

通訊系統

第一次小考輔導課

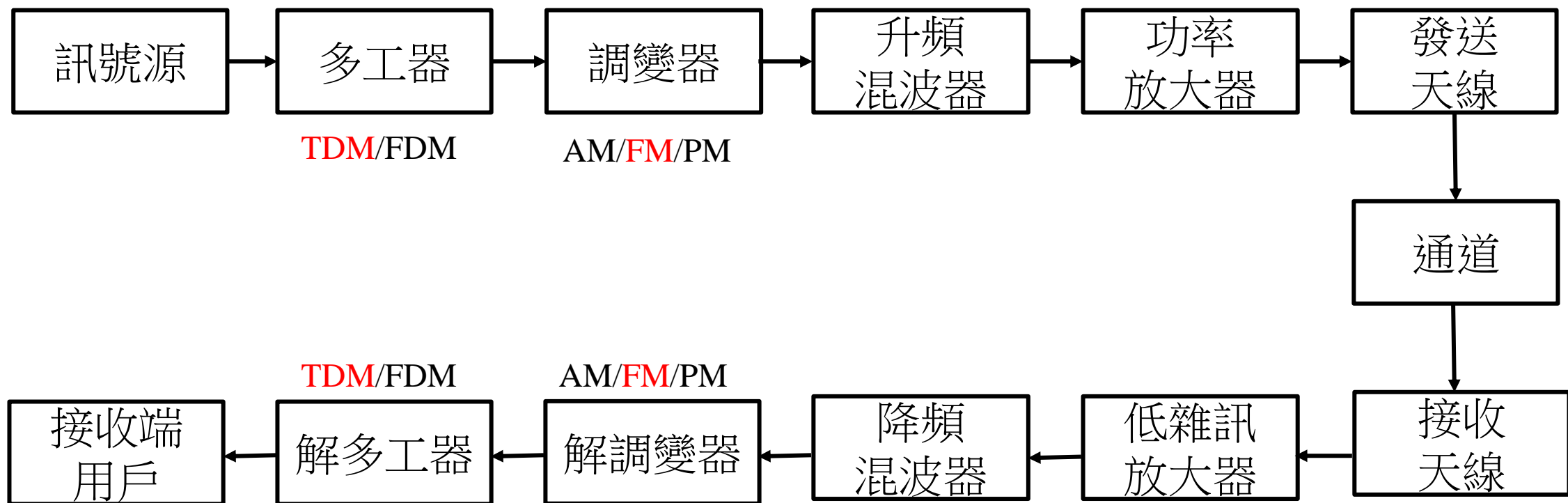
課後問答與補充

2020.10.12

助教: 徐靖憲

課後問答1

通訊系統方塊圖



Q: Please list the equations of antenna gain and beamwidth to explain how to increase antenna gain and decrease the antenna beamwidth if antenna efficiency is fixed? What are three major antenna functions and two antenna specifications?

$$G = \eta \frac{4\pi A_e}{\lambda^2} = \eta \left(\frac{\pi D f_c}{c} \right)^2$$

$\therefore \eta$ 固定, 当 $D \uparrow$, $f_c \uparrow$, 则 $G \uparrow$

$$\theta_{3dB} = 3dB \text{ beamwidth} = \frac{70\lambda}{D} = \frac{70c}{Df_c}$$

\therefore 当 $D \uparrow$, $f_c \uparrow$, 则 $\theta_{3dB} \downarrow$

天線增益

波束寬度

天線效益固定

three major antenna functions

1. 指定方向發射或接收信號
2. 增加天線增益
3. 防止干擾

three antenna specification

1. antenna gain
2. 3dB beamwidth
3. side lobe level

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$$G = \eta \frac{4\pi}{\lambda^2} A_e$$

$$\lambda = \text{wavelength}(m) = \frac{c}{f_c}$$

c : light velocity = $3 \times 10^8 m/sec$ f_c : carrier frequency

$$G = \eta \left(\frac{\pi D}{\lambda} \right)^2 = \eta \left(\frac{\pi D f_c}{c} \right)^2$$

$$A_e: \text{Antenna aperture } \frac{\pi D^2}{4} = \pi \left(\frac{D}{2} \right)^2$$

D : diameter of parabolic dish ant.

