通訊系統

第一次小考輔導課

課後問答與補充

2020.10.12

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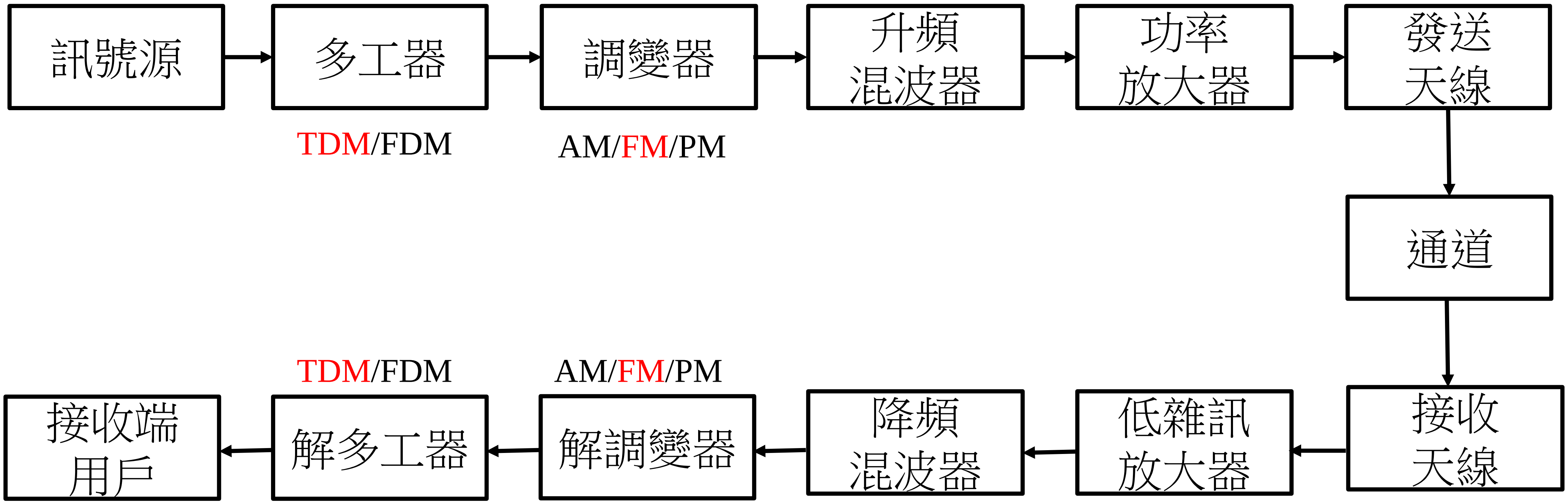
課後問答1

通訊系統方塊圖


訊號源
多工器
TDM/FDM
調變器
AM/FM/PM
升頻
混波器
功率
放大器
發送
天線
通道
接收
天線
低雜訊
放大器
降頻
混波器
AM/FM/PM
解調變器
TDM/FDM
解多工器
接收端
用戶

Q: Please list the equations of antenna gain (天線增益) and beamwidth (波束寬度) to explain how to increase antenna gain and decrease the antenna beamwidth if antenna efficiency is fixed (天線效益固定)? What are three major antenna functions and two antenna specifications?

- $G = \eta \dfrac{4\pi A_e}{\lambda^2} = \eta \left(\dfrac{\pi D f_c}{C}\right)^2$

  ∴ η 固定，当 $D\uparrow$, $f_c\uparrow$, 則 $G\uparrow$

- θ_{3dB} = 3dB beamwidth $= \dfrac{70\lambda}{D} = \dfrac{70C}{Df_c}$

  ∴ 当 $D\uparrow$, $f_c\uparrow$, 則 $\theta_{3dB}\downarrow$

three major antenna function

1、指定方向發射 or 接收信号
2、增加天線增益
3、防止干擾

Three antenna specification

1. antenna gain
2、3dB beamwidth
3、side lobe level

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$$G = \eta \frac{4\pi}{\lambda^2} A_e$$

$$\lambda = wavelength(m) = \frac{C}{f_c}$$

C : light velocity $= 3 \times 10^8 m/\sec$ $\quad f_c$:carrier frequency

$$G = \eta \left(\frac{\pi D}{\lambda}\right)^2 = \eta \left(\frac{\pi D f_c}{C}\right)^2$$

A_e: Antenna aperture $\dfrac{\pi D^2}{4} = \pi \left(\dfrac{D}{2}\right)^2$

D : diameter of parabolic dish ant.

