IEOR 240 Case 2
Giant Motor Company: Final Report
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FORMULATION

Currently, Giant Motor Company (GMC) has three manufacturing plants in the United States. Each plant is dedicated to producing a single type of cars. In its planning for the coming year, GMC is considering the retooling of its Lyra and/or Libra plants. The linear program developed in this case is intended to provide a recommendation for the retooling, as well as its production plan at each plan in the coming year. First, the following assumptions are necessary for defining the problems presented in the case as well as developing the linear program:

Assumptions:

1. Assuming the goal of Giant Motor Company is to maximize its profit for the coming year.
2. Assuming in the retooled factories, the production capacity of all types of cars are interchangeable, meaning that any combination of production is possible as long as the total number of production is less than or equal to the overall capacity.
3. Assuming the retooling of factories will be completed before the start of the next fiscal year and interruption of production in the current year can be ignored.
4. Assuming all the cars produced within the demand can be sold.
5. Assuming if the plant is not operating, there's is still fixed cost incurred at that plant, as fixed cost consists of mostly property tax, insurance, payments on the loan that was taken out to construct the plant and etc.

Parameters:

Below is a list of all relevant parameters of this linear program:

- M, number of types of cars M = 3 (Lyras, Libras, Hydras)
- N, types of manufacturing plants N = 5 (Lyras, Libras, Hydras, New Lyra, New Libra)
- $Capacity_n$, n = 1...N, the capacity(unit of cars) of production for manufacturing plant n.

$$Capacity_1 = 1000,000$$
$$Capacity_2 = 800,000$$
$$Capacity_3 = 900,000$$
$$Capacity_4 = 1600,000$$
$$Capacity_5 = 1800,000$$

- FC_n , n = 1...N, fixed cost (in dollars) for manufacturing plant n.

$$FC_1 = 2000,000,000$$
$$FC_2 = 2000,000,000$$
$$FC_3 = 2600,000,000$$
$$FC_4 = 3400,000,000$$
$$FC_5 = 3700,000,000$$

- PM_{mn}, m = 1...M, n = 1...N, profit margin (in dollars) for producing car type m at manufacturing plant n.

$$PM_{11} = 2000$$
$$PM_{12} = 0$$
$$PM_{13} = 0$$
$$PM_{14} = 2500$$
$$PM_{15} = 2300$$
$$PM_{21} = 0$$
$$PM_{22} = 3000$$
$$PM_{23} = 0$$
$$PM_{24} = 3000$$
$$PM_{25} = 3500$$
$$PM_{31} = 0$$
$$PM_{32} = 0$$
$$PM_{33} = 5000$$
$$PM_{34} = 0$$
$$PM_{35} = 4800$$

- $Demand_m$, m = 1...M, demand for car type(unit of cars) m

$$Demand_1 = 1400,000$$
$$Demand_2 = 1100,000$$
$$Demand_3 = 800,000$$

- $Diversion_{mm}$, m = 1...M, demand diversion percentage of unsatisfied demand for car type m be diverted to demand for car type m.

$$Diversion_{11} = 0$$
$$Diversion_{12} = 0.3$$
$$Diversion_{13} = 0.05$$
$$Diversion_{21} = 0$$
$$Diversion_{22} = 0$$
$$Diversion_{23} = 0.1$$
$$Diversion_{31} = 0$$
$$Diversion_{32} = 0$$
$$Diversion_{33} = 0$$

Decision Variable:

Below is a list of decision variables that are used in this linear program:

- X_{mn} , m = 1...M, n = 1...N, number of car type m to produce at manufacturing plant n.
- $Y_n = \begin{cases} 1 & \textit{if manufacturing plant n operates} \\ 0 & \textit{otherwise} \end{cases}$, n = 1...N
- A_m, m = 1...M, number of unsatisfied demand for car type m.