η : surface efficiency = 55%

*reason for modulation and up-conversion $f_c \uparrow, G \uparrow$

$$\theta_{3dB} = 3dB \text{ beamwidth} = \frac{70\lambda}{D}, D \uparrow, G \uparrow, \theta_{3dB} \downarrow$$

Q3. If transmit power P_t ^{發射功率} bandwidth B ^{頻寬} are fixed, how do you improve the signal to noise ratio (SNR) ^{訊號雜訊比} of the receiver? Please state the reasons with link budget equation.

$$SNR = \frac{S}{N} = \frac{P_r}{kT_s B} = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2 kT_s B}$$

$\therefore P_t, B$ 固定
 $\nless G_t \uparrow, G_r \uparrow, \lambda \uparrow, d \downarrow, T_s \downarrow$
 $SNR \uparrow$

$$P_r = \frac{P_t G_t G_r}{L_p}$$

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The path loss between two antennas is $L_p = \left(\frac{4\pi R}{\lambda}\right)^2$

The received signal-to-noise power ratio is $SNR = \frac{S}{N} = \frac{P_r}{kT_s B}$

$$SNR = \frac{S}{N} = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2 kT_s B} \quad (\text{Link budget equation})$$

Q4: Please write the formula of maximum data transmission rate for a reliable wireless communication system.

最大數據傳輸率

Shannon's channel capacity theorem I

$$C = B \log_2 (1 + \text{SNR})$$

$$= B \log_2 \left(1 + \frac{S}{N_0 B}\right)$$

*通道容量 $C = B \log_2 (1 + \text{SNR})$ (bit/sec)

通訊系統最大傳輸率 R (bit/sec)

$\therefore \text{當 } R < C \Rightarrow \text{reliably communication}$

B = channel bandwidth

SNR = average signal to noise power ratio

N_0 = noise power spectral density (watts/Hz)

2. Channel bandwidth, B (Hz): 傳輸訊號之頻帶

Shannon's channel capacity theorem (I)

$$- C = B \log_2 (1 + \text{SNR}) (\text{bits/sec}) = B \log_2 \left(1 + \frac{S}{N_0 B}\right) (\text{bit/sec})$$

- $T_s \equiv$ system effective temperature

- $C \equiv$ channel capacity (bits/sec)

\equiv the maximum rate at which information may be transmitted without error through the channel.

- $B =$ channel bandwidth

- $\text{SNR} =$ average signal to noise power ratio.

- $N_0 \equiv$ noise power spectral density watts/Hz.

$$= kT_0, T_0 = 290^\circ K \quad (\text{IEEE STD}),$$

$$k = \text{Boltzmann's constant} = 1.38 \times 10^{-23} \text{ watts/Hz} \cdot ^\circ K$$

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擇優設計

Q5 What are the design trade-offs of communication system?

What are two constraints in the design of communications system?

(1) 減少發射功率 P_t
 \Rightarrow 降低功率消耗

P_t

channel
bandwidth

(2) 減少頻寬 B
 \Rightarrow 可增加用戶數

Trade
off

Two constraints:

- (1) 有限發射功率
- (2) 有限頻寬

(4) 增加複雜度

達到有效利用
發射功率與頻寬
的目的

系統複雜度

(3) 增加複雜度
 \Rightarrow 提高成本(價格)

天線增益的參數定義

Q6: Please list the antenna gain formula with the definition of parameters, and state the reasons for using the modulation and up-conversion mixer in the transmitter

$$G = \eta \frac{4\pi A_e}{\lambda^2} = \eta \left(\frac{\pi D f_c}{c} \right)^2$$

η = surface efficiency

A_e = Antenna aperture $\frac{\pi D^2}{4}$

D = diameter of parabolic dish ant.

$$G = \eta \left(\frac{\pi D f_c}{c} \right)^2$$

$\therefore D \uparrow, f_c \uparrow, G \uparrow$

不用使用大的天線(D), 藉由升頻混波器提高載波頻率(f_c)即可達到增加 antenna gain (G)的目的

Q8] Please state three major system parameters of mobile communication system.

