

Exercises on solving $Ax=0$: pivot variables, special solutions

January 6, 2017

7.1

(a) To find the reduced form of A:

$$\begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 2 & -2 & 11 & -3 \end{bmatrix}, \rightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & -12 & -3 & -21 \end{bmatrix}, \rightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 4 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \rightarrow \begin{bmatrix} 1 & 5 & 7 & 9 \\ 0 & 1 & \frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \rightarrow \begin{bmatrix} 1 & 0 & \frac{23}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} & \frac{7}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) The rank is 2.

(c) The special solutions are $\begin{bmatrix} -\frac{23}{4} \\ \frac{1}{4} \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -\frac{1}{4} \\ \frac{7}{4} \\ 0 \\ 1 \end{bmatrix}$.

7.2

If A_1 and A_2 are 2 by 2 matrices: $A_1 = I$ so that $(A_1 B) = B$ and rank of B is 1; $A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ so that rank of $(A_2 B)$ is 0.