulsion term described as a vector of ship propulsion parameters valid between the points i and $i-1$, consisting of fixed parameters such as propeller and stern geometry, and variable parameters such as rudder angle, number of propeller revolutions per minute (rpm), and propeller pitch (P) for ships with CPP.

While proving the physical validity of Equation (1) is trivial, tracing the function $\boldsymbol{f}$ is analytically impossible and numerically (using mechanistic simulations) difficult. However, the hypothesis of this study is that the function $\boldsymbol{f}$ can be successfully traced using supervised ML if trained on a sufficient amount of data. The data, consisting of the parameters as described above, should be gathered from the full-scale tests preferably in different ice conditions for different ships sailing in different modes, in order to maximise the generalisation.

Thus, based on the training dataset, the ML algorithm learns the functional dependency between independent parameters and the dependent parameter (i.e. traces the function $\boldsymbol{f}$), which gives it the capability to predict v_i once presented with a new set of independent parameters that it has not seen before. Consequently, such an algorithm can be used to simulate the ship speed profile along a track for which the ice conditions and ship parameters are known (contained in 'simulation dataset' in Figure 1). This is achieved by manually setting the initial ship speed when entering the ice field (v_{init}), and iteratively applying Equation (1) until the end of the track has been reached. The proposed approach is schematically presented in Figure 1 and tested on a case study in the next section.

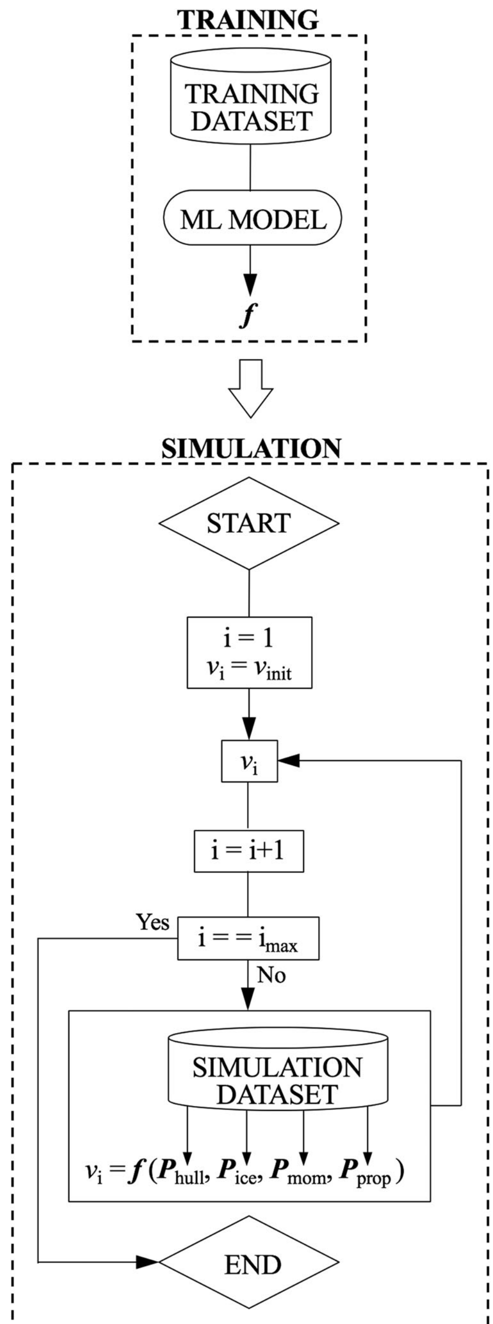

Figure 1. The concept of a ML-based simulator (i_{max} is the last point along the track). (This figure is available in colour online.)

4. Case study

4.1. Data

The dataset used in this case study was acquired during the full-scale ice tests of icebreaker S.A. Agulhas II (particulars given in Table 1) in the Bothnian Bay during 21 and 22 March 2012 (Suominen et al. 2013).