Data rate, Latency, Number of connections

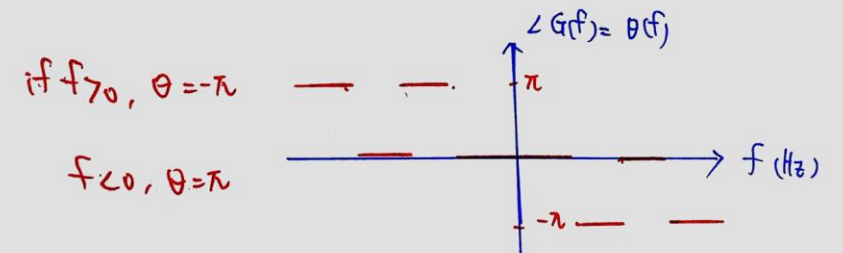
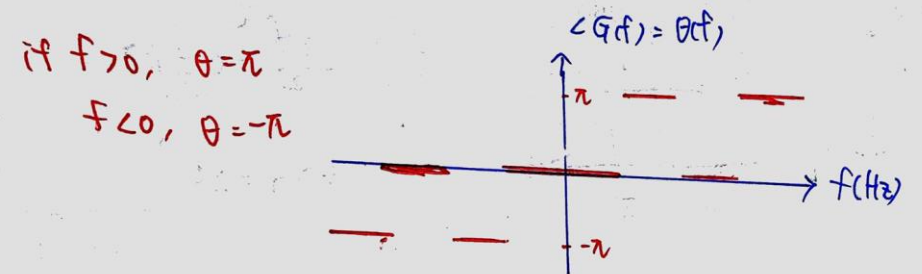
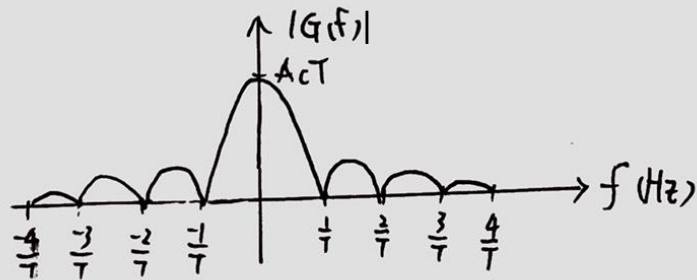
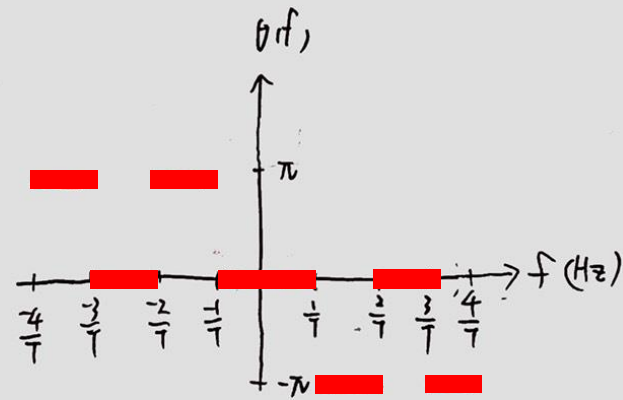
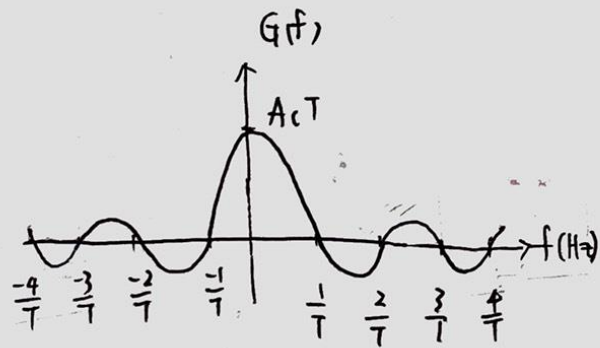
Qq: What are the future applications of 5G? Why 5G can be applied for intelligent transportation system?