η : surface efficiency =55%

*reason for modulation and up-conversion $f_c\uparrow, G\uparrow$

$$\theta_{3dB} = 3dB\ beamwidth = \frac{70\lambda}{D}, D\uparrow, G\uparrow, \theta_{3dB}\downarrow$$

發射功率 頻寬

Q3. If transmit power P_t bandwidth B are fixed, how do you improve

訊號雜訊比

the signal to noise ration (SNR) of the receiver? Please state the reasons with link budget equation.

$$SNR = \frac{S}{N} = \frac{P_r}{kT_sB} = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2 kT_sB}$$

∴ P_t、B 固定

當 $G_t\uparrow$, $G_r\uparrow$, $\lambda\uparrow$, $d\downarrow$ $T_s\downarrow$

$SNR\uparrow$ 。

$$P_r = \frac{P_t G_t G_r}{L_P}$$

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The path loss between two antennas is $L_P = (\frac{4\pi R}{\lambda})^2$

The received signal-to-noise power ratio is $SNR = \frac{S}{N} = \frac{P_R}{kT_sB}$

$$SNR = \frac{S}{N} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2 kT_sB}$$ (Link budget equation)

最大數據傳輸率

Q4: Please write the formula of maximum data transmission rate for a reliable wireless communication system.

Shannon's channel capacity theorem I

$$C = B\log_2(1+SNR)$$

$$= B\log_2\left(1+\frac{S}{N_0 B}\right)$$

*通道容量 $C = B\log_2(1+SNR)$ (bit/sec)

通訊系統最大傳輸率 R (bit/sec)

∴ 當 $R < C$ => reliably communication

B = channel bandwidth

SNR = average signal to noise power ratio

N0 = noise power spectral density (watts/Hz)

2.Channel bandwidth , B(Hz)： 傳輸訊號之頻帶

Shannon's channel capacity theorem (I)

- $C = B\log_2(1+SNR)(bits/\sec) = B\log_2\left(1+\frac{S}{N_0 B}\right)$ (bit/sec)
- $T_s \equiv$ system effective temperature
- $C \equiv$ channel capacity $(bits/sec)$
  $\equiv$ the maximum rate at which information may be transmitted without error through the channel.
- B =channel bandwidth
- SNR =average signal to noise power ratio.
- $N_0 \equiv$ noise power spectral density watts/Hz.
  $= kT_0, T_0 = 290^o K$ (IEEE STD),
  k=Boltz mann's constant=$1.38*10^{-23}$ walts/Hz-$^\circ K$

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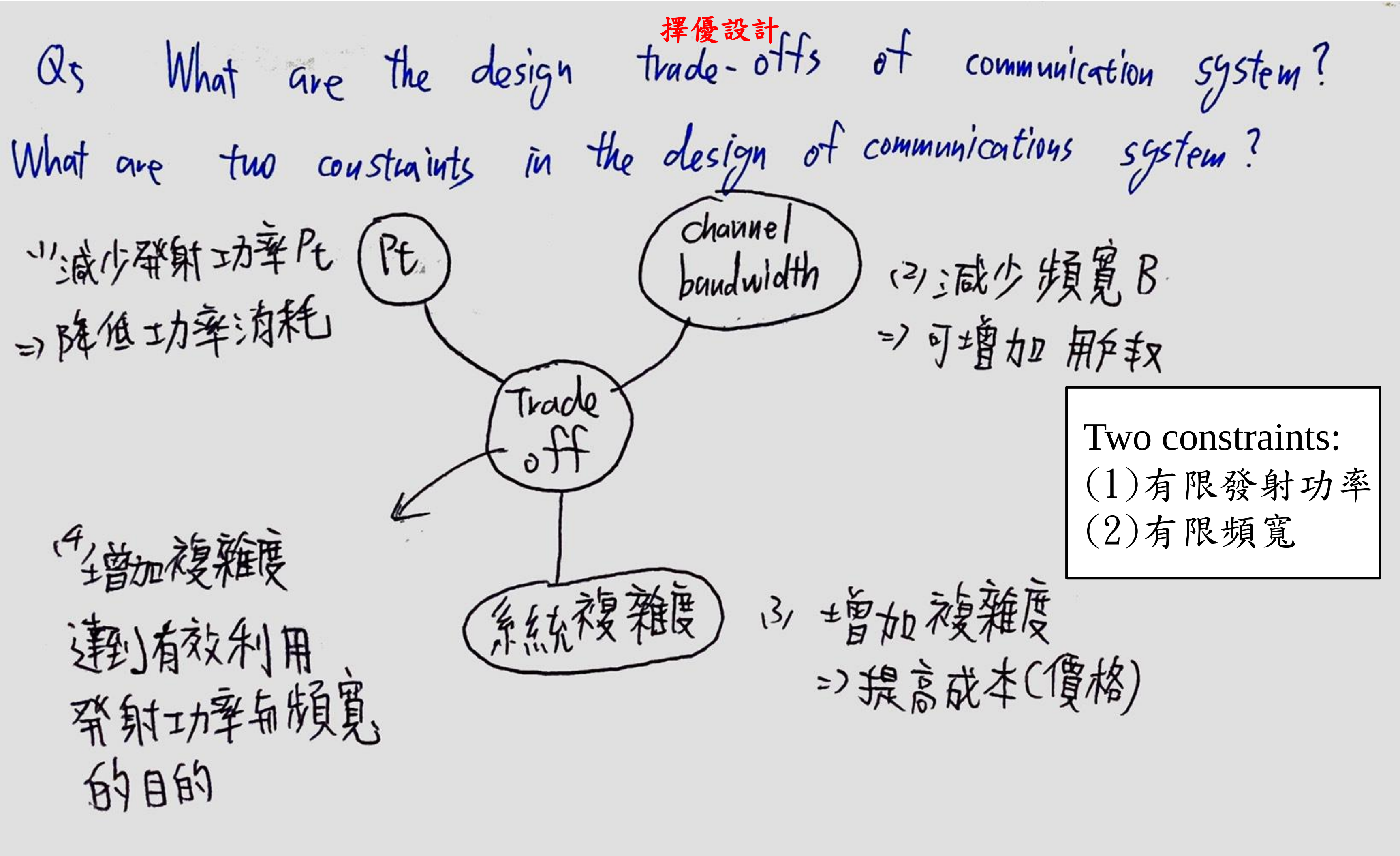
擇優設計



