Exercises on solving Ax=0 and row reduced form R

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8.1

(a) The coefficient of the particular solution is 1.

(b) The complete solution includes solution(s) to $Ax = 0$. The complete solution $x_p + cx_n$ can be more than one.

(c) $Ax = 0$ always has the zero vector as a solution.

8.2

$$\left[\begin{array}{c|c} U & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Therefore, we have $R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. To solve for $Rx = 0$, $x = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ when $x_2 = 1$.

$$\left[\begin{array}{c|c} U & c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & 4 & 8 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Therefore, we have $R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $d = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. To solve for $Rx = d$, $x = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$ when $x_2 = 1$.

8.3

If $A = C$, then Ay has to equal Cy for all vectors y. Since $Ax = b$ has solutions to every b, then y has to be one of the solutions. Similarly, it follows that y is also one of the solutions for $Cx = b$. So, $Ay = b = Cy$.