1. 雲端、大數據
2. 車聯網(自駕車)
3. 工業4.0

課後問答2

Q1: Determine the Fourier transform of $g(t) = A_c \text{rect}(\frac{t}{T})$ and draw its amplitude spectrum and phase spectrum. Assume that $\theta(f) = -\pi$, if $G(f)$ is less than zero and $f > 0$

$$G(f) = A_c T \text{sinc}(fT)$$



Q2: (1) Determine the spectrum of rectangular pulse $g(t) = A_c \text{rect}(\frac{t}{T})$

$$G(f) = A_c T \text{sinc}(Tf)$$

(2) Determine the bandwidth of rectangular pulse

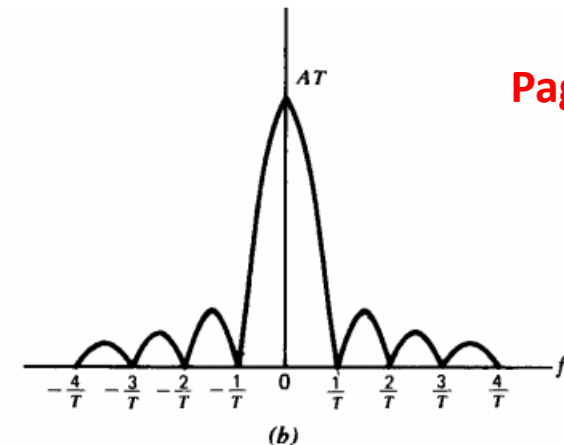
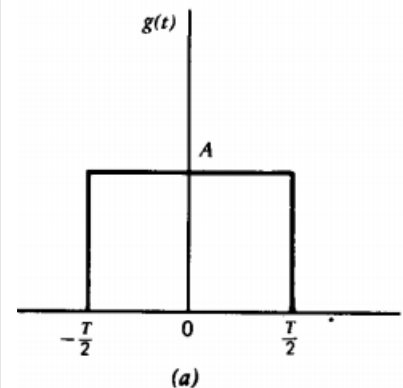
$$\text{B.W} = \frac{1}{T} \text{ (Hz)}$$

Bandwidth

(a) Null to null bandwidth \equiv B.W. provides a measure of the extent of significant spectral content of the signal for positive frequency.

* in Fig 2.2 the rectangular pulse ,B.W.=1/T.

* in Fig 2.8 RF pulse B.W.=2/T.



Q3: (1) Determine the spectrum of RF pulse $g(t) = \text{rect}(\frac{t}{T}) \cos 2\pi f_c t$

$$g(t) = \text{rect}(\frac{t}{T}) \cos 2\pi f_c t$$

$$\therefore \cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \quad (\text{Euler's Formula})$$

$$\therefore \cos(2\pi f_c t) = \frac{1}{2} (e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$

$$\Rightarrow g(t) = \frac{1}{2} \text{rect}(\frac{t}{T}) (e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$

$$= \frac{1}{2} e^{j2\pi f_c t} \text{rect}(\frac{t}{T}) + \frac{1}{2} e^{-j2\pi f_c t} \text{rect}(\frac{t}{T})$$

① P1-13 Time Scaling 性質

$$g(at) \xleftrightarrow{FT} \frac{1}{|a|} G(\frac{f}{a})$$

$$\Rightarrow \text{rect}(\frac{t}{T}) \xleftrightarrow{FT} T \text{sinc}(fT)$$

② P1-15 Frequency Shifting 性質

$$e^{j2\pi f_c t} g(t) \xleftrightarrow{FT} G(f - f_c)$$

$$\Rightarrow e^{j2\pi f_c t} \text{rect}(\frac{t}{T}) \xleftrightarrow{FT} T \text{sinc}[T(f - f_c)]$$

