

## Exercises on Column Space and Nullspace

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**6.1**

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(a) Suppose there is another vector in the space  $S + T$ :  $s' + t'$  where  $s'$  is in  $S$  and  $t'$  is in  $T$ .  $(s + t) + (s' + t') = (s + s') + (t + t')$  and  $c(s + t) = cs + ct$ .  $S + T$  is a vector space because it satisfies the two requirements.

(b) If  $S$  and  $T$  are lines, then  $S + T$  is a plane and  $S \cup T$  is two lines. The plane contains all vectors in the two lines.  $S + T$  contains all combinations of the vectors in  $S$  and  $T$ . Therefore,  $S + T$  is a span of  $S \cup T$ .

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**6.2**

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We can rewrite the first equation to  $x = 12 + 3y + z$ . So the solution is 12, 3, 1.

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**6.3**

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$N(C)$  is all the solutions to the equation

$$Nx = \begin{bmatrix} Ax \\ Bx \end{bmatrix} = 0$$

$N(C)$  has to satisfy both  $Ax = 0$  and  $Bx = 0$ . Therefore,  $N(C) = N(A) \cap N(B)$ .