The ship was manoeuvred through level ice, ridged ice fields, and ice channels both on straight course and turning, during which ship and ice data was collected. Ship data consists of measurements of ship speed and course (both by AIS and by onboard measuring systems), as well as of measurements of machinery parameters (engine power, propeller pitch and revolutions). Ice data was obtained from three sources: visual observations; stereo camera measuring thickness of level ice turned by the ship side; EM measuring device which measured the ice thickness (h) from the ship side at a frequency of 20 Hz (see Suominen et al. 2014 for the discussion about shipborne ice thickness measuring methods).

In order to use the data for the purpose of this study, several transformations of the original data are needed. First, the data is transformed from time to space domain and interpolated to 1 m spacing. This results in a dataset covering a total length of 121,882 m. Interpolation to 1 m spacing is done in order to retain the flexibility to choose the desired resolution of the simulator by tuning the parameter d. Second, the parts of the data where the ship speed change is the result of the crew actions are removed, and only the parts with the constant P and rpm are kept, thus accounting for the ship speed change which occurs only as a result of ship–ice interaction. This simplification is done as capturing the effect of abrupt changes in the propulsion parameters is deemed unfeasible on a dataset of this size. Third, the values of P and rpm are averaged between the port and starboard propellers (although the variation between them is only slight for the selected parts of the data). After applying these transformations, the final dataset covers the length of 76,986 m (L_{data}), with values of P, rpm, h and v for each point at 1 m spacing.

Table 1. Ship particulars of S.A. Agulhas II.

Parameter	Value
Ship type	Icebreaker
Ice class	PC 5
Propulsion type	CPP
Number of propellers	2
Installed power	9 MW
Open-water speed	16 kn
Breadth	22 m
Draught	7.55 m
Waterline length	124.7 m
Ship weight (loaded)	12,500 t

4.2. Train-test split

The full dataset is split into training (to estimate the function $\boldsymbol{f}$) and test datasets (to test the simulator). For testing, five different tracks each of 3000 m in length (L_{test}) are randomly selected from the full dataset. Due to the limited amount of data, training and testing are performed for each of the five test tracks individually, where in each iteration the test dataset consists of the data for the current test track, and the training dataset consists of all of the remaining data. Therefore, the number of samples in training (N_{train}) and test datasets (N_{test}) for each test track are given by

$$N_{\mathrm{train}} = \frac{L_{\mathrm{data}} - L_{\mathrm{test}}}{d} \tag{2}$$

and

$$N_{\mathrm{test}} = \frac{L_{\mathrm{test}}}{d} \tag{3}$$

4.3. Method implementation

Due to the limitations of the available dataset, several simplifications of the generalised simulator described in Equation (1) are needed. First, as the measurements are performed during a short period of time (two days) in the first-year ice, and also due to the lack of additional data, it is considered that the ice properties are constant throughout the entire dataset. Also, data on ice compression is not available, which is therefore also omitted. This reduces $\boldsymbol{P}_{\mathrm{ice}}$ to a measure of ice volume between points i and i − 1, which is here described by the average thickness of ice between these two points, denoted by $\bar{h}$. Second, since the data is available only for one ship, fixed ship parameters are omitted. This pertains to the ship and added mass from $\boldsymbol{P}_{\mathrm{mom}}$ and the entire $\boldsymbol{P}_{\mathrm{hull}}$ vector, while $\boldsymbol{P}_{\mathrm{prop}}$ thus consists only of variable parameters such as average values of P and rpm between the points i and i − 1 ($\bar{P}$ and $\overline{rpm}$). Therefore, generalised simulator from Equation (1) is reduced to

