ELECENG 3TR4: Communication Systems
Lab 4: Random Processes

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Numerical Experiment #1: Evaluation of Autocorrelation and Power Spectral Density

(i). Comparison between Theoretical Results and MATLAB Results

Theoretical autocorrelation function of the output

$$B = 250\,Hz$$

$$h(t) = 2Bsinc(2Bt) = 500sinc(500t)$$

$$R_y(\tau) = E[Y(\tau)Y(t+\tau)] = \sigma^2\big(h(t) \times h(-t)\big) = \sigma^2\big(500sinc(500\tau) \times 500sinc(-500\tau)\big)$$

Since sinc function is symmetry

$$R_y(\tau) = \big(500sinc(500\tau)\big)^2\sigma^2$$

Theoretical PSD

$$h(t) \leftrightarrow H(f)$$

$$2 \times 250sinc(2 \times 250t) \leftrightarrow rect(\frac{f}{2 \times 250})$$

$$S_y(f) = k|H(f)|^2 = k \times rect\left(\frac{f}{2 \times 250}\right) = k \times rect(\frac{f}{500})$$

The theoretical and MATLAB results display similar patterns in both the autocorrelation functions and the power spectral densities (PSD). The autocorrelation functions exhibit sinc functions of the same frequency, and the PSDs present rectangular functions within the identical bandwidth. However, the MATLAB plot introduces some noise, indicating a slight divergence from the theoretical model, which can be attributed to the non-ideal nature of the white noise signal used in the simulation.

(ii). Varying Maxlag

Autocorrelation Results: As maxlag increases from 100 to 500, the autocorrelation plots become more spread out in time. The sinc function become clearer with a larger maxlag, which allows more precise estimation of the signal's correlation properties.

PSD Results: The PSD plots show more peaks as maxlag increases, providing longer time window for the autocorrelation function before FFT. The PSD peaks are sharper, offering a clearer distinction of the signal's frequency content within the bandwidth from 100 to 500. However, increasing maxlag also reveals more fluctuations or "noise" in the passband of the PSD plot. These are likely due to the finite sample effects and the inherent noise in the numerical simulation.

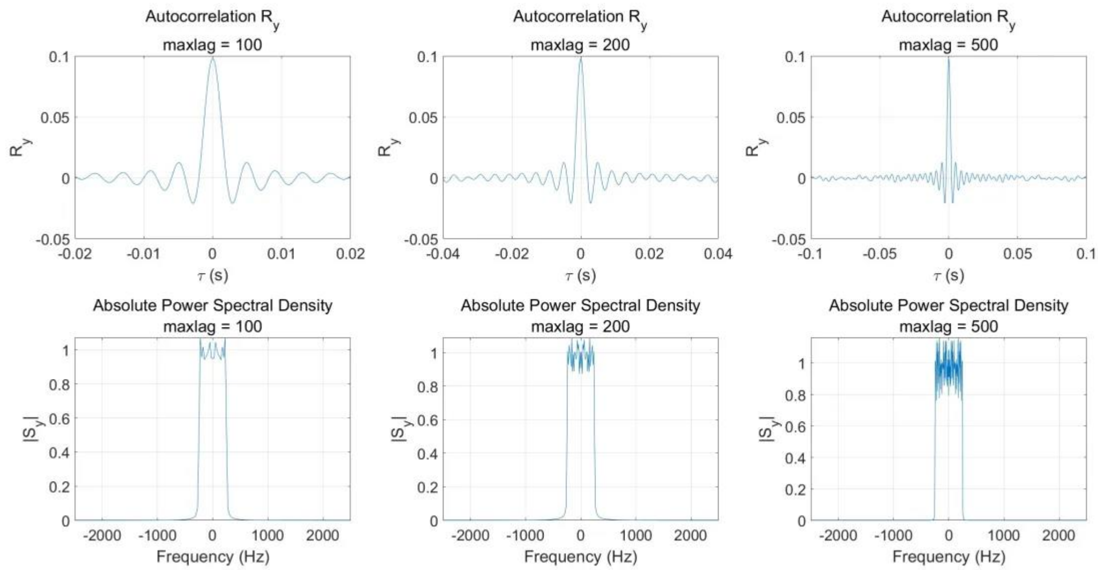

Figure 1. Impact of Maxlag on Autocorrelation and PSD

(iii). Bandwidth of Filter by Autocorrelation Plot

$$\tau = 0.002s, R_y = 0.00024$$

$$B = \frac{1}{|\tau|} = \frac{1}{0.002} = 500Hz$$

The first zero-crossing of the sinc function occurs at $\tau = 0.002$ seconds, where R_y approaches 0, indicating a double-sided bandwidth of approximately 500 Hz. Therefore, an ideal low-pass filter with a single-sided bandwidth of 250Hz, aligning with the expected filter design. The maxlag parameter does not affect the bandwidth estimation as it only extends the time window for the autocorrelation analysis without altering the fundamental frequency characteristics of the signal.