Two constraints:
(1)有限發射功率
(2)有限頻寬

天線增益的參數定義

Q6: Please list the antenna gain formula with the definition of parameters, and state the reasons for using the modulation and up-conversion mixer in the transmitter

$$G = \eta \frac{4\pi A_e}{\lambda^2} = \eta \left(\frac{\pi D f_c}{c}\right)^2$$

Fc = 載波頻率

η = surface efficiency

A_e = Antenna aperture $\frac{\pi D^2}{4}$

D = diameter of parabolic dish ant.

λ = 波長

$$G = \eta \left(\frac{\pi D f_c}{c}\right)^2$$

$\because D\uparrow, f_c\uparrow, G\uparrow$

不用使用大的天線(D)，藉由升頻混波器提高載波頻率(fc)即可達到增加 antenna gain (G)的目的

Q8 Please state three major system parameters of mobile communication system.

Data rate、Latency、Number of connections

Q9: What are the future applications of 5G? Why 5G can be applied for intelligent transportation system?

1、雲端、大數據

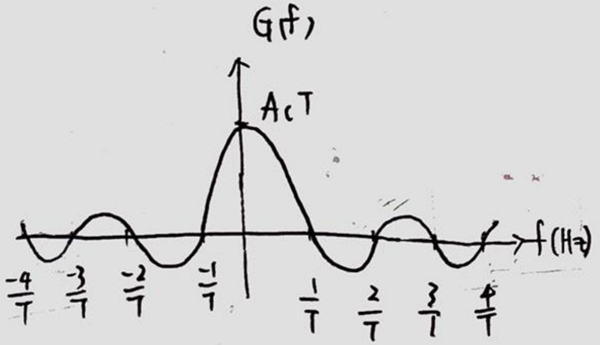
2、車聯網(自駕車)

