Exercises on Markov matrices; Fourier series

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24.1

(a)

$$\begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & b \\ 0 & 1-b^2 \end{bmatrix}$$

$(1+b)(1-b) < 0 \rightarrow b < -1$ or $b > 1$.

One symmetric matrix that has a negative eiganvalue is $\begin{bmatrix} 1 & -2 \\ 0 & -3 \end{bmatrix}$.

(b) Since one of the pivots is 1 after elimination, the other pivot (eiganvalue) must be negative.

(c) It can't have two negative eigan values because there are only two cases, $b < -1$ or $b > 1$. No matter which case is b in, one of the terms in $(1+b)(1-b)$ is negative and one is positive.

24.2

A belongs to the following classes invertible, orthogonal, permutation, diagonalizable and Markov. A is not a projection because A times itself does not result A.

B belongs to diagonalizable and Markov. B is not invertible because there are dependent columns. B is not orthogonal and is not a permutation matrix. B is a projection because $B^2 = B$.

$QR, S\Lambda S^{-1}, Q\Lambda Q^T$ is possible for A; and $LU, S\Lambda S^{-1}, Q\Lambda Q^T$ is possible for B.

24.3

Since A is a Markov matrix, every column of A sums up to 1.

$$A = \begin{bmatrix} .7 & .1 & .2 \\ .1 & .6 & .3 \\ .2 & .3 & .5 \end{bmatrix}$$

We know that one of A's λ is 1. And the eiganvector when λ is one is the steady state.

$$(A - \lambda I)x = \begin{bmatrix} -.3 & .1 & .2 \\ .1 & -.4 & .3 \\ .2 & .3 & -.5 \end{bmatrix} x = 0 \rightarrow x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

When A is a symmetric Markov matrix, not only every column sums up to one, every row also sums up to one. So when computing the eiganvector from $(A - \lambda I)$, each row of that matrix sums up to zero. Therefore $x_1 = (1, ..., 1)$.