Exercises on orthogonal matrices and Gram-Schmidt

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17.1

$$Qx = 0$$
$$Q^TQx = 0$$
$$Ix = 0$$
$$x = 0$$

17.2

$$A = a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$B = b - \frac{A^Tb}{A^TA}A = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{(-1)}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \\ 0 \end{bmatrix}$$

$$C = c - \frac{A^Tc}{A^TA}A - \frac{B^Tc}{B^TB}B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - 0 - \frac{(-1)}{\frac{3}{2}}\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ -1 \end{bmatrix}$$

We can observe that $a^Td = b^Td = c^Td = 0$. d is one-dimensional so the space perpendicular to d should be three-dimensional and a, b, c are linearly independent, so a, b, c are bases for the space perpendicular to d. The same thing can be said for A, B, C.