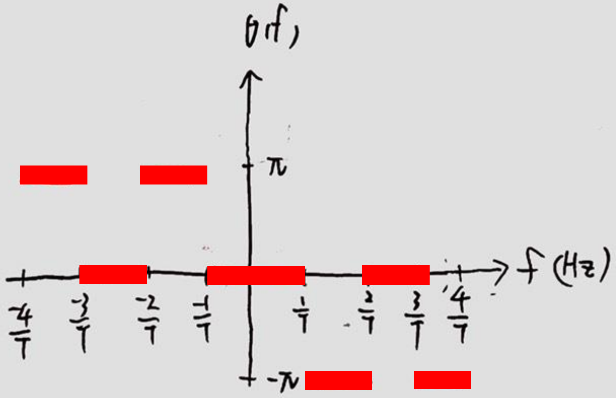
3、工業4.0

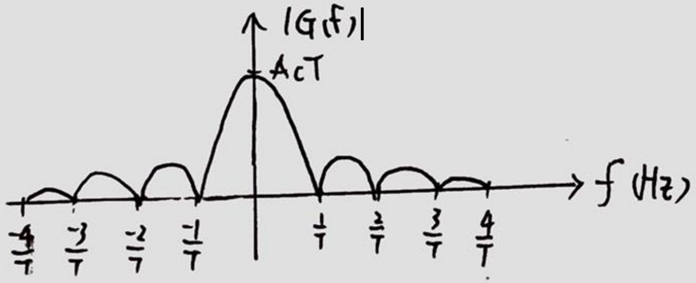
課後問答2

Q1: Determine the Fourier transform of $g(t) = A_c \, \text{rect}\left(\frac{t}{T}\right)$ and draw its amplitude spectrum and phase spectrum. Assume that $\theta(f) = -\pi$, if $G(f)$ is less than zero and $f > 0$

$$G(f) = A_c T \, \text{sinc}(fT)$$

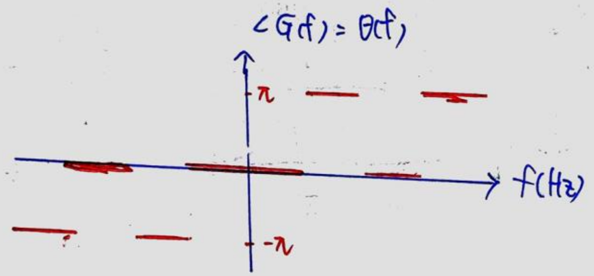






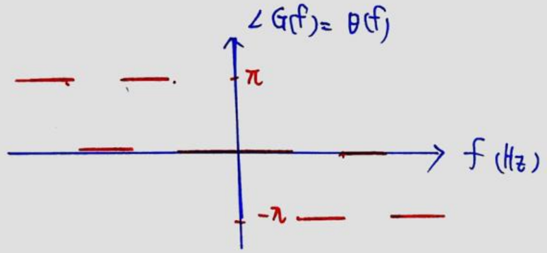
if $f > 0$, $\theta = \pi$

$f < 0$, $\theta = -\pi$



if $f > 0$, $\theta = -\pi$

$f < 0$, $\theta = \pi$



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☆ $g(t) = A\,rect(\frac{t}{T}) \longleftrightarrow G(f) = AT\,sinc(fT)$

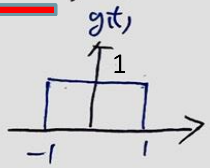
＊ 時域壓縮，頻域擴張

☆ $g(t) = rect(at) \longleftrightarrow G(f) = \frac{1}{a} sinc(\frac{f}{a})$

① $a = \frac{1}{2}$, $g(t) = rect(\frac{t}{2}) \longleftrightarrow G(f) = 2 sinc(2f) = 2\frac{\sin(2\pi f)}{2\pi f}$

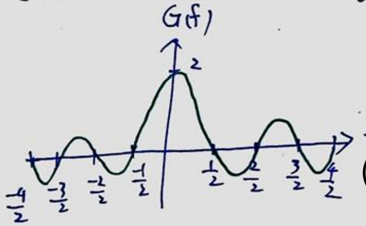
$-\frac{1}{2} \le \frac{t}{2} \le \frac{1}{2}$

$\Rightarrow -1 \le t \le 1$



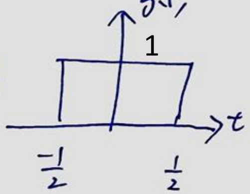
$2\pi f = 0 \pm k\pi$

$\Rightarrow f = \frac{k}{2}$



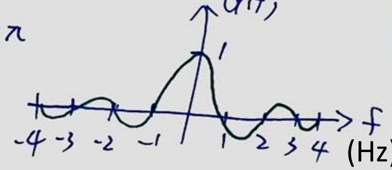
② $a = 1$, $g(t) = rect(t) \longleftrightarrow G(f) = sinc(f) = \frac{\sin(\pi f)}{\pi f}$

$-\frac{1}{2} \le t \le \frac{1}{2}$



$\pi f = 0 \pm k\pi$

$\Rightarrow f = k$

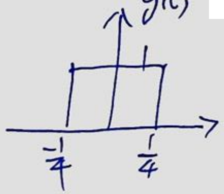


③ $a = 2$, $g(t) = rect(2t) \longleftrightarrow G(f) = \frac{1}{2} sinc(\frac{f}{2}) = \frac{1}{2}\frac{\sin(\frac{\pi f}{2})}{\frac{1}{2}\pi f}$

