Exploring the Influence of Topography on Wildfire Behavior

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Abstract

Wildfires pose significant threats to communities and ecosystems worldwide, influenced by factors such as weather, topography, and fuel composition. Our study aims to explore the influence of topography on wildfire propagation through mathematical modeling. Our model represents a probabilistic approach where one cell propagates to an adjacent cell, incorporating terrain slope and computed probabilities. The results demonstrate that varying the slope sensitivity factor leads to increased asymmetry and stochasticity in fire behavior. This study advances our understanding of how topography influences wildfire spread, offering insights to inform wildfire management strategies.

1 Introduction

Wildfires, a growing global concern, have left an indelible mark on communities worldwide, threatening lives, ecosystems, and landscapes. They occur in regions with hot and dry climates and are fueled by high winds and flammable vegetation. Over the past decade, an alarming annual average of 61,410 wildfires and the relentless spread across 7.2 million acres annually underscore the urgent need for comprehensive understanding and effective management strategies ("Wildfire Statistics", 2023). While existing fire behavior models encompass climatic factors like wind speed, temperature, and humidity, this paper zeroes in on the pivotal role of topography.

Topography, often overlooked in traditional models, holds a significant impact on fire propagation dynamics. Scientifically, elevation and slope emerge as crucial factors, with fire exhibiting accelerated spread in ascending terrain and a more gradual propagation in descending landscapes. When a fire ignites at the bottom of a steep slope it will spread more quickly uphill because heat rises. As that hot air rises, it preheats fuels that are further uphill, causing them to readily ignite once the fire reaches them (Moore, 2021).

In this paper, we aim to address a fundamental question: How can we model the influence of topography on fire propagation, and how does the behavior of fire spread vary across different parameter values? Our objective is to develop various models that show the correlation between directional slope and the spread of wildfires—a pivotal stride toward enhancing the effectiveness of wildfire management strategies. Section 2 outlines the underlying model, Section 3 details the experimental methodology, Section 4 presents the validation, and Section 5 provides both qualitative and quantitative results.

2 Mathematical Model

In constructing our fire propagation model, we deliberately omitted external variables, including temperature, wind speed, humidity, and soil composition. Additionally, within this simulation, we assume a perpetual existence of fire, foregoing the possibility of it going out. We considered that fire can only propagate from one cell to neighboring cells.

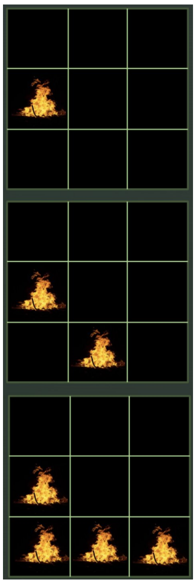


Figure 1: In the top figure, the fire strikes the middle left cell of a 3x3 matrix at time T. In the middle matrix, after time $2T$ the fire propagates diagonally, bottom middle. In the bottom matrix, after time $3T$ the fire further propagates to its neighboring cells. Source: Garaud, 2023

Figure 1 illustrates the basic idea of our model, which uses a probabilistic approach to fire propagation. After a certain amount of time T, fire propagates from one cell to the next cell. The neighboring cells have some probability p to catch on fire. We will establish a fire matrix M that depicts the landscape, where entries are denoted as 1 when the cell is on fire and 0 when it is not. To account for topography, an elevation matrix will be introduced, aiding in determining whether a neighboring cell is uphill, downhill, or on level ground by computing the slope between two cells. The slope between two cells is determined by taking the elevation difference

$$x = \frac{\Delta E}{d} \tag{1}$$

between the current cell and its neighboring cell (represented by ΔE) and dividing it by the distance (d) between the cells. When the fire propagates upwards, left, right, or downwards the distance is considered as 1. However, when the fire propagates diagonally, the distance is calculated as $\sqrt{2}$.

As previously mentioned, neighboring cells carry a certain probability of catching fire. This probability of propagation will be the sigmoid function of the slope

$$f(x) = \frac{1}{1 + e^{-ax}} \tag{2}$$

where a is a constant and x is the computed slope.

