The Basics

Instructor: Dr. Christy Hazel (she/her)¹

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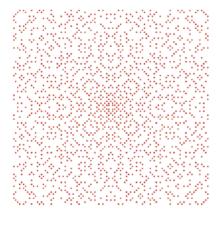
Student Hours: (Also known as "Office Hours") Mondays 1-2PM, Tuesdays 3-5PM, Fridays 12-1PM, & by appointment

Class Mentor: Isidora Bailly-Hall Mentor Email: baillyha@grinnell.edu

Class Meetings: 11:00-11:50AM MWF in Noyce 2245

Textbook: Explorations in Number Theory by Jeanne Agnew

(available for free on PWeb)



What is Number Theory?

Number theory, in its most basic form, studies patterns within the positive integers:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, \dots$$

The subject is especially interested in the relationships between these numbers. For example, how are the prime numbers (3, 5, 7, 11, 13, 17, ...) distributed amongst the positive integers? Or given a particular polynomial equation, does it have integer solutions? And if it does, how do we best describe those integer solutions? As you might observe, many of the questions asked in elementary number theory are simple to state. However, as we develop the tools needed to answer these questions, we will discover that they can be quite deep and mysterious.

Course Goals

The primary goal of this course is to develop your proof-writing skills and to expand your creative problem-solving strategies. We will use number theory as a backdrop, but the main objective is to prepare you for the 300-level foundations courses. Even if you decide not to pursue mathematics after this course, many of these skills are helpful in other fields.

A successful student in this course will...

- Strengthen technical writing and communication skills, especially in mathematics.
- Hone problem-solving skills. In particular, learn how to approach problems that you have never seen before, including those in a new mathematical area.
- Develop individualized study skills in preparation for foundations courses by developing personalized plans for how to...

¹Feel free to call me Christy (that is my preferred way to be addressed by students). Or if it makes you more comfortable, you are welcome to call me Professor Christy or Professor Hazel.

- get the most out of class time and office hours,
- best utilize time spent on homework,
- best read a mathematical textbook and use this as a supplement to in-class material,
- prepare for exams in upper level mathematics courses.
- Learn about mathematical induction, equivalence relations, and basic counting techniques. (These are foundational topics that will appear in many mathematics courses.)
- Learn about the key concepts in elementary number theory including divisibility, congruences, the Euler ϕ function, quadratic residues, and multiplicative functions.

Note the style of mathematical writing is likely different from what you are used to. Most people feel uncomfortable and unsure when they first start writing proofs—this is completely okay and expected! One of our course goals is to learn to work through this discomfort and to become more confident in our proof-writing skills.

Grading Scheme: How Your Learning Will be Assessed

Course grades will be weighted according to the following scheme:

- Weekly Homework 25%
- Reading Assignments 10%
- Course Engagement 5%
- Two Best Midterm Exams 30% (15% each)
- Lowest Midterm Exam 10%
- Final Exam 20%

More information about these categories is given later on in the syllabus.

How to Contact Me

- Student Hours. If you have math or course questions, you are encouraged to come to Student Hours/Office Hours to ask me in person.
 - This is my dedicated time to help students from my classes! I like to call these "Student Hours" instead of "Office Hours" to emphasize that.
 - Come by for any amount of time during these scheduled hours.
 - While it's often more productive if you come with a specific question, you are also welcome to come listen to other students' questions and/or quietly work on assignments and then ask questions as they pop up.
- Email. You are welcome to email me anytime with any questions or comments about mathematics or about the course (hazelchristy@grinnell.edu).

- Note I usually don't check my email between 7PM-8AM on weekdays and only occasionally on the weekends. You are welcome to email me whenever is most convenient for you, but note I might not respond until normal working hours.
- I will try to always respond within 24 hours (though if you email Friday night, there's a
 decent chance I won't get back to you until Monday).

More About Assessments

Homework

Homework will be assigned each week on PWeb. Each assignment will have two parts. Both parts will be submitted through Gradescope. The first part of the assignment will be **Tuesday at 6PM**. The second part will be due **Friday at 6pm**.