$$v_i = \boldsymbol{f}(\bar{h},\ v_{i-1}, \bar{P}, \overline{rpm}) \tag{4}$$

In order to estimate the function $\boldsymbol{f}$, supervised ML is used on the training dataset for each of the five cases as described in Section 4.2, with simplifications in accordance with Equation (4). Implementation of the ML models is done using scikit-learn (Pedregosa et al. 2011), a free ML library for the Python programming language. Here, several ML models for regression (see Hastie et al. 2009 or Bishop 2006 for details about the theory of ML) are tested, namely: linear regressor, random forest regressor, support vector regressor, and artificial neural network (ANN) regressor. After the implementation of each of the regression models, it is found that ANN regressor (conceptualised in Figure 2) outperforms the others in terms of a relative error. ANNs are computing systems that mimic biological brain consisting of interconnected neurons which communicate through synapses. In ANNs, this principle is used to create a network of artificial neurons which is capable to learn from the data by adjusting the strength of the synaptic connections, called weights (see Haykin 2011 about the details of theory of ANNs). For ANN used in this study, different hyperparameter configurations are tested and it is found that ANN with optimal architecture for this purpose has two hidden layers with 14 neurons in each and the remaining hyperparameters as listed in Table 2.

Additionally, different values of spacing between the two adjacent simulated points (d) are tested and the value of 20 m is found to yield the most accurate results. According to Equation (2) and Equation (3), this results in roughly N_{train} = 3700 and N_{test} = 150. In other words, for each of the test tracks, ship speed profile is obtained by iteratively calculating the ship speed at 150 consecutive points with the distance between each being equal to 20 m.

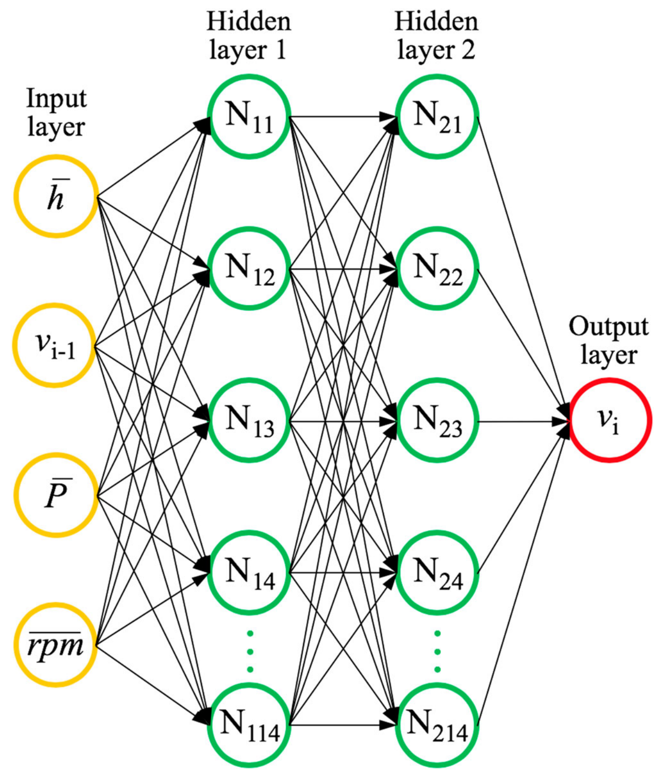

Figure 2. Model of ANN (N_{ij} denotes the j-th neuron in the i-th hidden layer). (This figure is available in colour online.)

4.4. Results

The trained ML algorithm is tested on the data from test tracks which it has not seen during the training, according to the procedure presented in Figure 1, with 'simulation dataset'

Table 2. Hyperparameters of ANN.

Parameter	Value
Number of hidden layers	2
Number of neurons in hidden layer 1	14
Number of neurons in hidden layer 2	14
Activation function	ReLU
Solver for weight optimisation	Adam
L2 penalty parameter	0.001
Batch size	Auto
Learning rate	Constant
Initial learning rate	0.01
Maximum number of epochs	100,000
Shuffle samples in each iteration	True
Tolerance for the optimisation	0.0001
Warm start	False
Momentum for gradient descent update	0.9
Early stopping	False
Exponential decay rate for estimates of first moment vector in Adam	0.9
Exponential decay rate for estimates of second moment vector in Adam	0.999
Value for numerical stability in Adam	1e − 8

becoming the 'test dataset'. The results are presented in Figure 3. For each test track, the accuracy of the simulator is quantified with an average relative error of the simulated ship speed against the measured one, given by

$$\varepsilon = \frac{100 \sum_{\mathrm{i}=0}^{N_{\mathrm{test}}} \frac{|v_{\mathrm{meas_i}} - v_{\mathrm{sim_i}}|}{v_{\mathrm{meas_i}}}}{N_{\mathrm{test}} + 1} \tag{5}$$