$\frac{t}{\frac{1}{2}}$

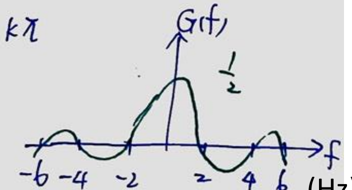
$-\frac{1}{2} \le 2t \le \frac{1}{2}$

$\Rightarrow -\frac{1}{4} \le t \le \frac{1}{4}$



$\frac{\pi f}{2} = 0 \pm k\pi$

$\Rightarrow f = 2k$



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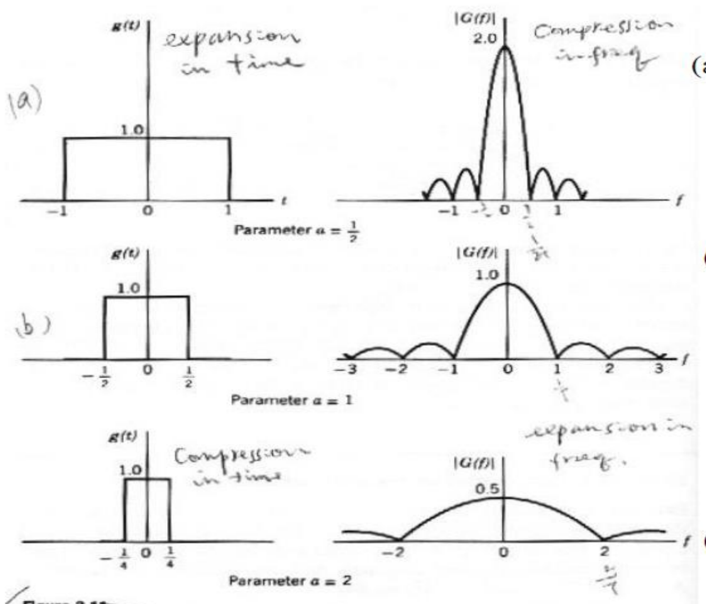


Figure 2.11 The inverse relation between time- and frequency-domain descriptions of rectangular pulse g(t) = rect(at).

(a) if $a = \frac{1}{2}, rect(\frac{t}{2}) \xleftrightarrow{F} 2\sin c(2f)$

(b) $= \begin{cases} 1, -\frac{1}{2} < t < \frac{1}{2} \\ 0, otherwise \end{cases}$

(c) if $a = 2, rect(2t) \xleftrightarrow{F} \frac{1}{2}\sin c(\frac{f}{2})$

Ex: g(t) = 9 rect(3t/4)，求G(f)?

Q2: (1) Determine the spectrum of rectangular pulse $g(t) = A_c \, rect\left(\frac{t}{T}\right)$

$$G(f) = A_c \, T \, sinc(Tf)$$

(2) Determine the bandwidth of rectangular pulse

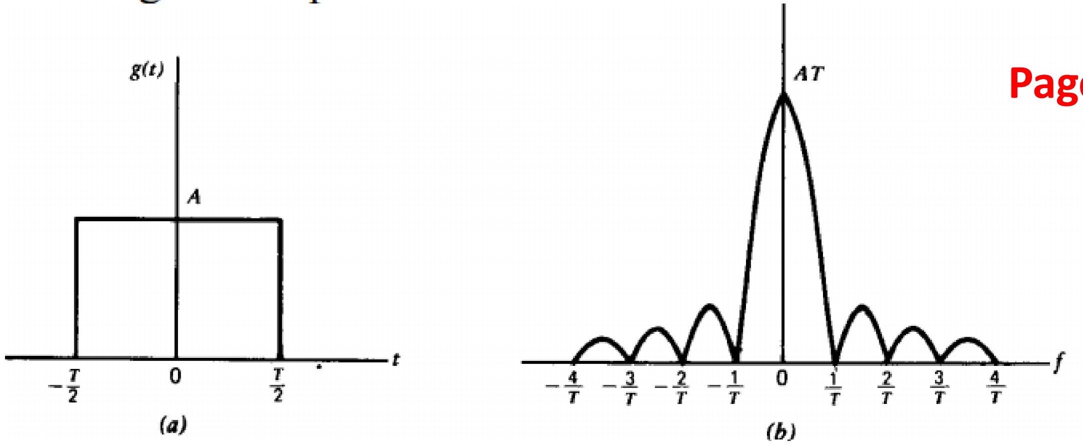
$$B.W = \frac{1}{T} \; (Hz)$$

1. 正頻率
2. 能量集中的範圍

Bandwidth

(a)Null to null bandwidth $\equiv$ B.W. provides a measure of the extent of significant spectral content of the signal for <u>positive frequency</u>.

