

## Exercises on the four fundamental subspaces

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**10.1**

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(a) If there are right hand sides  $b$  for which  $Ax = b$  has no solutions, it means that there are right hand sides for which combinations of the columns of  $A$  can't represent. If so, the rank  $r$  is smaller than the number of rows  $m$ .

(b) The solutions to  $A^T y = 0$  are the left nullspace of  $A$  whose dimension is  $m - r$ . Since from part (a) we concluded that  $r < m$ , it leads to  $m - r > 0$  so there are always solutions.

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**10.2**

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$A^T y = d$  is solvable when  $d$  is in row space of  $A$ . The solution  $y$  is unique when the left nullspace of  $A$  contains only the zero vector.