## Exercises on Transposes, Permutations, Spaces

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## 5.1

(a) If we can rotate the three rows of the permutation matrix in one direction, we can get  $P^3 = I$ . One example is that if  $P = \begin{bmatrix} 010 \\ 001 \\ 100 \end{bmatrix}$ , then  $P^2 = \begin{bmatrix} 000 \\ 100 \\ 010 \end{bmatrix}$  and  $P^3 = \begin{bmatrix} 100 \\ 010 \\ 001 \end{bmatrix}$ .

(b) If we can exchange the rows in a 4 by 4 matrix in an non-orderly fashion, unlike P in part

(a), then we will get 
$$P^4 \neq I$$
. For example, if  $\hat{P} = \begin{bmatrix} 0100 \\ 0001 \\ 1000 \end{bmatrix}$ , then  $\hat{P}^2 = \begin{bmatrix} 0001 \\ 1000 \\ 0100 \\ 0100 \end{bmatrix}$ ,  $\hat{P}^3 = \begin{bmatrix} 1000 \\ 0100 \\ 0010 \\ 0001 \end{bmatrix}$  and

$$\hat{P}^4 = \begin{bmatrix} 0100\\0001\\0010\\1000 \end{bmatrix} \neq I.$$

## 5.2

(a) If A is symmetric, then we can represent A by:

$$\begin{bmatrix} a & b & c & d \\ b & e & h & i \\ c & h & f & j \\ d & i & j & g \end{bmatrix}$$

There are 10 independent entries.

(b) If A is skew-symmetric  $(A^T = -A)$ , we can represent A by:

$$\begin{bmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{bmatrix}$$

There are 6 independent entries.

**5.3** 

- (a) True: With  $A^T = A$  and  $B^T = B$ , it leads us to  $(A + B)^T = A^T + B^T = A + B$  and  $(cA)^T = cA^T$ . The addition and scalar multiplication results are both in the subspace.
- **(b) True**: With  $A^T = -A$ , it leads to  $(A + B)^T = A^T + B^T = -A B = -(A + B)$  and  $(cA)^T = cA^T = -cA$ .
- (c) Flase: With  $A^T \neq A$  and  $B^T \neq B$ , it leads us to  $(A+B)^T = A^T + B^T \neq A + B$ . An example is:

$$\begin{bmatrix} 10\\10 \end{bmatrix} + \begin{bmatrix} 01\\01 \end{bmatrix} = \begin{bmatrix} 11\\11 \end{bmatrix}$$

It does not form a subspace, because the result of addition is a symmetrical matrix.