* in Fig 2.2 the rectangular pulse ,B.W.=1/T.
* in Fig 2.8 RF pulse B.W.=2/T.



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Q3: (1) Determine the spectrum of RF pulse $g(t) = \text{rect}\left(\frac{t}{T}\right)\cos 2\pi f_c t$

$$g(t) = \text{rect}\left(\frac{t}{T}\right)\cos 2\pi f_c t$$

$$\because \cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) \quad \text{(Euler's Formula)}$$

$$\therefore \cos(2\pi f_c t) = \frac{1}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

$$\Rightarrow g(t) = \frac{1}{2}\text{rect}\left(\frac{t}{T}\right)\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

$$= \frac{1}{2}e^{j2\pi f_c t}\,\text{rect}\left(\frac{t}{T}\right) + \frac{1}{2}e^{-j2\pi f_c t}\,\text{rect}\left(\frac{t}{T}\right)$$

① P1-13 Time Scaling 性質

$$\boxed{g(at) \xleftrightarrow{FT} \frac{1}{|a|}G\left(\frac{f}{a}\right)} \Rightarrow \boxed{\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} T\,\text{sinc}(fT)}$$

② P1-15 Frequency Shifting 性質

$$\boxed{e^{j2\pi f_c t}g(t) \xleftrightarrow{FT} G(f-f_c)} \Rightarrow \boxed{e^{j2\pi f_c t}\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} T\,\text{sinc}[T(f-f_c)]}$$

$$\Rightarrow \boxed{e^{-j2\pi f_c t}\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{FT} T\,\text{sinc}[T(f+f_c)]}$$

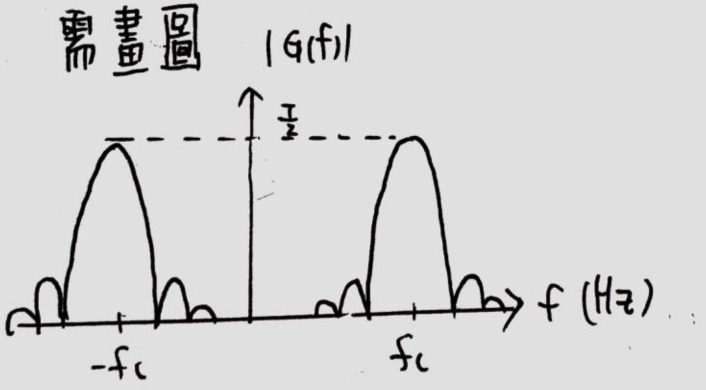
By ①, ②

$$G(f) = \frac{1}{2}T\,\text{sinc}[T(f-f_c)] + \frac{1}{2}T\,\text{sinc}[T(f+f_c)]$$

$$\Rightarrow G(f) = \begin{cases} \frac{T}{2}\,\text{sinc}[T(f-f_c)], & f>0 \\ 0, & f=0 \\ \frac{T}{2}\,\text{sinc}[T(f+f_c)], & f<0 \end{cases}$$

(2) Bandwidth of RF pulse

需畫圖



$$BW = \frac{2}{T}\,(Hz)$$

Q4: Determine the spectrum of $g(t) = A_c \cos(2\pi f_c t)$

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$\because \cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$ (Euler's formula)

$\therefore \cos(2\pi f_c t) = \frac{1}{2}(e^{j2\pi f_c t} + e^{-j2\pi f_c t})$

$\Rightarrow g(t) = A_c \cdot \frac{1}{2}(1 \times e^{j2\pi f_c t} + 1 \times e^{-j2\pi f_c t})$

① P 1-15 Frequency Shifting 性質

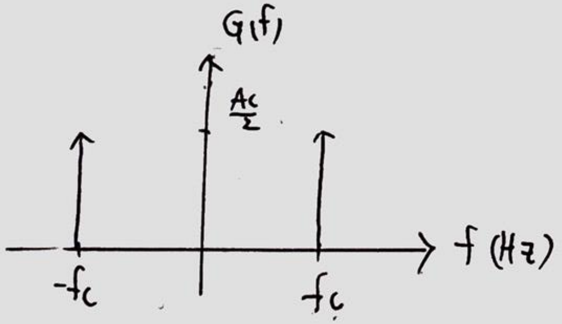
$e^{j2\pi f_c t} g(t) \xleftrightarrow{FT} G(f-f_c)$ $\quad$ $e^{-j2\pi f_c t} g(t) \xleftrightarrow{FT} G(f+f_c)$