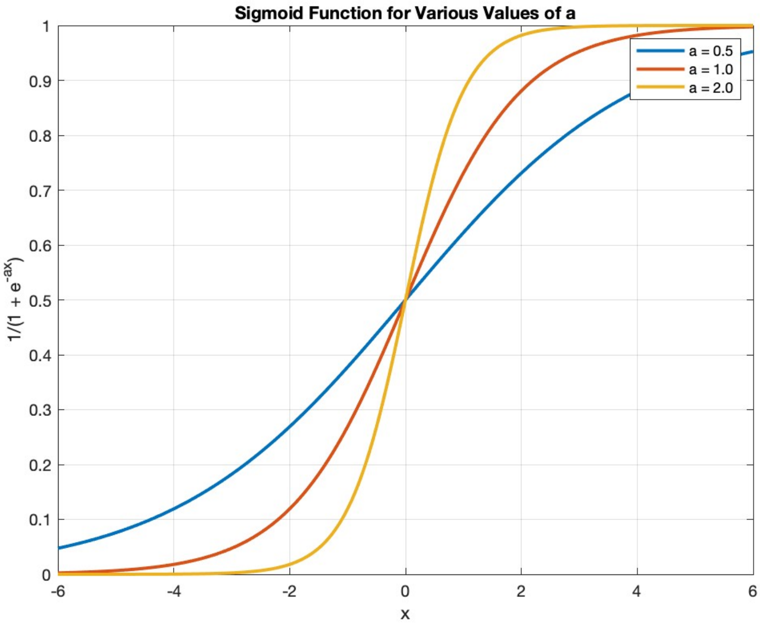


Figure 2: Sigmoid graphs for various values of a. Blue curve represents a=0.5, red curve represents a=1, and yellow curve represents a=2.

The parameter a controls the steepness of the curve. In Figure 2, we illustrate three distinct sigmoid curves corresponding to different values of a. For $0 < a < 1$, the sigmoid curve becomes shallower, while $a > 1$ makes the curve steeper. Increasing a amplifies the rate at which the function transitions approximately from zero to one. The use of the sigmoid function for probability determination stems from its characteristics, with a range limited between 0 and 1, aligning well with the probabilistic nature of fire spread. The sigmoid function accommodates both negative and positive x values, allowing for a comprehensive assessment of slopes. Negative slopes, representing downhill terrain, result in lower probabilities of catching on fire. For flat terrains (0 slope), the probability converges to 0.5, and positive slopes, denoting uphill landscapes, yield higher probabilities of catching fire.

3 Method

In this study, we conducted a series of experiments to investigate the dynamics of fire propagation over an elevated terrain. The experiment is designed to run for 10 discrete time steps (maxt), with each step comprising 1,000 realizations (maxr). Fire propagation is facilitated by a fire propagate function. To begin, the function loops through the rows and columns of the fire matrix, M, checking if the cell is on fire. The next step is calculating the slope between the adjacent cells. Then, we compute the probability of propagation using the sigmoid function based on the slope. By varying a, we explore different scenarios, reflecting how the rate of fire spread responds to changes in terrain steepness. The final stage of the fire propagation function involves deciding whether the fire should propagate to neighboring cells. This decision is contingent upon comparing the computed probability with a random probability between 0-1. If the computed probability is greater than the randomized probability, the neighboring cell is ignited.

To incorporate elevation into our simulation, we iterate over the rows and columns of the elevation matrix. For each row, we define a mean elevation and mean slope

$$E_{mean} = \frac{i}{n} \tag{3}$$

$$S_{mean} = \frac{1}{n} \tag{4}$$

where i represents the row index and n is the total number of rows. This mean elevation serves as a key parameter, ensuring that as we move down the matrix rows, the mean elevation progressively increases, effectively creating a consistent uphill slope in the terrain. Within each row, we calculate the elevation for each column using the formula

$$E_{ij} = E_{mean} + \sigma \tag{5}$$

where σ is some random noise. The random noise is specified to be within a range of 0.05, uniformly distributed between elevation values. Increasing the noise range causes the terrain to become more rugged and uneven, leading to greater elevation fluctuations across the landscape. In contrast, reducing the noise range results in a smoother and more uniform terrain, with fewer fluctuations in elevation across the landscape.