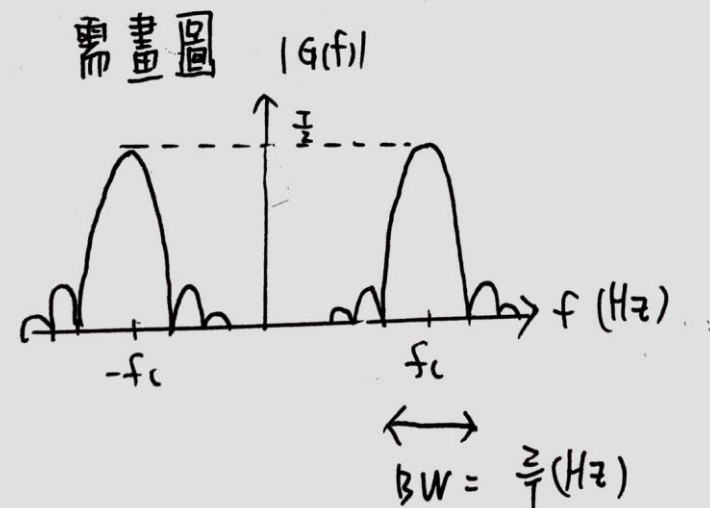
By ①, ②

$$G(f) = \frac{1}{2} T \text{sinc}[T(f - f_c)] + \frac{1}{2} T \text{sinc}[T(f + f_c)]$$

$$\Rightarrow G(f) = \begin{cases} \frac{T}{2} \text{sinc}[T(f - f_c)] & , f > 0 \\ 0 & , f = 0 \\ \frac{T}{2} \text{sinc}[T(f + f_c)] & , f < 0 \end{cases}$$

12) Bandwidth of RF pulse

需畫圖



Q4: Determine the spectrum of $g(t) = A_c \cos(2\pi f_c t)$

$$\therefore \cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \quad (\text{Euler's formula})$$

$$\therefore \cos(2\pi f_c t) = \frac{1}{2} (e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$

$$\Rightarrow g(t) = A_c \cdot \frac{1}{2} (e^{j2\pi f_c t} + e^{-j2\pi f_c t})$$

① P 1-15 Frequency Shifting 性質

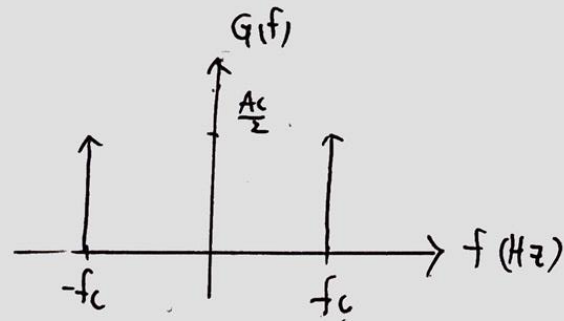
$$e^{j2\pi f_c t} g(t) \xleftrightarrow{FT} G(f - f_c) \quad e^{-j2\pi f_c t} g(t) \xleftrightarrow{FT} G(f + f_c)$$

