Exercises on eigenvalues and eigenvectors

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21.1

(a) There are two nonzero eigenvalues, so the rank of A is greater or equal than 2. Also, since A has 0 as an eiganvalue, A is singular so the rank has to be smaller than three. Therefore, the rank of A is two.

(b) $\det(B) = 0 \times 1 \times 2 = 0$

(c) Not sure if we have sufficient information, let's look at the next one.

(d) When $Bv = \lambda v$, $B^{-1}v = \frac{1}{\lambda}v$ and $B^2 v = \lambda^2 v$. It leads to $(B^2 + I)^{-1}v = \frac{1}{\lambda^2+1}v$

21.2

$$\det(A - \lambda I) = 0 = \begin{bmatrix} 1-\lambda\ 2\ 3 \\ 0\ 4-\lambda\ 5 \\ 0\ 0\ 6-\lambda \end{bmatrix} = (1-\lambda)(4-lambda)(6-\lambda) \rightarrow \lambda = 1, 4, 6$$

$$\det(B - \lambda I) = 0 = \begin{bmatrix} -\lambda\ 0\ 1 \\ 0\ 2-\lambda\ 0 \\ 3\ 0\ -\lambda \end{bmatrix} = -\lambda(-\lambda)(2-\lambda) + (-3)(2-\lambda) = \lambda^2(2-\lambda) - 3(2-\lambda) =$$
$$(2-\lambda)(\lambda+\sqrt{3})(\lambda-\sqrt{3}) \rightarrow \lambda = 2, \sqrt{3}, -\sqrt{3}$$

$$\det(C - \lambda I) = 0 =$$
$$\begin{bmatrix} 2-\lambda\ 2\ 2 \\ 2\ 2-\lambda\ 2 \\ 2\ 2\ 2-\lambda \end{bmatrix} = (2-\lambda)((2-\lambda)^2-4) - 2\cdot(2(2-\lambda)-4) + 2(4-2(2-\lambda)) = (2-\lambda)(4+\lambda^2-4\lambda-4) - 2(4-$$
$$2\lambda-4) + 8 - 4(2-\lambda) = 8 + 2\lambda^2 - 8\lambda - 8 - 4\lambda - \lambda^3 + 4\lambda^2 + 4\lambda + 4\lambda + 8 - 8 + 4\lambda = -\lambda^3 + 6\lambda^2 = \lambda^2(6-\lambda)$$