Exercises on Cramer's rule, inverse matrix, and volume

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20.1

$$C = \begin{bmatrix} 6 & -3 & 0 \\ 3 & 1 & -1 \\ -6 & 2 & 1 \end{bmatrix}$$

$$AC^T = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 3 & -6 \\ -3 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I = det(A)I$$

$$det(A) = 3$$

Changing A_{13} to 100 wouldn't change the determinant because on the matrix factor the position of that entry is 0, so it doesn't affect the determinant.

20.2

The matrix of partial derivatives is:

$$\begin{bmatrix} sin\phi cos\theta \ \rho cos\theta cos\phi \ -\rho sin\ thetasin\ phi \\ sin\phi sin\theta \ \rho sin\theta cos\phi \ \rho sin\phi cos\theta \\ cos\phi \ -\rho sin\phi \ 0 \end{bmatrix}$$

Using cofactor formula to find the determinant of the above matrix:
$J = sin\phi cos\theta(\rho^2 sin^2\phi cos\theta) - \rho cos\theta cos\phi(-\rho sin\phi cos\phi cos\theta) - (-\rho sin\theta sin\phi)(-\rho sin^2\phi sin\theta - \rho sin\theta cos^2\phi) = \rho^2 sin^3\phi cos^2\theta + \rho^2 sin\phi cos^2\phi cos^2\theta + \rho^2 sin^3\phi sin^2\theta + \rho^2 sin^2\theta sin\phi cos^2\phi = \rho^2 sin^3\phi + \rho^2 sin\phi cos^2\phi = \rho^2 sin\phi$