② P 1-23

$$1 \xleftrightarrow{FT} \delta(f)$$

By ①, ②

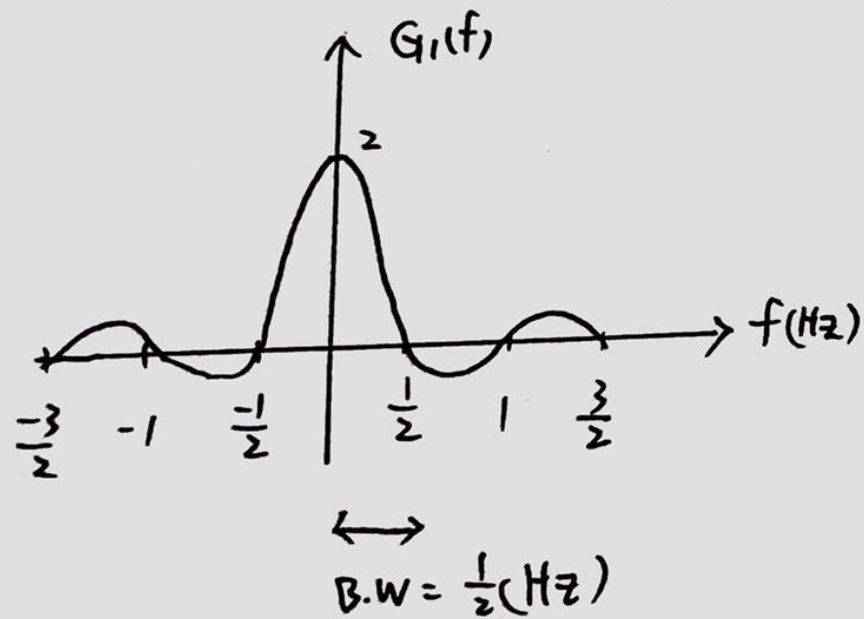
$$G(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)]$$



Q5: Determine the spectrum and bandwidth of (1) $g_1(t) = \text{rect}(\frac{t}{2})$
 (2) $g_2(t) = \text{rect}(2t)$

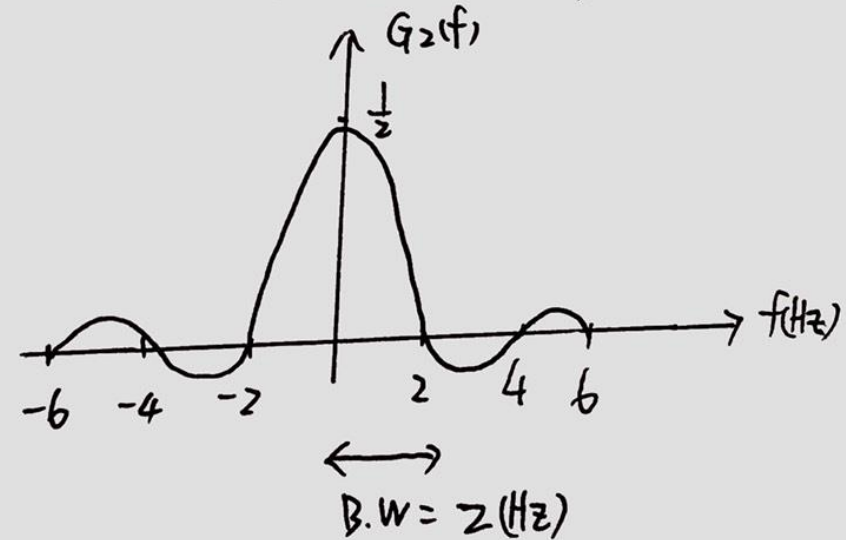
(1)

$$G_1(f) = 2 \text{sinc}(2f)$$



(2)

$$G_2(f) = \frac{1}{2} \text{sinc}(\frac{f}{2})$$

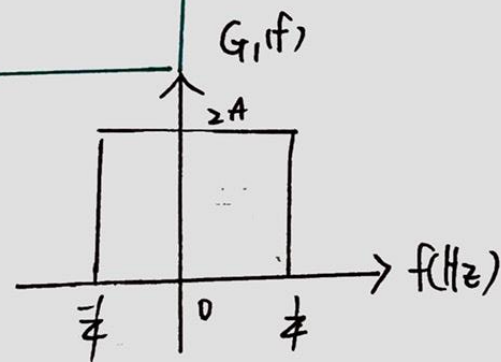


Q6 Determine and draw the spectrum of (1) $g_1(t) = A \operatorname{sinc}(\frac{t}{2})$
 (2) $g_2(t) = A \operatorname{sinc}(2t)$

