Exercises on orthogonal vectors and subspaces

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14.1

We can observe that if we have $y_1 = 1$, $y_2 = 1$ and $y_3 = -1$, then on the left hand, the sum of all equations is 0 because they cancel out, while on the right we get 1.

This is because we can rewrite the second part of $Fredholm's Alternative$:

$y^T b = 1 = y^T (Ax) = 0$ when there is no solution x

14.2

(a) Since $N(A)$ is perpendicular to $C(A^T)$, we have $r^T n = 0$. Since $C(A^T)$ is perpendicular to $N(A^T)$, we have $c^T l = 0$. Also, since these vectors are all nonzero vectors while we know that $N(A) + C(A^T) = N(A^T) + C(A) = m = n = 2$ in this case, it leads to $m = n = 1$.

(b) c is the basis for columns of A and r is the basis for rows of A. Therefore, A is in the form of cr^T.