Exercises on positive definite matrices and minima

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27.1

Starting with the equation $Ax = \lambda x$, we want to prove that $\lambda > 0$.

$$(ABx)^T Bx = (\lambda x)^T Bx$$

$$(Bx)^T A(Bx) = \lambda(x^T Bx)$$

Since both A and B are positive definite, we know that $x^T Ax > 0$ and $x^T Bx > 0$. Given the left hand side is positive and one of the factors of right hand side is positive, it leads us to $\lambda > 0$.

27.2

$$x^T Ax = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 7 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 7y & 5x + 9y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x^2 + 12xy + 9y^2 = (x + 3y)^2 + 6xy$$

The energy of A is sometimes positive sometimes negative, because the first term is always going to be equal or greater than zero while the second term is not.