Exercises on Transposes, Permutations, Spaces

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5.1

(a) If we can rotate the three rows of the permutation matrix in one direction, we can get $P^3 = I$. One example is that if $P = \begin{bmatrix} 010 \\ 001 \\ 100 \end{bmatrix}$, then $P^2 = \begin{bmatrix} 000 \\ 100 \\ 010 \end{bmatrix}$ and $P^3 = \begin{bmatrix} 100 \\ 010 \\ 001 \end{bmatrix}$.

(b) If we can exchange the rows in a 4 by 4 matrix in an non-orderly fashion, unlike P in part (a), then we will get $P^4 \neq I$. For example, if $\hat{P} = \begin{bmatrix} 0100 \\ 0001 \\ 0010 \\ 1000 \end{bmatrix}$, then $\hat{P}^2 = \begin{bmatrix} 0001 \\ 1000 \\ 0010 \\ 0100 \end{bmatrix}$, $\hat{P}^3 = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$ and $\hat{P}^4 = \begin{bmatrix} 0100 \\ 0001 \\ 0010 \\ 1000 \end{bmatrix} \neq I$.

5.2

(a) If A is symmetric, then we can represent A by:

$$\begin{bmatrix} a \ b \ c \ d \\ b \ e \ h \ i \\ c \ h \ f \ j \\ d \ i \ j \ g \end{bmatrix}$$

There are 10 independent entries.

(b) If A is skew-symmetric ($A^T = -A$), we can represent A by:

$$\begin{bmatrix} 0 \ a \ b \ c \\ -a \ 0 \ d \ e \\ -b \ -d \ 0 \ f \\ -c \ -e \ -f \ 0 \end{bmatrix}$$

There are 6 independent entries.

5.3

(a) True: With $A^T = A$ and $B^T = B$, it leads us to $(A+B)^T = A^T + B^T = A + B$ and $(cA)^T = cA^T$. The addition and scalar multiplication results are both in the subspace.

(b) True: With $A^T = -A$, it leads to $(A+B)^T = A^T + B^T = -A - B = -(A+B)$ and $(cA)^T = cA^T = -cA$.

(c) False: With $A^T \neq A$ and $B^T \neq B$, it leads us to $(A+B)^T = A^T + B^T \neq A + B$. An example is:

$$\begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 01 \\ 01 \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \end{bmatrix}$$

It does not form a subspace, because the result of addition is a symmetrical matrix.