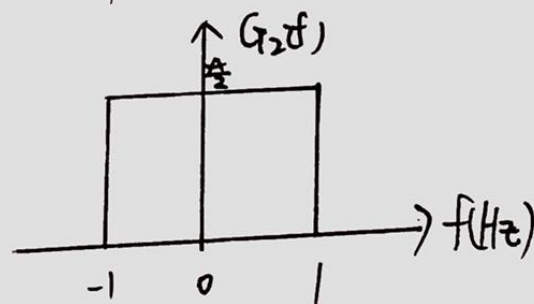
P 1-14

$$A \operatorname{sinc}(t) \xleftrightarrow{FT} A \operatorname{rect}(-f) = A \operatorname{rect}(f) \quad (\text{偶对称})$$

$$G_1(f) = 2A \operatorname{rect}(2f)$$



$$G_2(f) = \frac{A}{2} \operatorname{rect}\left(\frac{f}{2}\right)$$



Q7 (1) If the total energy of $g(t)$ is, please prove $E_t = \int_{-\infty}^{\infty} |G(f)|^2 df$

See 1-20



(2) If $g(t) = \text{sinc}(Bt)$, please determine the total energy of $g(t)$,

$$G(f) = \frac{1}{B} \text{rect}\left(\frac{f}{B}\right)$$

$$= \frac{1}{B}, \quad -\frac{B}{2} \leq f \leq \frac{B}{2}$$

$$E_t = \int_{-\infty}^{\infty} |G(f)|^2 df = \frac{1}{B^2} \int_{-\frac{B}{2}}^{\frac{B}{2}} df = \frac{1}{B} \text{ (watt-sec)}$$

The total energy delivered by the voltage source Page: 1-20

$$E_t \equiv \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df \quad v(t) = i(t) = g(t)$$

Proof:

$$E_t \equiv \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} g(t) g^*(t) dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df \right] g^*(t) dt$$

$$= \int_{-\infty}^{\infty} G(f) \left[\int_{-\infty}^{\infty} g^*(t) e^{j2\pi ft} dt \right] df$$

$$= \int_{-\infty}^{\infty} G(f) \left[\int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \right]^* df$$

$$= \int_{-\infty}^{\infty} G(f) G^*(f) df$$

$$= \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$\therefore E_t = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

Q8: The signal $s(t) = \text{sinc}(3t)$ inputs LTI system impulse response $h(t)$
 $= \text{sinc}(2t)$

(1) Determine the output spectrum of $y(t)$

$$\begin{cases} y(t) = s(t) * h(t) & \text{時域做 convolution} \\ Y(f) = S(f) H(f) & \rightarrow \text{頻域做相乘} \end{cases}$$

$$\begin{aligned} S(f) &= \frac{1}{3} \text{rect}\left(\frac{f}{3}\right) \\ &= \frac{1}{3}, \quad -\frac{3}{2} \leq f \leq \frac{3}{2} \end{aligned}$$

$$\begin{aligned} H(f) &= \frac{1}{2} \text{rect}\left(\frac{f}{2}\right) \\ &= \frac{1}{2}, \quad -1 \leq f \leq 1 \end{aligned}$$

$$\Rightarrow Y(f) = \frac{1}{6}, \quad -1 \leq f \leq 1 \quad (\text{取交集})$$

$$= \frac{1}{6} \text{rect}\left(\frac{f}{2}\right) \#$$

(2) Determine the output signal $y(t)$

$$y(t) = \frac{1}{3} \text{sinc}(2t)$$

Q9: If $g(t)$ is a complex signal (1) Determine the spectrum of $\text{Re}\{g(t)\}$ and $j\text{Im}\{g(t)\}$