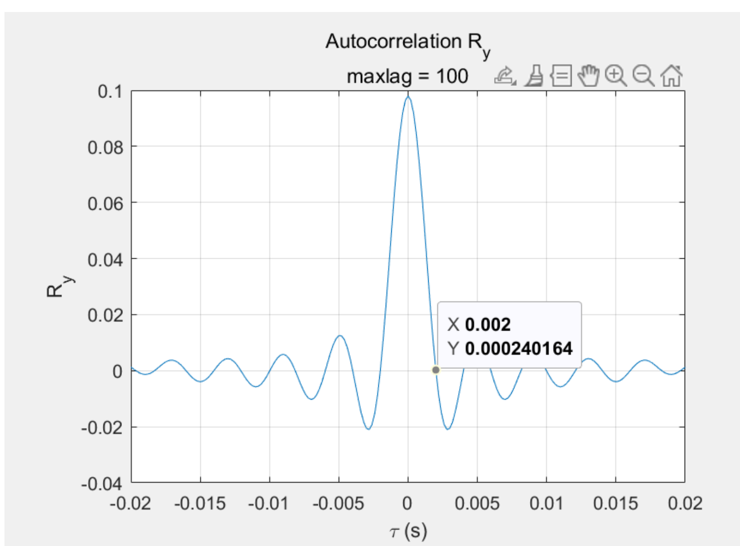

Figure 2. First Zero-Crossing of Sinc Function

Numerical Experiment #2: A sinusoid buried in noise

(i). Peak at Zero

$$y(t) = Asin(2\pi f_c t + \theta) + n(t)$$

$$R_y(t) = E[y(t)y(t+\tau)] = E\{[Asin(2\pi f_c t + \theta) + n(t)] + [Asin(2\pi f_c (t+\tau) + \theta) + n(t+\tau)]\}$$

$$R_y(t) = E[A^2 \sin(2\pi f_c t + \theta) \sin(2\pi f_c (t+\tau) + \theta) + Asin(2\pi f_c t + \theta) n(t+\tau) + Asin(2\pi f_c\,(t+\tau) + \theta) n(t) + n(t) n(t+\tau)]$$

$$Since\ n(t) is\ white\ Gaussian\ noise, E[n(t)] = 0\ and\ E[n(t)n(t+\tau)] = \sigma^2 \delta(\tau)$$

$$R_y(t) = A^2 E[sin(2\pi f_c t + \theta) sin(2\pi f_c (t+\tau) + \theta)] + \sigma^2 \delta(\tau) = \frac{A^2}{2} \cos\,(2\pi f_c \tau) + \sigma^2 \delta(\tau)$$

The peak at zero lag in an autocorrelation plot shows the total power of the signal, as autocorrelation at zero lag is power of the signal. The peak is in the presence of a sinusoidal signal plus white noise.

(ii). Varying Maxlag

The maxlag parameter extends from 100 to 200 and then to 20,000 has an impact on the frequency resolution. When estimating the frequency c_f in the frequency domain, a larger maxlag results in a more refined frequency resolution, which is indicated with narrower lobes as maxlag increases from the PSD plots. There is the inverse relationship between the time window of the autocorrelation function and the frequency resolution. As maxlag increases, the frequency bins become finer in the frequency domain to get more precise estimation of the signal's frequency components.

