

Exercises on linear transformations and their matrices

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30.1

(a) Yes. It is a linear transformation.

(b) $T(x, y) = (2x, 2y)$

(c) This transformation simply doubles the coordinates of x and y . So, if A is $\begin{bmatrix} x \\ y \end{bmatrix}$ and $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$, $T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

30.2

If we want to obtain a non-linear transformation, we could define it to be $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ y^2 \end{bmatrix}$. In this case, $T\left(2\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x \\ 4y^2 \end{bmatrix}$, which does not equal to $2T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 4x \\ 2y^2 \end{bmatrix}$.

$T = \begin{bmatrix} 2 & 0 \\ 0 & y \end{bmatrix}$ in this example.