Exercises on complex matrices; fast Fourier transform

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26.1

$$F_2 = \begin{bmatrix} 1 & 1 \\ 1 & w \end{bmatrix}$$

Since $w = e^{\frac{2\pi}{2}i}$, $w = -1$. So $F_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

26.2

$$F_4 = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_2 & 0 \\ 0 & F_2 \end{bmatrix} P$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

$$P = \begin{bmatrix} 1000 \\ 0010 \\ 0100 \\ 0001 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & i \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1000 \\ 0010 \\ 0100 \\ 0001 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & i \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}$$

The product of those three matrices are indeed F_4.