5. Discussion

The test results presented in Figure 3 show a very good accuracy of the ML-based simulator, especially for the test tracks #1, #2, and #3, where the simulator approximates the measured ship speed profile excellently with the average relative error between 2.6% and 4.3%. Although these results are based on a case study with several simplifications, they clearly demonstrate the potential of ML-based simulations which are seemingly capable of simulating the ship speed profile with high accuracy, without the need to explicitly model complex ship–ice interaction mechanisms, which is required by the traditional mechanistic approaches.

However, for some of the test tracks (#4 and #5), the simulator error is larger reaching 9.4%, although still capturing the qualitative shape of the speed profile correctly in most parts. A larger error in these test tracks is mainly attributed to the limited size of the training dataset, but could also be a result of the simplifications used in Equation (4). This mainly pertains to the ice properties vector $\boldsymbol{P}_{\mathrm{ice}}$, which is therein described only by the ice volume. However, based on the test results showing high accuracy of the simulator, the authors believe that describing the $\boldsymbol{P}_{\mathrm{ice}}$ only by ice volume is a justified simplification, as long as the mechanical properties of ice within the field are constant. In other words, the ML algorithm is seemingly able to learn about the difference between geometrically differentiable ice features implicitly, merely from the difference in ice volume. This seems reasonable, since e.g. ridges contain a significantly larger volume of ice than the level ice for the same distance, thus allowing the ML-algorithm to distinguish between the 'high-volume' (ridges) and 'low-volume' (level ice) features. In turn, this simplification eliminates the necessity to separately classify different ice features from the original ice profile such as level ice, ridges, ice channels, and open water before the simulations are run, which significantly reduces the required effort in comparison to the conventional methods. On the other hand, this capability of implicit learning about the effect of the ice cover on the ship speed based only on the ice volume obviously does not apply to certain ice properties, which are uncharacterizable merely by the ice volume, and variation of which could be one of the sources of larger error in test tracks #4 and #5. These can be e.g. variation of the ridge consolidated layer thickness within the ice field, or inclusions of ice types with significantly different mechanical properties (e.g. multi-year ice), which then need to be explicitly included as parameters into $\boldsymbol{P}_{\mathrm{ice}}$ or modelled in some other way. However, due to the lack of data about any other ice parameters besides the ice volume, such analysis is omitted in this study.