Objective Function:

The objective for this linear program is to maximize total profit with a particular production plan, as well as the decision for retooling, which is given by:

$$\text{Max } \sum_{m=1}^{M} \sum_{n=1}^{N} X_{mn} * PM_{mn} - \sum_{n=1}^{N} FC_n * Y_n$$

Constraints:

The linear program is subjected to the following constraints:

1. GMC needs to decide whether to retool the Lyra plant; if retooling, $Y_4 = 1, Y_1 = 0$
$$Y_1 + Y_4 = 1$$

2. GMC needs to decide whether to retool the Libra plant; if retooling, $Y_5 = 1, Y_2 = 0$
$$Y_2 + Y_5 = 1$$

3. Make sure the fixed cost of the Hydra plant is taking into consideration
$$Y_3 = 1$$

4. GMC cannot produce more cars than the total capacity at each manufacturing plant n.
$$\sum_{m=1}^{M} X_{mn} \leq Capacity_n * Y_n, \ n = 1...N$$

5. The number of unsatisfied demand for Lyra equals to demand minus units sold.
$$\sum_{n=1}^{N} X_{1n} + A_1 = Demand_1$$

6. The number of unsatisfied demand for Libra equals to the original unsatisfied demand plus the diverted demand from Lyra.

$$\sum_{n=1}^{N} X_{2n} + A_2 = Demand_2 + Diversion_{12} * A_1$$

7. The number of unsatisfied demand for Hydra equals the original unsatisfied demand plus the diverted demand from Lyra and Libra.

$$\sum_{n=1}^{N} X_{3n} + A_3 = Demand_3 + Diversion_{13} * A_1 + Diversion_{23} * A_2$$

8. Libra and Hydra cannot be produced at current Lyra manufacturing plant

$$X_{21} = X_{31} = 0$$

9. Lyra and Hydra cannot be produced at current Libra manufacturing plant

$$X_{12} = X_{32} = 0$$

10. Lyra and Libra cannot be produced at current Hydra manufacturing plant

$$X_{13} = X_{23} = 0$$

11. Hydra cannot be produced at new Lyra manufacturing plant

$$X_{34} = 0$$

12. Non-negativity of decision variables X

$$X_{mn} \geq 0, m = 1...M, n = 1...N$$

13. Binary nature of decision variable Y

$$Y_n \in \{0,1\}, n = 1...N$$

14. Non-negativity of decision variables A

$$A_m \geq 0, m = 1...M$$

SOLUTIONS

The optimal solution is to retool Lyra Plant but not Libra Plant. The recommended production plan is 1,600,000 at New Lyra Plant, 800,000 at Libra Plant, and 807,143 at Hydra Plant. The maximum profit (optimal objective value) will be $ 2,607,140,000. Production by cars would be 1,257,150 Lyra, 1,142,850 Libra, and 807,143 Hydra. Table 1 presents productions by plants and by cars.

Table 1: Production by Plant and Car Types

Cars	New Lyra Plant	Old Libra Plant	Hydra Plant	Car Production
Lyra	1,257,150	0	0	1,257,150
Libra	342,850	800,000	0	1,142,850
Hydra	0	0	807,143	807,143
Plant Production	1,600,000	800,000	807,143	

DISCUSSIONS
Diversion
Based on projected demand and the optimal amount of production, unsatisfied demand for Lyra is 142,850. As 30% of unsatisfied demand for Lyra is diverted to the demand for Libras, 42,855 (=142,850*0.3) will materialize as demand for Libras. Five percents of unsatisfied demand for Lyra is diverted to the demand for Hydra; hence, 7,143 (=142,850*0.05) will materialize as demand for Hydra. There will be no unsatisfied demand for Libra and Hydra.

Even though diversion helps GMC achieve the optimal profitability, it has negative impacts on GMC's market share. In other words, GMC **loses market share to its competitors**.

Table 2: Diversion by Car Types

Cars	Projected Demand	Unsatisfied Demand	Diversion
Lyra	1,400,000	142,850	
Libra	1,100,000		42,855
Hydra	800000		7,143

Profitability
In order to discuss the profitability of the solutions, clarification is needed on the parameters and terminologies.

The **Profit Margin** provided in the case prompt is, in fact, the **Contribution Margin**, which is the selling price of each car minus the variable cost. To maintain consistency with the case prompt, the word profit margin and contribution margin will be used interchangeably in this report. The product of profit margin and unit sold will give us the **Profit before Fixed Cost** because the profit margin only takes into account the variable cost.

