ASSIGNMENT 1
PM569 Spatial Statistics, Fall 2018

Part 1 Due September 14, 2018
Part 2 Due September 21, 2018

1 Theoretical Semivariograms and Covariance

For the following theoretical semivariogram functions:

- Power model: $\gamma(h) = \tau^2 + \sigma^2 h^\lambda$ if $h > 0; 0$ otherwise
- Rational quadratic: $\gamma(h) = \tau^2 + \sigma^2 \frac{h^2}{(1+\phi h^2)}$ if $h > 0; 0$ otherwise
- Powered exponential: $\gamma(h) = \tau^2 + \sigma^2(1 - exp(-|\phi h|^\lambda)$ if $h > 0; 0$ otherwise
- Wave: $\gamma(h) = \tau^2 + \sigma^2(1 - \frac{sin(\phi h)}{\phi h})$ if $h > 0; 0$ otherwise

1. Plot the semivariograms (on the same plot) fixing $\tau^2 = 1, \sigma^2 = 5, \phi = 2, \lambda = 0.5$. Choose your own distances. Identify which of the parameters represent the nugget, sill, and range.
2. Using the plot from 1), provide a description of the differences in the four models despite having the same fixed parameter values. Also explain whether or not each semivariogram corresponds to a stationary process.
3. For the two power models, vary the value λ and plot the different semivariograms. What does the power parameter λ represent?
4. For the rational quadratic and wave models, vary the values of τ^2, σ^2, ϕ (one at a time) and describe how the semivariogram changes.
5. Find/calculate and plot the **covariogram** (covariance as a function of distance) for the exponential, spherical and Gaussian functions shown in class.
6. The Matern covariance function is widely used in the spatial statistical literature. The function is:

$$C(h) = \sigma^2 \frac{1}{\Gamma(\kappa)2^{\kappa-1}}(\sqrt{2\kappa}\frac{h}{\phi})^\kappa K_\kappa(\sqrt{2\kappa}\frac{h}{\phi})$$

Where κ is a smoothness parameter, Γ is a gamma function, and K_κ is a modified Bessel function. Using the functions gamma() and besselK(), write the above Matern covariance function in R and test and plot it with your choice of parameter values. What do you see changing for small κ (i.e. $\kappa \to 0$) versus large κ (i.e. $\kappa \to \infty$)?

2 Empirical Semivariograms

For this question we will use meteorological data from weather stations in Southeastern Texas at the time of Hurricane Harvey, August 25, 2017 (averaged over hourly measurements from 8pm to 11pm). We will focus on wind speed (m/s) and atmospheric pressure (millibars) to examine the strength and location of landfall of the hurricane. Use the geoR(), proj4(), maps(), ggplot2(), and ggmaps() packages in R.
The data were acquired from the National Oceanic and Atmospheric Administration. Here is some background on the hurricane:
`http://www.weather.gov/crp/hurricane_harvey`.

1. Perform exploratory data analysis: examine the data distributions of wind speed and atmospheric pressure and provide summary statistics. Create two maps showing the locations and color gradients for the values of the meteorological parameters (use lat/lon coordinates for ggmaps). Please describe any spatial trends that can be visualized in the data.

2. Project the latitude and longitude to UTM coordinates. Use these x,y values in the subsequent questions.

3. Create maps for each variable in UTM projected coordinates using ggplot.

4. Plot empirical semivariograms (robust) for wind speed and atmospheric pressure. Try two different binning schemes for each (e.g. change max distance, number of bins). Describe the spatial structure, and give an eyeball estimate of the nugget, sill and range. Compare the spatial aspects of the two variables. Do you think one or the other is a better indicator of the hurricane strength and scope?

5. Determine if there is a linear spatial trend in either variable.

6. Determine if there is anisotropy in either variable.