If $g(t)$ is a complex signal

$$g(t) \longleftrightarrow G(f)$$
$$g^*(t) \longleftrightarrow G^*(-f)$$

$$g(t) = \text{Re}\{g(t)\} + j\text{Im}\{g(t)\} \quad (1)$$

$$g^*(t) = \text{Re}\{g(t)\} - j\text{Im}\{g(t)\} \quad (2)$$

$$(1) + (2) \Rightarrow \text{Re}\{g(t)\} = \frac{1}{2} [g(t) + g^*(t)]$$

$$(1) - (2) \Rightarrow \text{Im}\{g(t)\} = \frac{1}{2j} [g(t) - g^*(t)]$$

$$\times j \hookrightarrow j\text{Im}\{g(t)\} = \frac{1}{2} [g(t) - g^*(t)]$$

$$\Rightarrow \text{Re}\{G(f)\} = \frac{1}{2} [G(f) + G^*(-f)]$$

$$\text{Im}\{G(f)\} = \frac{1}{2j} [G(f) - G^*(-f)] \quad \#$$

(2) Prove the spectrum of $\text{Re}\{g(t)\}$ is even symmetric and the spectrum of $j\text{Im}\{g(t)\}$ is odd symmetric

$$\text{Re}\{G^*(-f)\} = \frac{1}{2} [G^*(-f) + G(f)] = \text{Re}\{G(f)\} \quad \left(\begin{array}{c} \text{Even} \\ \text{Symmetry} \end{array} \right)$$

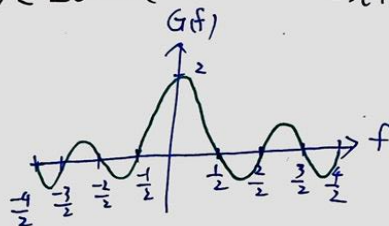
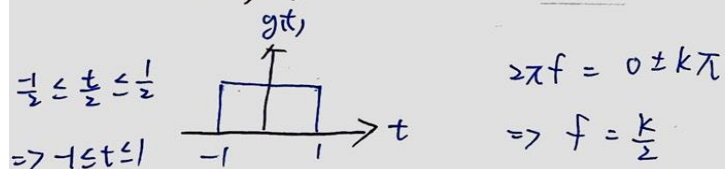
$$\text{Im}\{G^*(-f)\} = \frac{1}{2j} [G^*(-f) - G(f)] = -\text{Im}\{G(f)\} \quad \left(\begin{array}{c} \text{Odd} \\ \text{Symmetry} \end{array} \right)$$

★ $g(t) = A \text{rect}\left(\frac{t}{T}\right) \longleftrightarrow G(f) = AT \text{sinc}(fT)$

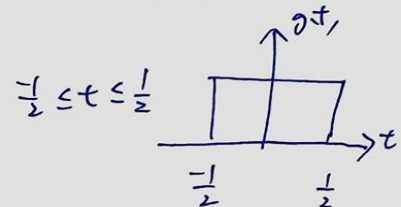
* 時域壓縮，頻域擴張

★ $g(t) = \text{rect}(at) \longleftrightarrow G(f) = \frac{1}{a} \text{sinc}\left(\frac{f}{a}\right)$

① $a = \frac{1}{2}$, $g(t) = \text{rect}\left(\frac{t}{2}\right) \longleftrightarrow G(f) = 2 \text{sinc}(2f) = \frac{2 \sin(2\pi f)}{2\pi f}$



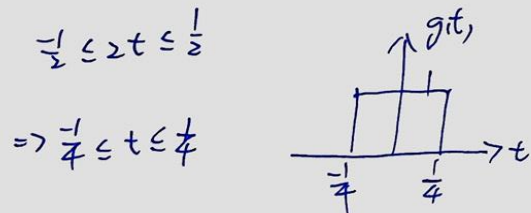
② $a = 1$, $g(t) = \text{rect}(t) \longleftrightarrow G(f) = \text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$



$\pi f = 0 \pm k\pi \Rightarrow f = k$



③ $a = 2$, $g(t) = \text{rect}(2t) \longleftrightarrow G(f) = \frac{1}{2} \text{sinc}\left(\frac{f}{2}\right) = \frac{1}{2} \frac{\sin(\frac{\pi f}{2})}{\frac{1}{2}\pi f}$



$\frac{\pi f}{2} = 0 \pm k\pi \Rightarrow f = 2k$

