Exercises on independence, basis, and dimension

January 8, 2017

9.1

To find the largest possible number of independent vectors, we can put all vectors in a matrix and then perform elimination to find out the number of pivots.

$$\begin{bmatrix} 1\,1\,1\,0\,0\,0 \\ -1\,0\,0\,1\,1\,0 \\ 0\;-1\,0\;-1\,0\,1 \\ 0\,0\;-1\,0\;-1\;-1 \end{bmatrix} \rightarrow \begin{bmatrix} 1\,1\,1\,0\,0\,0 \\ 0\,1\,1\,1\,1\,0 \\ 0\,0\,1\,0\,1\,1 \\ 0\,0\,0\,0\,0\,0 \end{bmatrix}$$

There are three pivots so there are three independent vectors.

9.2

An obvious basis for the plane is $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$. When it intersects with the xy plane, $z = 0$, and the intersection would be a line. A basis for the line is $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. A basis perpendicular to the plane is the cross product of the two bases, $\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$, because they are all on the plane.