Table 2 lists the calculation of how much Profit before Fixed Cost is generated by each car type. Interestingly, although Hydra has the lowest sales volume, it generates the most Profit before Fixed Cost, because its Profit Margin is much higher than the other two, 100% higher than Lyra and 66.67% higher than Libra.

Table 2: Profitability of each car type.

Car Type	Production (Unit Sold)	Profit Margin ($)	Profit before Fixed Cost ($)
Lyra	1,257,150	2,500	3,142,875,000
Libra	1,142,850	3,000	3,428,550,000
Hydra	807,143	5,000	4,035,715,000
Total Profit before Fixed Cost ($)			10,607,140,000

Table 3 lists the calculation of how much Overall Profit is generated by each factory. The Profit before Fixed Cost shows the combined number from all types of cars manufactured at each factory. Although the Hydra plant and the new Lyra plant have similar Profit before Fixed Costs, the Hydra plan has a much higher Overall Profit after considering Fixed Cost. This is not surprising, because the Fixed Cost of the new Lyra plant takes into account the cost of the retooling, whereas the Hydra plant does not have such cost because it is not retooled.

Table 3: Profitability of each factory.

Factory	Profit before Fixed Cost ($)	Fixed Cost ($)	Overall profit ($)
Old Libra Plant	2,400,000,000	2,000,000,000	400,000,000
Hydra Plant	4,035,715,000	2,600,000,000	1,435,715,000
New Lyra Plant	4,171,425,000	3,400,000,000	771,425,000
Total	10,607,140,000	8,000,000,000	2,607,140,000

SUMMARY

Based on the prompt and assumptions, GMC should retool the Lyra Plant and follow the production plan listed in Table 1. Although this production plan will optimize profit, GMC will lose part of the Lyra market share to its competitors.

APPENDIX

1. Model File

```
# Parameters
param M; # Number of lines of cars (Lyras, Libras, and Hydras)
param N; # Types of manufacturing plant (Lyras, Libras, Hydras, New
Lyra, and New Libra)
param Capacity{n in 1..N}; # The capacity of production for
manufacturing plant n.
param FC{n in 1..N}; # Fixed cost (in dollars) for manufacturing plant
n.
param PM{m in 1..M, n in 1..N}; # Profit margin (in dollars) for
producing car lien m at manufacturing plant n.
param Demand{m in 1..M}; # Demand for car line m.
param Diversion{m in 1..M, p in 1..M}; # Demand diversion percentage
of unsatisfied demand for car line m be diverted to demand for car
line p.

# Decision Variable
var X{m in 1..M, n in 1..N} integer >= 0; # Number of car line m to
produce at manufacturing plant n.
var Y{n in 1..N} integer >= 0, <= 1; # 1 if manufacturing plant n
operates, 0 otherwise
var A{m in 1..M} integer >= 0;; # Number of unsatisfied demand for car
line m.

# Objective Function
maximize totalprofit: # Maximize total profit (dollar) for the coming
year
	sum{m in 1..M, n in 1..N} X[m,n]*PM[m,n] - sum{n in
1..N}FC[n]*Y[n];

# Constraints
subject to if_retool_Lyra_plant: # Need to decide whether to retool
the Lyra plant, if retool, Y4=1,Y1=0
	Y[1] + Y[4] = 1;

subject to if_retool_Libra_plant: # Need to decide whether to retool
the Libra plant, if retool, Y5=1,Y2=0
	Y[2] + Y[5] = 1;

subject to Hydra_plant_fixed_cost: # Make sure the fix cost of the
Hydra plant is taking into consideration
```

```
    Y[3] = 1;

subject to capacity_limit {n in 1..N}: # Cannot manufacture cars
exceed the total capacity at each manufacturing plant n. Can only
manufacture cars at plant n when the manufacturing plant n operates.
    sum{m in 1..M} X[m,n] <= Capacity[n]*Y[n];

subject to demand_diversion_Lyra: # Number of unsatisfied demand for
Lyra equals to demand minus units sold.
    sum{n in 1..N} X[1,n] + A[1] = Demand[1];

subject to demand_diversion_Libra: # Number of unsatisfied demand for
Libra equals to the original unsatisfied demand plus the diverted
demand from Lyra.
    sum{n in 1..N} X[2,n] + A[2] = Demand[2] + Diversion[1,2] * A[1];

subject to demand_diversion_Hydra: # Number of unsatisfied demand for
Hydra equals to the original unsatisfied demand plus the diverted
demand from Lyra and Libra.
    sum{n in 1..N} X[3,n] + A[3] = Demand[3] + Diversion[1,3] * A[1]
+ Diversion[2,3] * A[2];

subject to line_limit_Lyra_1: # Libra and Hydra cannot be produced at
current Lyra manufacturing plant
    X[2,1] = 0;

subject to line_limit_Lyra_2: # Libra and Hydra cannot be produced at
current Lyra manufacturing plant
    X[3,1] = 0;

subject to line_limit_Libra_1: # Lyra and Hydra cannot be produced at
current Libra manufacturing plant
    X[1,2] = 0;

subject to line_limit_Libra_2: # Lyra and Hydra cannot be produced at
current Libra manufacturing plant
    X[3,2] = 0;

subject to line_limit_Hydra_1: # Lyra and Libra cannot be produced at
current Hydra manufacturing plant
    X[1,3] = 0;

subject to line_limit_Hydra_2: # Lyra and Libra cannot be produced at
current Hydra manufacturing plant
```

```
      X[2,3] = 0;

subject to line_limit_new_Lyra: # Hydra cannot be produced at new Lyra
manufacturing plant
      X[3,4] = 0;
```

