Exercises on left and right inverses; pseudoinverse

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32.1

$$A_{right}^{-1} = A^T(AA^T)^{-1}$$

$$AA^T = \begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 1\end{bmatrix}$$

$$(AA^T)^{-1} = \frac{1}{2}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$

$$A_{right}^{-1} = \frac{1}{2}\begin{bmatrix}1 & 0\\0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & 0\\0 & 2\\1 & 0\end{bmatrix}$$

32.2

By looking at A, we can see that A is not invertible because the second column is a multiple of the first column. To find the pserdoinverse of A, first we have to find the decomposition of A it self.

$$A^TA = \begin{bmatrix}4 & 8\\3 & 6\end{bmatrix}\begin{bmatrix}4 & 3\\8 & 6\end{bmatrix} = \begin{bmatrix}80 & 60\\60 & 45\end{bmatrix}$$

$$det(\begin{bmatrix}80-\lambda & 60\\60 & 45-\lambda\end{bmatrix}) = (80-\lambda)(45-\lambda) - 360 = \lambda^2 - 125\lambda = 0$$

$$\lambda = 0, 125$$

When $\lambda = 125$, the eiganvector of A^TA is $\begin{bmatrix}0.8\\0.6\end{bmatrix}$. When $\lambda = 0$, the eiganvector is $\begin{bmatrix}0.6\\-0.8\end{bmatrix}$.

$u_1 = Av_1 = \begin{bmatrix}1\\2\end{bmatrix}$ and $u_2 = Av_2 = \begin{bmatrix}2\\-1\end{bmatrix}$

$$A^+ = V\Sigma^+U^T = \begin{bmatrix}0.8 & 0.6\\0.6 & -0.8\end{bmatrix}\begin{bmatrix}\sqrt{125} & 0\\0 & 0\end{bmatrix}\frac{1}{\sqrt{5}}\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix} = \frac{1}{125}\begin{bmatrix}4 & 8\\3 & 6\end{bmatrix}$$