Exercises on properties of determinants

January 12, 2017

18.1

Since each row of A adds up to zero, this means that for each row $a_i : a_i x = 0$. Another way to look at this is that the nullspace of A can be nonzero vectors; therefore, A is singular and determinant of A is 0.

If each row of A adds up to one, this means that each row of $A - I$ adds up to zero, so $A - I$ is singular and determinant of $A - I$ is zero. However, this does not mean that determinant of A is zero in this case because it does not follow the addition rule. For example, if $A = \begin{bmatrix} 01 \\ 10 \end{bmatrix}$, det$(A - I)$ is 0 and det(A) is -1.

18.2

$$\det \begin{bmatrix} 1 \; a \; a^2 \\ 1 \; b \; b^2 \\ 1 \; c \; c^2 \end{bmatrix} = \det \begin{bmatrix} 1 \; a \; a^2 \\ 0 \; b-a \; (b+a)(b-a) \\ 0 \; c-a \; (c+a)(c-a) \end{bmatrix} = \det \begin{bmatrix} 1 \; a \; a^2 \\ 0 \; b-a \; (b+a)(b-a) \\ 0 \; 0 \; (c-a)(c-b) \end{bmatrix} = \text{(b-a)(c-a)(c-b)}$$