② P 1-23

$1 \xleftrightarrow{FT} \delta(f)$

By ①, ②

$G(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)]$

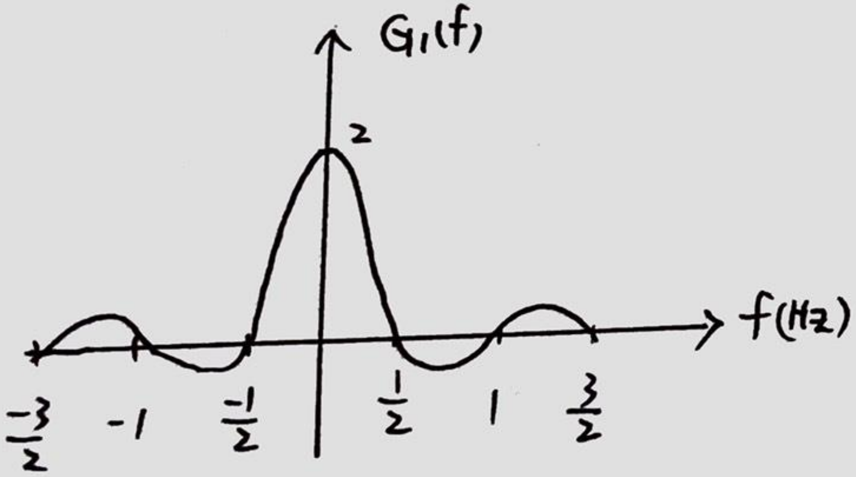


Q5: Determine the spectrum and bandwidth of (1) $g_1(t) = \text{rect}\left(\frac{t}{2}\right)$

(2) $g_2(t) = \text{rect}(2t)$

(1)

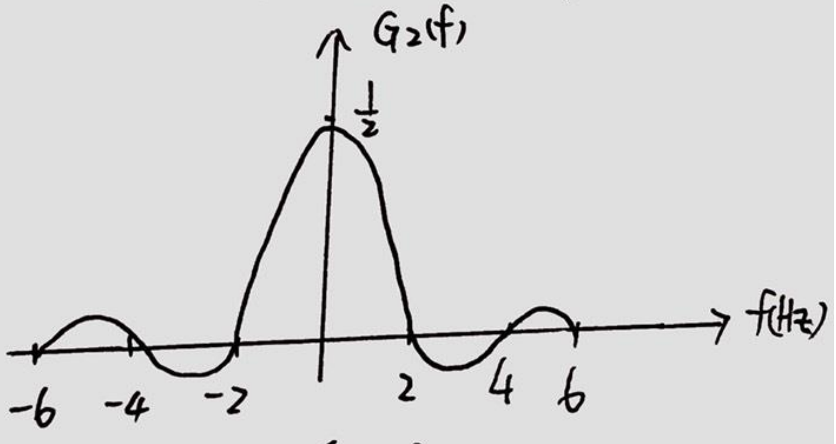
$$G_1(f) = 2\ \text{sinc}(2f)$$



$\text{B.W} = \frac{1}{2}(\text{Hz})$

(2)

$$G_2(f) = \frac{1}{2}\ \text{sinc}\left(\frac{f}{2}\right)$$



$\text{B.W} = 2(\text{Hz})$

Q6 Determine and draw the spectrum of (1) $g_1(t) = A\,\mathrm{sinc}(\frac{t}{2})$

(2) $g_2(t) = A\,\mathrm{sinc}(2t)$

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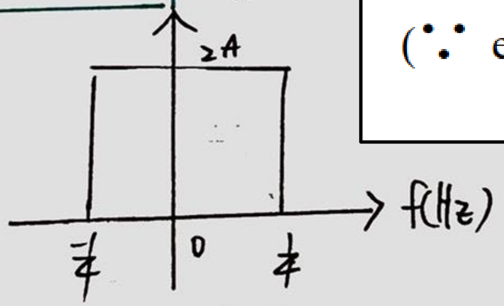
P 1-14

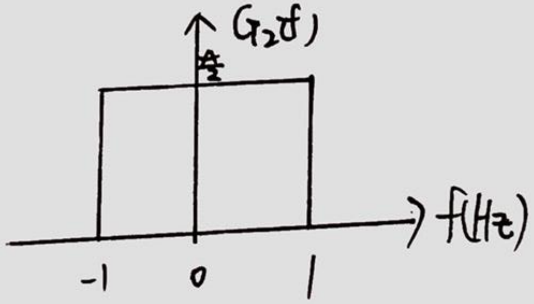
$$\boxed{A\ \mathrm{sinc}(t) \xleftrightarrow{FT} A\,\mathrm{rect}(-f) = A\,\mathrm{rect}(f) \quad (偶对称)}$$

$$A\sin c(2\omega t) \xleftrightarrow{F} \frac{A}{2\omega} rect(-\frac{f}{2\omega})$$

$$(\because \text{ even function}) \ = \frac{A}{2\omega} rect(\frac{f}{2\omega})$$