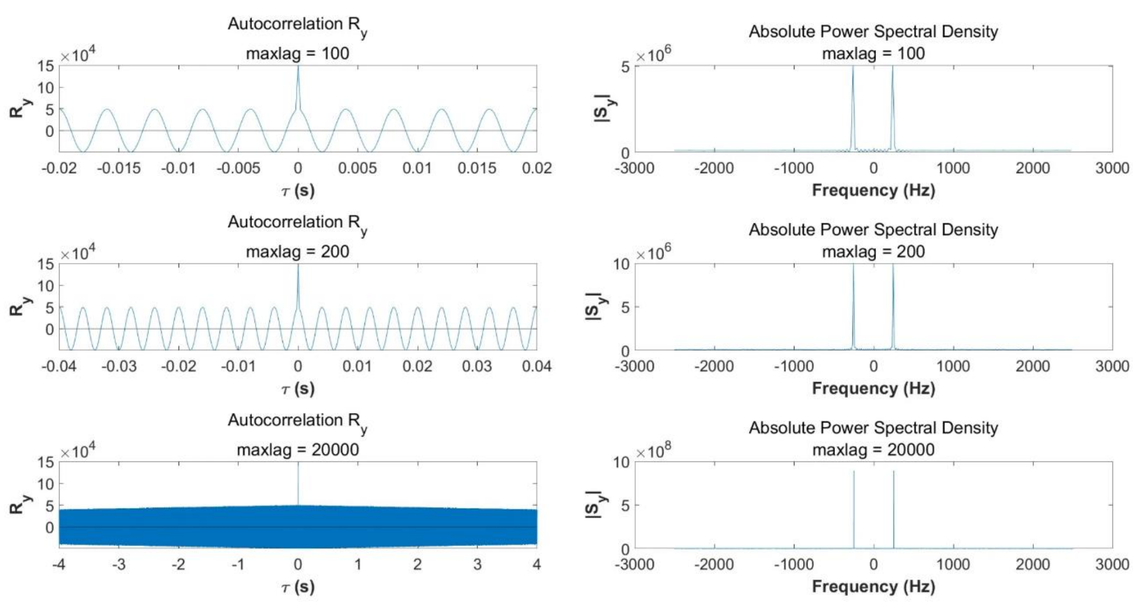

Figure 3. Impact of Maxlag on Frequency Resolution

(iii). Frequency Estimation

It is difficult to distinguish the signal's characteristics accurately, since y(t) consists of a sinusoidal signal buried in noise, and the frequency f_c is not practical. For a clean sinusoidal signal, it can be estimated the frequency directly from the time-domain plot by measuring the period and the calculate the frequency. However, when noise presents, the measure is not unreliable.

The reliable method to estimate the frequency is through spectral analysis. By taking the Fourier Transform of y_t, it can be observed the signal's frequency spectrum, which spreads the noise across a wide range of frequencies and may reveal the sinusoidal signal as a distinct peak. The frequency corresponding to this peak is the estimated frequency f_c of the sinusoidal component.

