Exercises on diagonalization and powers of A

January 15, 2017

22.1

In order to find the eigenvectors of A, we first need to find the eiganvalues by solving:

$$det(A - \lambda I) = 0$$

$$det \begin{bmatrix} 4-\lambda & 0 \\ 1 & 2-\lambda \end{bmatrix} = det \begin{bmatrix} 4-\lambda & 0 \\ 0 & 2 - lambda \end{bmatrix} = (4-\lambda)(2-\lambda) = 0$$

$$\lambda = 4, 2$$

When $\lambda = 4$, $(A - \lambda I)x = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

When $\lambda = 2$, $(A - \lambda I)x = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Therefore, S has the columns of multiples of the two eiganvectors above.

The matrices that diagonalize A^{-1} is the same because $A^{-1} = S\Lambda^{-1}S^{-1}$.

22.2

To find Λ and S of A, we can solve:

$$det \begin{bmatrix} 0.6-\lambda & 9 \\ 0.4 & 0.1-\lambda \end{bmatrix} = det \begin{bmatrix} 0.6-\lambda & 0.9 \\ 0 & (0.1-\lambda) - \frac{0.36}{0.6-\lambda} \end{bmatrix} = \frac{0.7 \pm \sqrt{1.69}}{2}$$

$$\lambda = 1, -0.3$$

When $\lambda = 1$, $(A - \lambda I)x = \begin{bmatrix} -0.4 & 0.9 \\ 0.4 & -0.9 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 0.9 \\ 0.4 \end{bmatrix}$.

When $\lambda = -0.3$, $(A - \lambda I)x = \begin{bmatrix} 0.9 & 0.9 \\ 0.4 & 0.4 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Therefore, $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -0.3 \end{bmatrix}$ and $S = \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix}$.

As $\Lambda^k \rightarrow \infty, \Lambda \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $S\Lambda^{\infty}S^{-1} \rightarrow \begin{bmatrix} 9 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} (-\frac{1}{13}) \begin{bmatrix} -1 & -1 \\ -4 & 9 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 9 & 9 \\ 4 & 4 \end{bmatrix}$. The columns of this matrix are steady state vectors.