

HW#1: q3 - answers Runtime Analysis

Christy Ko

a)

```
void f1(int n)
{
    int i=2;
    while(i < n){
        /* do something that takes O(1) time */
        i = i*i;
    }
}
```

$\sum k$	1	2	3	4	5	k^{th}
$i=2$	2^1	2^4	2^9	2^{16}	2^{25}	$i=2^{k^2}$

$$n = 2^{k^2}$$

$$\log n = 2^k$$

$$\log \log n = k$$

$$T(n) = \theta(1) + \theta(\log \log n) = \boxed{\theta(\log \log n)}$$

b)

```
void f2(int n)
{
    for(int i=1; i <= n; i++){
        if( (i % (int)sqrt(n)) == 0){
            for(int k=0; k < pow(1,3); k++){
                /* do something that takes O(1) time */
            }
        }
    }
}
```

j	i
1	\sqrt{n}
2	$2\sqrt{n}$
j^{th}	$j\sqrt{n}$

$$i = j\sqrt{n}$$

$$\sum_{j=1}^{\sqrt{n}} \theta(j\sqrt{n})$$

$$\sum_{j=1}^{\sqrt{n}} \theta(j^2)$$

$$\sum_{j=1}^{\sqrt{n}} \theta(j^2) = \frac{1}{4}n^2 \leftarrow \sum_{k=1}^n k^2 = 1^2 + 2^2 + \dots + n^2 \approx \frac{n^3}{3}$$

$$\sqrt{n^3} \cdot \frac{n^2}{4} = \frac{n^{5/2}}{4} \quad \boxed{\theta(n^{5/2})}$$

iteration	1	2	3	4
m	$m=2$	$2+2=4$	$3+3=6$	$4+4=8$
				$\dots m=2^k \quad k=\log m$

c)

```
for(int i=1; i <= n; i++){
    for(int k=1; k <= n; k++){
        if( A[k] == i){
            for(int m=1; m <= n; m=m+m){
                // do something that takes O(1) time
                // Assume the contents of the A[] array are not changed
            }
        }
    }
}
```

$$T(n) = \sum_{i=1}^n \left(\sum_{k=1}^n \theta(1) + O(\sum_m \theta(1)) \right)$$

$$\sum_{i=1}^n \left(\sum_{k=1}^n \theta(1) + \log m \right)$$

$$\sum_{i=1}^n (\theta(n) + \log m)$$

$$\theta(n^2 + \log n!)$$

$$\boxed{\theta(n^2)}$$

d)

```
int f(int n)
{
    int *a = new int [10];
    int size = 10;
    for(int i=0; i < n; i++)
    {
        if(i == size)
        {
            int newsize = 3*size/2;
            int *b = new int [newsize];
            for(int j=0; j < size; j++) b[j] = a[j];
            delete[] a;
            a = b;
            size = newsize;
        }
        a[i] = i*i;
    }
}
```

$$T(n) = \sum_{i=0}^{n-1} \theta(1) + O(\sum_i \theta(1))$$

iteration	1	2	3	4	5	6
i	10	15	22	38	50	75
					$n=10 \times 1.5^k$	$k = \log_{1.5} \frac{n}{10}$

$$\theta(n) + O(10 \sum_{j=0}^k (\frac{3}{2})^j)$$

$$\theta(n) + 10\theta(\frac{3}{2})^k$$

$$\theta(n) + 10\theta(\frac{3}{2}^{\log_{1.5} \frac{n}{10}})$$

$$\theta(n) + 10\theta(\frac{n}{10})$$

$$\boxed{\theta(n)}$$