CSCI 3278 Homework 2

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3.2.7
Prove that $(X^+)^+ = (X^+)$

Assume for the sake of contradiction that $(X^+)^+ \neq X^+$, meaning that there is at least ones more attribute that may be added to the set of attributes $\{A_1, A_2 ... A_n\}$ in X. However, X^+ is already a closure of X meaning that all true FD's have been claimed by the closure of X. Because all true FD's have been claimed by X^+, no further FD's will be claimed by $(X^+)^+$. This is a contradiction, proving that $(X^+)^+ = X^+$.

3.3.1 b)
Indicate all the BCNF violations and decompose the relations into collections of relations that are in BCNF.

R(A,B,C,D) with FD's B→C and B→D

Using BCNF Decomposition Algorithm 3.20 from textbook:

1) Check whether R is in BCNF:

   FD's = B→C and B→D
   $B^+ = \{B, C, D\}$ which is a BCNF violation.

2) If there are BCNF violations, let one of them be X→Y. Compute X^+. Choose $R_1 = X^+$ and let R_2 have attribute X and attributes of R that are not in X^+.

Using B$\rightarrow$C,
$B^+ = \{B, C, D\}$
$R_1 = \{B, C, D\}$ and $R_2 = \{B, A\}$

3) Compute sets of FD's for R_1 and R_2; let them be S_1 and S_2 respectively

   $S_1 = \{B \rightarrow C, C \rightarrow D\}$ and $S_2 = \{\}$

4) Recurse over R_1 and R_2. Return union of results of decomposition.

   R_2 is in BCNF because it has 2 attributes.
   R_1 has FD's B$\rightarrow$C and B$\rightarrow$D
   B$^+ = \{B, C, D\}$ which is no longer a BCNF violation because the left side of the FD's are now super keys.

   Therefore $\mathbf{R}_1 = \{B, C, D\}$ and $\mathbf{R}_2 = \{B, A\}$

3.5.1 b)
Indicate all the 3NF violations and decompose into collections of relations that are in 3NF.

R(A,B,C,D) with FD's B$\rightarrow$C and B$\rightarrow$D

1) Determine super key of relation.

   AB$^+ = \{A, B, C, D\}$

2) Examine FD's of R.

   B$\rightarrow$C: B is not a super key and C is not prime.
   B$\rightarrow$D: B is not a super key and D is not prime.
   These are both 3NF violations.

3) Decompose based on B$\rightarrow$C

$R_1 = \{B, C\}$ $R_2 = \{B, A, D\}$
R_1 fits 3NF because it contains 2 attributes.

4) Determine super key of new relation R_2

$BA^+ = \{B, A, D\}$

5) Examine FD's of R_2

$B \to D$: B is not a super key and D is not prime. This is a 3NF violation.

6) Decompose again based on $B \to D$
$R_3 = \{B, D\}$ and $R_4 = \{B, A\}$
Now we have 3 relations with 2 attributes each which all fit 3NF.

Therefore $\mathbf{R_1}=\{B, C\}$, $\mathbf{R_3}=\{B, D\}$ and $\mathbf{R_4}=\{B, A\}$