Exercises on the Geometry of Linear Equations

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1.1

If a combination exists such that $x_1w_1 + x_2w_2 + x_3w_3$ gives the zero vector, these vectors are dependent. Writing out the vectors in equations, we have:

$$x + 4y + 7z = 0$$

$$2x + 5y + 8z = 0$$

$$3x + 6y + 9z = 0$$

Subtracting 2 times equation 1 from equation 2 gives us $y + 2z = 0$. Subtracting 3 times equation 1 from equation 3 also gives us $y + 2z = 0$. If $y = 2$ and $z = -1$, it leads to $x = -1$.

Answer: One combination that gives the zero vector could be $-w_1 + 2w_2 - w_3 = 0$. These vectors are dependent and they lie in a plane.

1.2

$$3 \cdot \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + (-2) \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 11 \end{bmatrix}$$

1.3

This statement is true. The product of two matrices is a N by M matrix where N is the number of rows in the first matrix and M the number of columns in the second matrix.