Exercises on matrix spaces; rank1; small world graph

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11.1

$$I = \begin{bmatrix}100\\010\\001\end{bmatrix} = \begin{bmatrix}010\\100\\001\end{bmatrix} - \begin{bmatrix}010\\001\\100\end{bmatrix} + \begin{bmatrix}100\\001\\010\end{bmatrix} - \begin{bmatrix}001\\100\\010\end{bmatrix} + \begin{bmatrix}001\\010\\100\end{bmatrix}$$

If $c_1P_1 + ... + c_5P_5 = 0$, then $\begin{bmatrix}c_3 \; c_1 - c_2 \; c_5 - c_4\\ c_1 - c_4 \; c_5 \; c_3 - c_2\\ c_5 - c_2 \; c_3 - c_4 \; c_1\end{bmatrix} = 0$. It leads to $c_1 = c_2 = c_3 = c_4 = c_5 = 0$. Therefore, the five permutation matrices are linearly independent because the only combination of the matrices that gives the zero matrix is the zero matrix itself.

11.2

(a) $A = \begin{bmatrix}1\;0\;-1\\ -1\;1\;0\\ 0\;-1\;1\end{bmatrix} \rightarrow \begin{bmatrix}1\;0\;-1\\ 0\;1\;-1\\ 0\;0\;0\end{bmatrix}$

The rank of A is 2. The special solution to $Ax = 0$ is $\begin{bmatrix}1\\1\\1\end{bmatrix}$. Therefore, X have columns that are multiples of $\begin{bmatrix}1\\1\\1\end{bmatrix}$. X is in the form of $\begin{bmatrix}a\;b\;c\\a\;b\;c\\a\;b\;c\end{bmatrix}$.

(b) For X to have the form of AX, X has to be a combination of columns of A. Each column of A sums up to 0, so each column of X also sums up to 0. X is in the form of
$\begin{bmatrix}a\;b\;c\\ d\;e\;f\\ -a-d\; -b-e\; -c-f\end{bmatrix}$.

(c) The dimension of the the nullspace $(n-r)$of A is 3 and the dimension of the column space r is 6. They add up to 9 which is the dimension of M.