Unit 2 Exam

January 16, 2017

1

(a) ± 1

Orthogonal matrix has determinant of ± 1.

(b) The determinant is a linear function of each row; therefore,

$$det \begin{bmatrix} q_1 + q_2 & q_2 + q_3 & a_3 + q_1 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} + \begin{bmatrix} q_2 & q_3 & q_1 \end{bmatrix} = \pm 2$$

(c) The second term is the same as the first term because it requires two row exchanges (changes sign twice), so the product is always 1.

2

(a) We can rewrite the problem as

$$\begin{bmatrix} 1 & -10 \\ 1 & -9 \\ & \vdots \\ 1 & 0 \\ & \vdots \\ 1 & 10 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

Multiply both sides by A^T

$$\begin{bmatrix} 21 & 0 \\ 0 & 770 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} \frac{1}{21} \\ 0 \end{bmatrix}$$

(b) I am projecting the matrix onto the column space of above.

The two bases are: $\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -10 \\ -9 \\ \vdots \\ 0 \\ \vdots \\ 10 \end{bmatrix}$

The error vector is perpendicular to the subspace.

$$e = b - Pb = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{21} \\ \vdots \\ \frac{1}{21} \end{bmatrix} = \begin{bmatrix} -\frac{1}{21} \\ \vdots \\ \frac{20}{21} \\ \vdots \\ -\frac{1}{21} \end{bmatrix}$$

3

(a)

$$P_A = A(A^TA)^{-1}A^T$$
$$P_Q = Q(Q^TQ)^{-1}Q^T$$

(b) Because both projection matrices project onto the same subspace. P_QQ is the projection of Q on to the column space of Q. Determinant of P_Q is zero because there are only 3 independent vectors in R^5 aka P_Q is singular.

(c) Choice 3

4

(a) Since the determinant can only come from one entry per column per row, the largest possible degree of polynomial is two.

(b) det $A = x(1) - x(x) + x(-x) - x(-1)(-x) = x - 3x^2$

When x = 3 or x = $\frac{1}{3}$, det $A = 0$.