2. Data File

```
param M := 3; # Number of lines of cars (Lyras, Libras, and Hydras)
param N := 5; # Types of manufacturing plant (Lyras, Libras, Hydras,
New Lyra, and New Libra)
param Capacity :=
      1 1000000
      2 800000
      3 900000
      4 1600000
      5 1800000; # The capacity of production for manufacturing plant
n.
param FC :=
      1 2000000000
      2 2000000000
      3 2600000000
      4 3400000000
      5 3700000000; # Fixed cost (in dollars) for manufacturing plant
n.
param PM :=
      1 1 2000
      1 2 0
      1 3 0
      1 4 2500
      1 5 2300
      2 1 0
      2 2 3000
      2 3 0
      2 4 3000
      2 5 3500
      3 1 0
      3 2 0
      3 3 5000
      3 4 0
      3 5 4800; # Profit margin (in dollars) for producing car lien m
at manufacturing plant n.
param Demand :=
```

```
      1 1400000
      2 1100000
      3 800000; # Demand for car line m.
param Diversion :=
      1 1 0
      1 2 0.3
      1 3 0.05
      2 1 0
      2 2 0
      2 3 0.1
      3 1 0
      3 2 0
      3 3 0; # Demand diversion percentage of unsatisfied demand for car line m be diverted to demand for car line m.
```

3. Solution File

```
:     _objname        _obj        _varname     _var       :=
1    totalprofit   2607140000    'X[1,1]'         0
2         .             .        'X[1,2]'         0
3         .             .        'X[1,3]'         0
4         .             .        'X[1,4]'   1257150
5         .             .        'X[1,5]'         0
6         .             .        'X[2,1]'         0
7         .             .        'X[2,2]'     8e+05
8         .             .        'X[2,3]'         0
9         .             .        'X[2,4]'    342850
10        .             .        'X[2,5]'         0
11        .             .        'X[3,1]'         0
12        .             .        'X[3,2]'         0
13        .             .        'X[3,3]'    807143
14        .             .        'X[3,4]'         0
15        .             .        'X[3,5]'         0
16        .             .        'Y[1]'           0
17        .             .        'Y[2]'           1
18        .             .        'Y[3]'           1
19        .             .        'Y[4]'           1
20        .             .        'Y[5]'           0
21        .             .        'A[1]'      142850
22        .             .        'A[2]'           5
23        .             .        'A[3]'           0
;
```