The Tuesday part of the assignment will consist of 1-3 proof problems and will only be graded for completion. I will provide feedback on these proofs by Wednesday at 6PM, and then you will be expected to revise and resubmit these problems as part of your Friday assignment. The Friday part will contain rewrites of your Tuesday proofs and also include some additional proof problems. The Friday assignment will be graded on accuracy.

The homework is structured in this way to improve your revision skills. A very common concern students have at this level is: "I wrote something that sounds like a proof, but I'm not sure if it's complete. How can I tell if what if I've written is correct?" I want you to learn how to critically analyze your own proof-writing to help alleviate this concern. Regular revisions will help with this goal.

Your lowest homework assignment grade will be dropped. Late work will generally not be accepted. Instead the drop policy allows everyone to miss one assignment with no penalty, no questions asked. With that said, if something is going on that is causing you to miss multiple deadlines, then please reach out to me so we can discuss your options in the course.

Reading Assignments

Learning a new mathematical concept requires revisiting and reviewing the topic multiple times, ideally in multiple different ways. The ways you are likely most familiar with are through lecture and activities in class, working on homework assignments, and reviewing notes/assignments to prepare for exams. Another essential part of learning mathematics, especially in our foundations courses, is reading the textbook.

To encourage regular reading before coming to class, I will have you submit two reading assignments each week. It's okay to not understand everything you read the first time, but this at least gets our brains thinking about the material ahead of time, which makes class time much more productive. Each week one reading assignment will be due **Monday at 10:30am** (I am intentionally leaving a 30 minute gap between due time and class start so folks aren't finishing at start of class). The other will be due **Wednesday at 10:30am**. The assignment will be given as a Qualtrics form, and you'll be able to find the links on PWeb. You'll be asked a few written response questions to guide your reading, and it will be mostly graded for completion.

Your two lowest reading assignment grades will be dropped. Late work will generally not be accepted. Instead the drop policy allows everyone to miss two reading assignments with no penalty, no questions asked. With that said, if something is going on that is causing you to miss multiple deadlines, then please reach out to me so we can discuss your options in the course.

Course Engagement

The only way to learn mathematics is to regularly and thoughtfully engage with the material. Thus 5% of your final course grade will reflect your engagement with the course, both inside and outside of the classroom. Your contributions to in-class discussions, your questions in Isidora's mentoring sessions, and your questions in office hours can all positively impact this part of your grade.

Midterm Exams

We will have three in-class midterm exams, dates below.

- Exam 1: Wednesday, September 20th
- Exam 2: Wednesday, October 25th
- Exam 3: Monday, November 20th (Note this is the Monday of Thanksqiving week)

I will post information about each exam on PWeb the week before the exam. Your lowest exam score will count for 10% of your final grade, while your highest two exam scores will count for 15% of your final grade.

Make-up exams will not be given, unless agreed upon beforehand or in an emergency, so please contact me immediately if any issue arises with the scheduled exams.

Final Exam

We will have a cumulative final exam on Thursday, December 14th from 9am-12pm. More information about the exam will be given on PWeb when we get closer to that date.

Classroom Culture and Expectations

We will be spending a lot of time discussing mathematics together this term. We should all recognize that different people will have different mathematical and personal backgrounds. In order to foster and maintain a learning environment that is productive for everyone, it is essential that we create a space where people feel comfortable asking questions and discussing material. Here are a few concrete things we can do to build a productive and inclusive learning community:

• Be present. We have a limited amount of time together in the classroom, so let's use this time productively. For all of us, this means showing up on time, being ready to start at the beginning of class, and keeping distractions (cell phones, non-course related materials, etc.) to a minimum.

- Recognize that learning is a process and be open to making mistakes. I do not expect you to be able to do every proof on your first try. Be open to trying new problems and making mistakes. Recognize that mistakes are a natural part of learning, and find something to learn from your mistakes. With that said, if you find yourself stuck on a problem for a long time without making progress, reach out to your peers, mentor, or me to get some guidance.
- Listen to understand, not to judge. You will often be discussing mathematics with your classmates both inside and outside of class. Make sure everyone in your group gets a chance to share ideas, and listen to understand and learn from your classmates' ideas, not to judge their abilities. If you disagree with something, share your disagreement respectfully and make sure you are critiquing the ideas and not the person.

