Exercises on determinant formulas and cofactors

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19.1

If we switch row 1 and row 2, then row 2 and row 4 and lastly row 2 and row 3, we can go from A to I. Since there are three row exchanges, the determinant of A is -1.

19.2

We can rewrite the determinants as:

$$\det \begin{bmatrix} 1\ 1\ 1\ 1 \\ 1\ 2\ 3\ 4 \\ 1\ 3\ 6\ 10 \\ 1\ 4\ 10\ 20 \end{bmatrix} = \det \begin{bmatrix} 2\ 3\ 4 \\ 3\ 6\ 10 \\ 4\ 10\ 20 \end{bmatrix} \text{ - } \det \begin{bmatrix} 1\ 3\ 4 \\ 1\ 6\ 10 \\ 1\ 10\ 20 \end{bmatrix} + \det \begin{bmatrix} 1\ 2\ 4 \\ 1\ 3\ 10 \\ 1\ 4\ 20 \end{bmatrix} \text{ - } \det \begin{bmatrix} 1\ 2\ 3 \\ 1\ 3\ 6 \\ 1\ 4\ 10 \end{bmatrix} = 1$$

$$\det \begin{bmatrix} 1\ 1\ 1\ 1 \\ 1\ 2\ 3\ 4 \\ 1\ 3\ 6\ 10 \\ 1\ 4\ 10\ (20-1) \end{bmatrix} = \det \begin{bmatrix} 2\ 3\ 4 \\ 3\ 6\ 10 \\ 4\ 10\ (20-1) \end{bmatrix} \text{ - } \det \begin{bmatrix} 1\ 3\ 4 \\ 1\ 6\ 10 \\ 1\ 10\ (20-1) \end{bmatrix} + \det \begin{bmatrix} 1\ 2\ 4 \\ 1\ 3\ 10 \\ 1\ 4\ (20-1) \end{bmatrix} \text{ - } \det \begin{bmatrix} 1\ 2\ 3 \\ 1\ 3\ 6 \\ 1\ 4\ 10 \end{bmatrix}$$

We can observe the difference of the two determinants:

$$\det \begin{bmatrix} 1\ 1\ 1\ 1 \\ 1\ 2\ 3\ 4 \\ 1\ 3\ 6\ 10 \\ 1\ 4\ 10\ 20 \end{bmatrix} \text{ - } \det \begin{bmatrix} 1\ 1\ 1\ 1 \\ 1\ 2\ 3\ 4 \\ 1\ 3\ 6\ 10 \\ 1\ 4\ 10\ 19 \end{bmatrix} =$$

$2 \times 6 - 3 \times 3 - (1 \times 6 - 3 \times 1) + (1 \times 3 - 2 \times 1) = 12 - 9 - (6 - 3) + 1 = 1$

Therefore, the determinant of $\begin{bmatrix} 1\ 1\ 1\ 1 \\ 1\ 2\ 3\ 4 \\ 1\ 3\ 6\ 10 \\ 1\ 4\ 10\ 19 \end{bmatrix}$ is 1 - 1 = 0.