Exercises on Elimination with Matrices

January 4, 2017

2.1

This system of equations in matrix form is:

$$\begin{bmatrix} 2 & 3 \\ 6 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

When using the method of elimination to solve this problem, 3 is the multiple of the first equation to be subtracted from the second equation. Using 2 as the pivot, we can rewrite the matrix system as:

$$\begin{bmatrix} 2 & 3 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Then, it leads to $y = -\frac{1}{2}$ and $x = \frac{13}{4}$.

2.2

Looking at this equation,

$$E \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

it is in the form of $EA = b$.

We can observe that 1) the first row of b equals the first row of A, 2) the second row of b equals the first row of A subtracted from the second row of A, 3) the third row of b equals the second row of A subtracted from the first row of A and 4) the fourth row of b equals the third row of A subtracted from the fourth row of A. Therefore:

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

To reduce it all the way to I, we can eliminate the second column by multiplying the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

and b, to get to c:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

and we can eliminate the third column by multiplying

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix}$$

and c to get to the identity matrix I.

Multiplying these matrices together, we can get M:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$