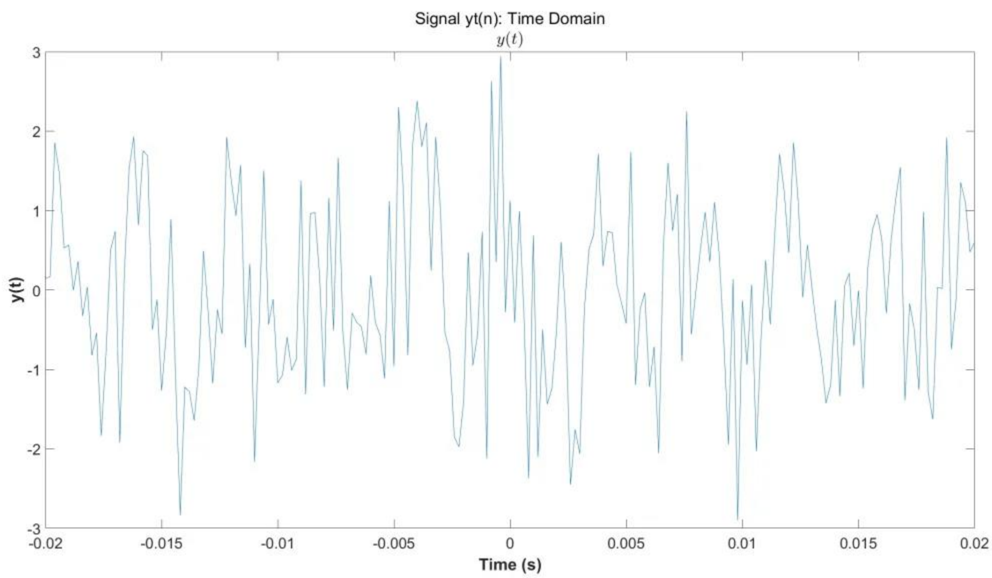

Figure 4. Sinusoidal Signal with White Noise on Time Domain

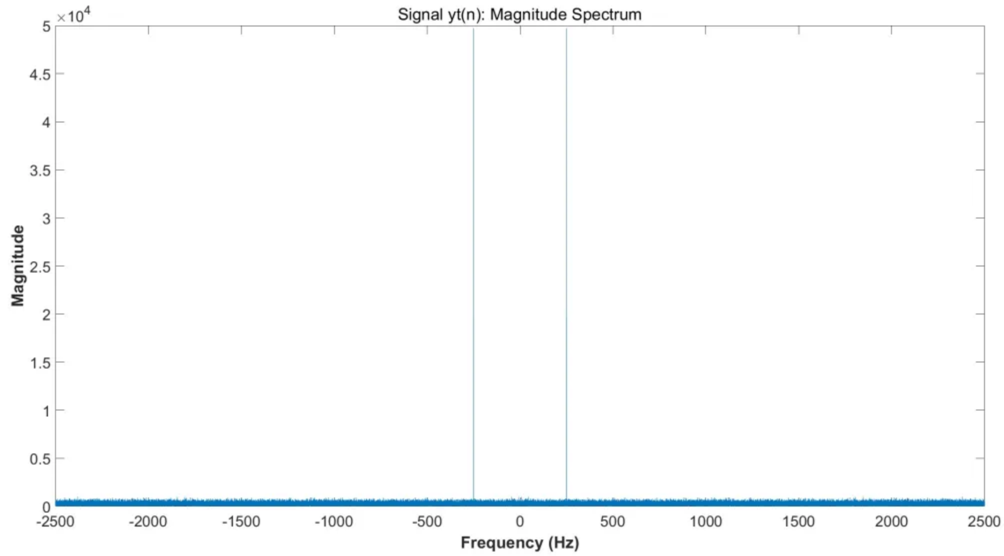

Figure 5. Sinusoidal Signal with White Noise on Frequency Domain

Numerical Experiment #3: Delay Estimation

In the third numerical experiment, which focuses on estimating delay, we examine the signals x(t) and y(t) graphically represented in the time domain, as depicted in Figure 5. It becomes challenging to identify the delay in y(t) since the signal is heavily masked by noise, making the location of its peak elusive.

To estimate the delay, one could plot the cross-correlation between x and y, and then find the τ value at which the autocorrelation function of y reaches its peak. This is illustrated in Figure 10, where the cross-correlation peaks at $\tau = 0.424$ seconds, a value that aligns with the delay observed in y(t)'s time domain graph.

However, using the autocorrelation of y (calculated as xcorr(y,y)) is ineffective for this purpose because its peak occurs at $\tau = 0$. This indicates no delay, which contradicts the observed delay in signal y.

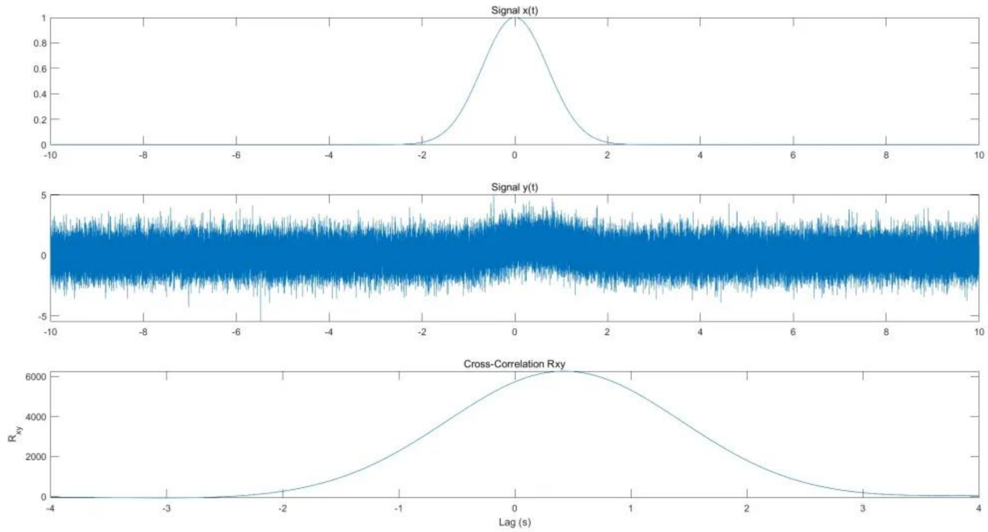

Figure 6. Plots of Signal x(t) and y(t) vs. Plot of Cross-correlation

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timeDelay =

     4.240000000000000e-01
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Figure 7. Time Delay Result on Matlab