$G_1(f) = 2A\,\mathrm{rect}(2f)$





$G_2(f) = \frac{A}{2}\,\mathrm{rect}(\frac{f}{2})$

Q7 (1) If the total energy of $g(t)$ is, please prove $E_t = \int_{-\infty}^{\infty} |G(f)|^2 df$

See 1-20

(2) If $g(t) = \text{sinc}(Bt)$, please determine the total energy of $g(t)$,

$$G(f) = \frac{1}{B}\text{rect}\left(\frac{f}{B}\right)$$

$$= \frac{1}{B}, \quad \frac{-B}{2} \le f \le \frac{B}{2}$$

$$E_t = \int_{-\infty}^{\infty} |G(f)|^2 df = \frac{1}{B^2}\int_{-\frac{B}{2}}^{\frac{B}{2}} df = \frac{1}{B}\ (\text{watt-sec})$$

The total energy delivered by the voltage source

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$$E_t \equiv \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df \qquad v(t) = i(t) = g(t)$$

Proof:

$$E_t \equiv \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} g(t)g^*(t)dt$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} G(f)e^{j2\pi ft} df\right] g^*(t)dt$$

$$= \int_{-\infty}^{\infty} G(f)\left[\int_{-\infty}^{\infty} g^*(t)e^{j2\pi ft} dt\right] df$$

$$= \int_{-\infty}^{\infty} G(f)\left[\int_{-\infty}^{\infty} g(t)e^{-j2\pi ft} dt\right]^* df$$

$$= \int_{-\infty}^{\infty} G(f)G^*(f)df$$

$$= \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$\therefore E_t = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

Q8: The signal $s(t) = \text{sinc}(3t)$ inputs LTI system impulse response $h(t) = \text{sinc}(2t)$

(1) Determine the output spectrum of $y(t)$

$$\left[\begin{array}{ll} y(t) = s(t) * h(t) & \text{時域做 convolution} \\ Y(f) = S(f)\,H(f) & \rightarrow \text{頻域做相乘} \end{array}\right.$$

$S(f) = \frac{1}{3}\,\text{rect}\left(\frac{f}{3}\right)$

$= \frac{1}{3}, \quad \frac{-3}{2} \le f \le \frac{3}{2}$

$H(f) = \frac{1}{2}\,\text{rect}\left(\frac{f}{2}\right)$

$= \frac{1}{2}, \quad -1 \le f \le 1$

$\Rightarrow Y(f) = \frac{1}{6}, \quad -1 \le f \le 1$ (取交集)

$= \frac{1}{6}\,\text{rect}\left(\frac{f}{2}\right)$ #

(2) Determine the output signal $y(t)$

$y(t) = \frac{1}{3}\,\text{sinc}(2t)$

Q9: If g(t) is a complex signal (1) Determine the spectrum of Re{g(t)} and jIm{g(t) ，(2) Prove the spectrum of Re{g(t)} is even symmetric and the spectrum of j Im{g(t)} is odd symmetric

$$g(t) = \operatorname{Re} g(t) + j \operatorname{Im} g(t) \text{.............}(1)$$

$$g^*(t) = \operatorname{Re} g(t) - j \operatorname{Im} g(t) \text{..........}(2)$$

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(1)+(2) $$\operatorname{Re} g(t) = \frac{1}{2}\left[g(t) + g^*(t)\right]$$

(1)-(2) $$\operatorname{Im} g(t) = \frac{1}{2j}\left[g(t) - g^*(t)\right]$$

$$\therefore \operatorname{Re} g(t) \xleftrightarrow{F} \frac{1}{2}\left[G(f) + G^*(-f)\right]$$

$$\operatorname{Im} g(t) \xleftrightarrow{F} \frac{1}{2j}\left[G(f) - G^*(-f)\right]$$

If $g(t)$ is real function $\Longrightarrow \operatorname{Im} g(t) = 0$

$$\Rightarrow G(f) = G^*(-f)$$

$$\Rightarrow G^*(f) = G(-f)$$


!YOU OVER THERE!
HAVE A NICE DAY