Exercises on similar matrices and Jordan form

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28.1

Suppose $M = \begin{bmatrix} m_{11} \ m_{12} \ m_{13} \ m_{14} \\ \cdots \end{bmatrix}$:

$$JM = MK = \begin{bmatrix} 0 \ m_{21} \ m_{22} \ 0 \\ 0 \ 0 \ 0 \ 0 \\ 0 \ m_{41} \ m_{42} \ 0 \\ 0 \ 0 \ 0 \ 0 \end{bmatrix}$$

From the form of the product we can tell that M is not invertible, so M^{-1} does not exist. Therefore, $J = M^{-1}KM$ does not stand and J is not similar to K.

28.2

(a) Given $M^{-1}AM = B$, it leads to:

$$M^{-1}(A^2)M = M^{-1}AMM^{-1}AM = B^2$$

(b) If A^2 and B^2 are similar and have $\lambda = 0, 0$, it leads to $S^{-1}A^2S = \begin{bmatrix} 0 \ 0 \\ 0 \ 0 \end{bmatrix} = S^{-1}B^2S$. However, this does not guarantee that $S^{-1}AS = S^{-1}BS$. An example is $A = \begin{bmatrix} 0 \ 0 \\ 0 \ 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \ 0 \\ 0 \ 1 \end{bmatrix}$.

(c)

$$M^{-1}AM = \begin{bmatrix} 3 \ 1 \\ 0 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 \ 0 \\ 0 \ 4 \end{bmatrix} \begin{bmatrix} m_{11} \ m_{12} \\ m_{21} \ m_{22} \end{bmatrix} = \begin{bmatrix} m_{11} \ m_{12} \\ m_{21} \ m_{22} \end{bmatrix} \begin{bmatrix} 3 \ 1 \\ 0 \ 4 \end{bmatrix}$$

$$\begin{bmatrix} 3m_{11} \ 3m_{12} \\ 4m_{21} \ 4m_{22} \end{bmatrix} = \begin{bmatrix} 3m_{11} \ m_{11} + 4m_{12} \\ 3m_{21} \ m_{21} + 4m_{22} \end{bmatrix}$$

One of the solutions is $M = \begin{bmatrix} 1 \ -1 \\ 0 \ 1 \end{bmatrix}$. Therefore, those two matrices are similar.

(d) The eiganvalues of $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ are 3, 3. The eiganvalues of $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$ are 4, 2.

$$det \begin{bmatrix} 3-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = (3-\lambda)^2 - 1 = 0$$

Those two matrices are similar because they have different eiganvalues.

(e) Given the conditions, it leads to $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = M^{-1}AM$. Therefore, A and B are similar.