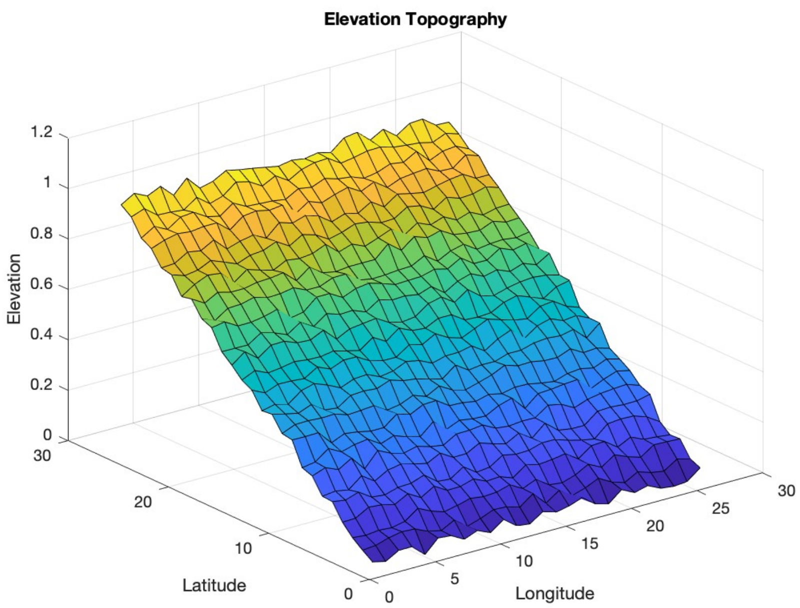


Figure 3: A 3D elevation terrain representation with a noise range of 0.05, with elevation values spanning the range from 0 to 1. The x-axis denotes longitude, the y-axis represents latitude, and the z-axis encapsulates elevation values.

Figure 3 portrays an elevated terrain, mirroring real-world landscapes with a random noise range of 0.05. The inclusion of random noise introduces variability to the terrain, emulating the irregularities, bumps, and fluctuations in natural landscapes. This contributes to a more realistic representation. As previously detailed in the discussion on elevation implementation, each row of elevation values is lower than the next, creating an uphill slope effect in the terrain.

4 Validation

In the validation process, probabilities are computed for specific slopes and compared with simulated probabilities, aiming for a close approximation. The calculated probability is the probability of propagation from the central cell to the neighboring cell. The simulation involves a single time step and 1000 realizations, with a fixed value of $a = 25$. Random noise was excluded from this comparison, which serves to assess the accuracy and reliability of our method.

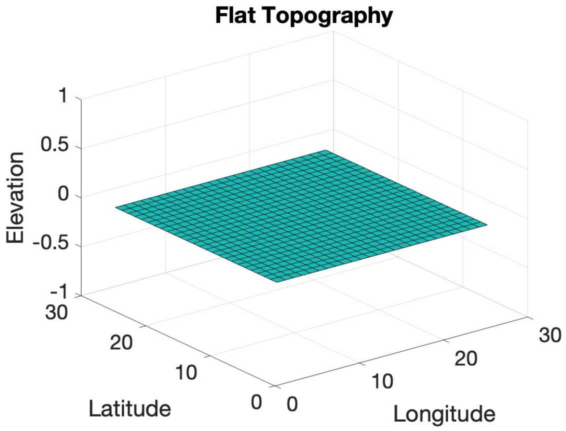
$$\text{a) Elevation Matrix: } \begin{bmatrix} 0.4615 & 0.4615 & 0.4615 \\ 0.5000 & 0.5000 & 0.5000 \\ 0.5385 & 0.5385 & 0.5385 \end{bmatrix}$$

$$\text{b) Calculated Probability: } \begin{bmatrix} 0.6639 & 0.7236 & 0.6639 \\ 0.5000 & 1.0000 & 0.5000 \\ 0.3361 & 0.2764 & 0.3361 \end{bmatrix}$$

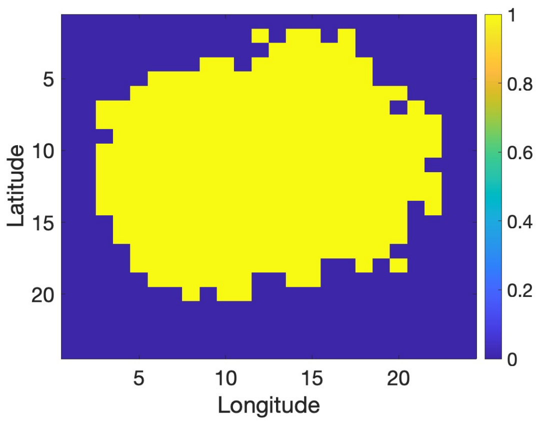
$$\text{c) Simulated Probability: } \begin{bmatrix} 0.6640 & 0.7020 & 0.6600 \\ 0.5020 & 1.0000 & 0.4950 \\ 0.3110 & 0.2770 & 0.3460 \end{bmatrix}$$