Class Mentor

Our course mentor, Isidora Bailly-Hall, is a senior math major who has taken both of our 300-level foundations courses and has participated in summer research projects. Her job is to help guide students as they transition to proof-based mathematics courses, including discussing her own experiences transitioning to 300-level courses. Isidora will also be available to talk about proofs from class or the readings, as well as discuss alternative proofs to and provide supplemental information about material in class. She can help with homework and your writing, particularly by pointing out where a completed solution goes wrong, or where a written solution gets hard to follow. She also can provide some basic LaTeX support. Isidora will have a regular schedule which will be posted on PWeb.

Grinnell Resources for Support and Learning

Below are just a few of the resources offered by Grinnell that could be helpful for you. Grinnell has a lot of resources available for students, so it can be hard to stay informed about all of them. The link https://www.grinnell.edu/students has more information on resources.

Academic Accommodations Based on a Disability

Everyone in this class deserves an equitable opportunity to learn and engage with the material. I encourage students to reach out to me with any distinctive learning needs. Students with documented disabilities should also communicate with and provide documentation to the Coordinator for Student Disability Resources, Jae Baldree, located on the 1st floor of Steiner Hall (x3089). If you have any questions about accommodations in this course, please feel free to reach out to me.

Counseling and Wellness

Student Health and Wellness (SHAW) provides a variety of services to support the physical, mental, and social well-being of Grinnell students. Find more information by visiting their website (https://www.grinnell.edu/about/leadership/offices-services/student-health/counseling), including information about counseling services that are available to Grinnell students. They also offer

a 24/7 counseling hotline at 641-269-4404 that connects you to a trained counseling professional who is also familiar with the range of resources available at Grinnell.

Academic Advising Support

If you have other needs that aren't addressed through the above resources, please let me know soon so that we can work together for the best possible learning environment. In some cases, I will recommend consulting with the Academic Advising staff. They are an excellent resource for developing strategies for academic success and can connect you with other campus resources as well: http://www.grinnell.edu/about/offices-services/academic-advising. If I notice that you are encountering difficulty, in addition to communicating with you directly about it, I will also likely submit an academic alert via Academic Advising's SAL portal. This reminds you of my concern, and it notifies the Academic Advising team and your adviser(s) so that they can reach out to you with additional offers of support.

Academic Honesty

Please familiarize yourself with the university's policies on academic honesty.

You are allowed (and strongly encouraged) to work on homework with your classmates. But you should make sure everyone is contributing ideas and that you write your final proof on your own.

To clarify, if you are brainstorming how to solve a problem with a group of students, and in the course of your conversation someone suggests a key piece of the solution, then everyone in your study group is welcome to use that key piece when you write up your own proofs by yourself later on. Additionally, you are allowed to discuss your written solutions for this problem with that group (since you already have an agreed-upon path through the proof that you discovered together).

On the other hand, if you or a classmate has already solved a problem (either on their own or with a different group), then it is not okay to share key pieces with other students in the class. It's very difficult to differentiate between a "key piece" and "just a hint". Thus if someone asks for help or hints on a homework problem that you've already solved, then you should tell them that you already solved the problem and instead recommend they ask me or Isidora.

Consulting any completed solution is academically dishonest. You should never look online or in another textbook for homework solutions. A key goal in this course is to learn how to navigate through being stuck. Reading a math solution is much easier than figuring it out yourself. It hurts your learning to find solutions online or in another book. I will intentionally assign some more difficult problems so that you can experience working through being stuck, and I do not assume students will ace every homework assignment. If you're feeling stuck and really don't know how to proceed, then you should talk with your classmates, Isidora, or me.

Class Schedule

See PWeb for a daily course schedule with readings and topics covered in class.