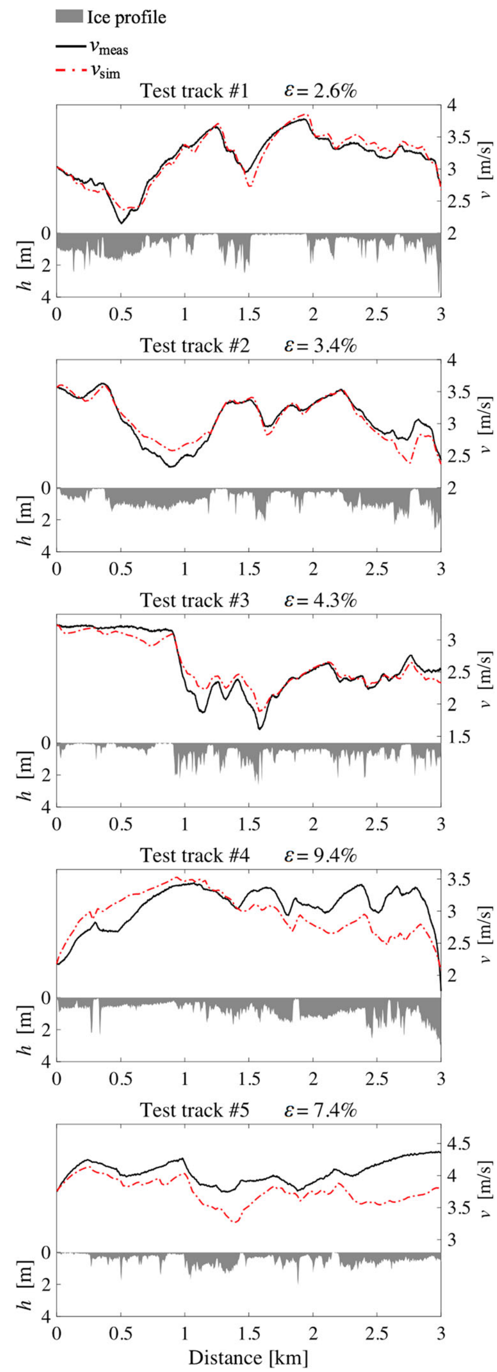

Figure 3. Test results of the ML-based simulator for each of the five test tracks. (This figure is available in colour online.)

A significant advantage of the presented approach compared to the traditional mechanistic numerical simulators is that once the algorithm training is completed, ML-based simulations are expected to be computationally much less expensive. This is expected to make them particularly suitable for the massive shipping efficiency optimisation frameworks such as the one developed in Lehtola et al. (2019). Also, the computational efficiency will become increasingly important in the upcoming age of autonomous ships when the operational decisions will have to be made constantly and automatically in very short time intervals based on a continuous data stream of dynamically changing environmental parameters.

Despite the discussed advantages and potentials, a disadvantage of the ML-based simulations in comparison with the traditional mechanistic ones is that they require large amounts of data for the training, gathering of which is expensive, both financially and time-wise. Even if the required data becomes available, a potential issue could arise due to combining datasets from different sources, as it is questionable if and how the ice profile measurements obtained by different methods can be combined. One way to overcome this is to develop semi-empirical models which could use data-driven approach for one ship (as presented in the case study) and then use the theoretical knowledge to generalise the results to other ships by some transformations according to parameters such as ship mass, hull shape, and propulsion characteristics.

Another source of uncertainty in this study is the simulator resolution d. While it is shown that the simulator performs with the highest accuracy at $d = 20$ m, this is somewhat an arbitrary number, found by manually testing different values. In reality, it is expected that there exists a limiting value of d at which the magnitude of the ship speed variation reaches a minimal value which can be captured by the ship speed measuring system. This depends on ship parameters (mainly mass), ship speed, and the severity of ice conditions. However, due to a limited amount of data used in this study, variations of ship speed at $d < 20$ m are too small to be reliably modelled.

It should be also mentioned that the ML models tested and used in this study are relatively simple compared to the cutting-edge deep learning algorithms, which could be more appropriate when the larger datasets are analysed.

6. Conclusions and future work

In this paper, a concept of a first machine learning-based simulator for the simulation of ship speed profile in a complex ice field has been presented. The developed methodology assumes that supervised machine learning can be used on the data to trace a function mapping several ship and ice parameters to the ship acceleration/deceleration between the two adjacent points along the route. The developed simulator has been tested on a case study, showing a high accuracy with an average relative error of simulated ship speed against the measured one ranging from 2.6% to 9.4%. Therefore, it is concluded that the developed methodology shows promising potential and presents a valuable alternative to the traditional mechanistic simulations, since high accuracy of the ship speed prediction was achieved despite simplifying the description of a complex ice cover by only taking the ice volume into account.

Future work consists of further development and training of the simulator based on a continually growing dataset, with an overall goal to develop a generalised model that would present a straightforward and computationally inexpensive method for the simulation of ship speed profile in an arbitrary ice field for an arbitrary ship.

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