5 Results

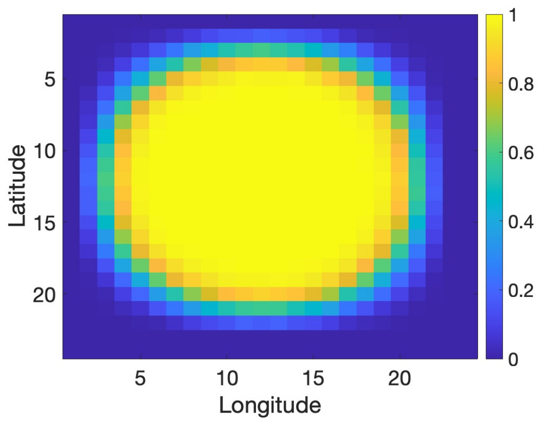
5.1 Qualitative Results



(a) Flat Terrain



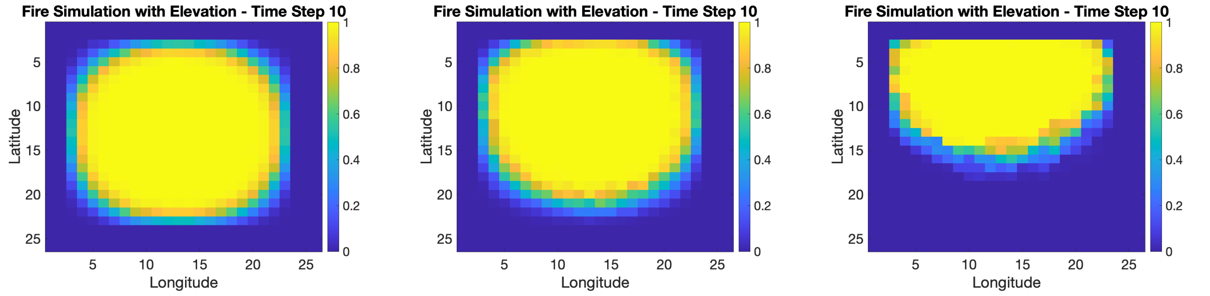
(b) Fire propagation: maxt=10 & maxr=1



(c) Fire propagation: maxt=10 & maxr=1,000

Figure 4: Fire propagation models on a flat terrain for various realizations. The probability ran for both models is 0.4. The x-axis is the longitude and the y-axis is the latitude.

Figure 4 presents two simulations of fire propagation models conducted over a flat terrain. The left figure represents a single realization, while the right figure illustrates the average behavior aggregated from 1,000 realizations. In the single realization, the model exhibits stochastic behavior, showcasing variability and unpredictability. In contrast, the average of the same simulations done over 1,000 realization reveals uniform and symmetrical patterns. The propagation appears radial, as the fixed probability results in an equally probable fire spread in all directions from the ignition point. As we shall see later when we experiment with an elevated terrain, the behavior of fire propagation is quite different and strongly asymmetric.



(a) $a = 1$ (b) $a = 25$ (c) $a = 100$

Figure 5: Three slope-dependent fire propagation models for various values of a. Simulated over 10 timesteps and 1,000 realizations. The geographical representation is presented with the x-axis denoting longitude and the y-axis representing latitude.

Figure 5 shows the fire propagation models based on varying values of a, for the same mean slope of $\frac{1}{n}$. We see that the fire becomes more asymmetrical with higher a values. Qualitatively speaking, the simulated propagation for $a = 100$ emphasizes a notable disparity, displaying a more substantial upward trend compared to downhill propagation, in contrast to $a = 1$ where the fire is almost radially symmetric, as in the flat case. This behavior can be attributed to the sigmoid function $\frac{1}{1+e^{-ax}}$ and its responsiveness to changes in a. When we increase a the sigmoid function becomes steeper and the transition from 0 to 1 is much quicker. This means that the function is more sensitive to changes and fire can be ignited uphill with much higher probabilities even for small slopes. This causes the fire to spread more extensively uphill than downhill.

5.2 Quantitative Results

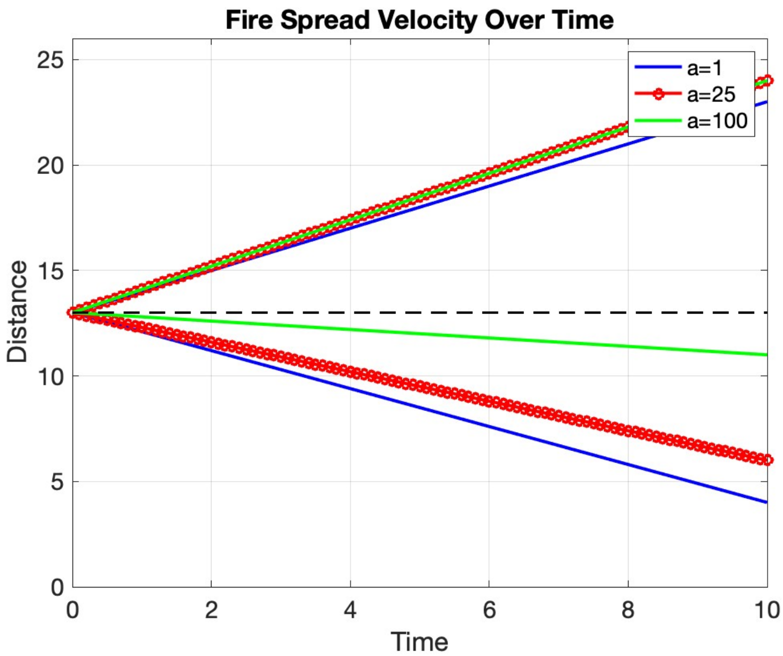


Figure 6: Velocity of fire spread over time for different values of a. Blue represents the velocity for $a = 1$, red represents the velocity for $a = 25$, and green represents the velocity for $a = 100$. The dashed line denotes the center of the fire matrix.

In order to better quantify the asymmetry of the fire, we observed the speed of fire propagation both uphill and downhill. To calculate the speed of propagation, entries in the mean matrix are close to 1 near the ignition point of the fire and drop to 0 further away from it. We define the upper and lower edges of the fire as the highest and lowest points where the mean matrix values are still above the threshold of 0.7. Figure 6, illustrates the velocity of fire spread for various values of a. The x-axis represents the time and the y-axis represents the distance. The dashed line signifies the center of the fire matrix, where propagation above it indicates uphill spread, and below it is downhill spread. For $a = 1$, the velocity of fire spread uphill is 1, and downhill is -0.9. For $a = 25$, the velocities are 1.1 uphill and -0.7 downhill. For $a = 100$, the velocities are 1.1 uphill and -0.2 downhill. These velocities are consistent with our qualitative observations. The velocity remains relatively stable for uphill propagation as a increases. However, for downhill propagation, the speed of fire spread slows down with increasing values of a.

6 Discussion

Our study focused on the effect of slope on fire propagation, but there were several limitations and caveats in our experiment. Firstly, our model did not account for the varying rate of fire spread resulting from changes in slope. According to the National Wildfire Coordinating Group, the rate of fire spread doubles with the first tripling of the slope, and the second tripling increases the rate by a factor of 4 or 6 depending on fuel conditions. If we considered this, our results would have a more pronounced propagation upwards, as steep slopes promote faster fire spread in uphill directions.

Additionally, our model provides a simplified representation of fire propagation, excluding external factors such as wind or the possibility of fire extinguishing. Moreover, the elevation data generated in our experiment doesn't reflect real-life topography. Instead, real-life data could be used for more realistic simulations.

Looking ahead, there are different ways to elevate our research. Future investigations could involve modeling fire propagation over different landscapes, such as valleys or mountainous regions. Also, integrating external factors like wind speed and precipitation into the simulation would enable a more realistic representation of wildfire behavior in various environmental conditions. We could even use machine learning techniques, such as neural networks, to optimize model parameters based on historical wildfire data to improve the accuracy and predictive capability of wildfire simulation models.

7 Conclusion

Our research highlights the influence of topography on fire behavior. By varying the slope sensitivity factor a in $\frac{1}{1+e^{-ax}}$, we observed diverse fire propagation patterns, showing the relationship between directional slope and fire spread. Our study lays the groundwork for more sophisticated fire propagation modeling, offering insights into wildfire management strategies.