Mathematics of Air Combat

Mathematics' Influence on the USAAF in World War II

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In World War II, the war effort needed people from all walks of life to keep it going, whether it was the man on the front line fighting to protect his country, the farmer at home growing the food to feed the troops, or the assembly line worker making the ammunition to fill the guns. Academics were no exception. In particular, mathematicians made some vital contributions to the war effort. Most notable to the general public was the cracking of Enigma, solved by Alan Turing when creating the first computer. Mathematicians, though, made many widely unknown contributions to the war effort. Some of the more significant contributions that greatly affected the United States Army Air Force (USAAF) were improving bombardier-gunner and air-to-air missile accuracy through the use of mathematics to properly model the problems so they could develop targeting sights and improve tactics.

The life of bomber crews in the war was a hard one. The casualty rate of bomber crews was quite high because their planes were very slow and not very manoeuvrable which made them easy targets for Axis fighters. One of the ways the Allies tried to counter this was to give fighter escorts to the bombers. This was not a complete solution because for most of the war, fighters had a shorter flight range than bombers which meant that for long range bombing missions, the bombers didn’t have escorts the whole way, leaving them vulnerable for a period of time. To compensate for this opening, bombers were outfitted with more gun turrets to give them some offensive capability against fighters. The USAAF adopted this tactic heavily by adding more gun turrets on their bomber models. The B-17 is a great example of this with its two most produced variants, the F and G models, going from seven guns spread amongst their turrets on the F variant to thirteen guns on the later G variant.

Adding more guns was not the perfect solution they were looking for. Many bombardier-gunners found it difficult to hit the fighters, effectively making them useless. This is where the mathematicians come into the story. Emerging in Britain was the mathematics field of operations research (OR). OR is a combination of mathematical modeling, statistical analysis, and mathematical optimization that searches for optimal or near-optimal solutions to complex decision-making problems. An OR group for the Royal Air Force (RAF) noticed that the bombardier-gunners would aim like they would if they were hunting birds. That is, they would aim in front of the fighter to account for bullet travel time.[1] This proved to be ineffective because they were shooting from a moving platform in a three dimensional space which has more influences than just bullet travel time to account for. Using vector algebra, which is the mathematics used for the analysis of angles and directions, and looking at bullet trajectory charts, mathematicians were able to model the optimum location to aim. John McCleary's paper, *Airborne Weapons Accuracy: Topologists and the Applied Mathematics Panel*, summarizes well the complex mathematics behind the needed model that was produced by the Applied Mathematics Group (AMG-C) at Columbia University.[2] What was found from the model was counterintuitive to what one would think in the majority of cases: it is best to aim directly at the tail of the fighter. This explains why the bombardier-gunners were missing the target in many situations. Because this is counterintuitive, one can imagine how hard it would be to convince the bombardier-gunners to aim this way. Saunders Mac Lane, the "Technical Representative" for the AMG-C,[3] affirms this when he said "A major part of our problem was properly training machine gunners to aim toward the tail."[4]

1 J. Barkley Rosser, "Mathematics and Mathematicians in World War II." in *AMS History of Mathematics, Volume 1: A Century of Mathematics in America, Part I.*, (n.p.: American Mathematical Society, 1988), 304.
2 John McCleary. "Airborne Weapons Accuracy: Topologists and the Applied Mathematics Panel." *The Mathematical Intelligencer* 28, no.4 (2006): 18-19.
3 Mina Rees, "The Mathematical Sciences and World War II." *The American Mathematical Monthly* 87, no.8 (Oct. 1980): 612.
4 McCleary, "Airborne Weapons Accuracy: Topologists and the Applied Mathematics Panel." 19.

Since most bombardier-gunners had little to no mathematics background and each calculation takes time, it would be unrealistic to teach every gunner how to do these calculations and expect them to perform it in their head in a matter of a split second while in battle. One of the solutions around this was assigning zones to sections of the gunner's window so if a plane was in a zone, the gunner would shoot the given distance and direction in front of the plane. These distances and directions were given to each gunner on what they called "poop sheets" for them to memorize. The Harvard mathematics professor, Edwin Hewitt, was on the OR team of the Eighth Bomber Command which was a part of the first USAAF troops to be deployed in Britain. Hewitt was tasked with devising defensive tactics for the previously mentioned B-17 as well as the B-24. Seeing that there were no formal aiming instructions for the bombardier-gunners, and seeing the zone system presented to new British recruits, Hewitt thought to adapt the zone system for the B-17 and B-24. Since every plane and gun had different characteristics, Hewitt needed to perform all the calculations needed for gun placement for each variation of the two planes. This is where the USAAF picked up this technique. After these calculations, Hewitt was then tasked with lecturing the new American troops on how to use the "poop sheets."[5] He also ended up going on seven bombing missions as a bombardier-gunner. In late 1944, Hewitt was transferred back to the USA, spending some weeks with Mac Lane and then working in OR in the Pacific theatre.[6]

Another solution came in part from the Jam Handy Organization, a film producing company most known for creating the animated *Rudolph the Red-Nosed Reindeer* movie. During the war, the Jam Handy Organization was commissioned to make a variety of training videos for the United States Army. One of their projects was to make instructional videos of where

5 Rosser, "Mathematics and Mathematicians in World War II." 304-305.
6 Hewitt, Edwin. "So Far So Good: My Life Up To Now." The Mathematical Intelligencer 12, no.4 (1990): 60.

bombardier-gunners should aim. This was done by making a model of the fighter and moving the camera and fighter every frame to make an animation of the common manoeuvres the fighters would make. They hired the mathematicians William M. Borgman and Edwin W. Paxson to mark on each frame the correct spot on which to aim. These films were then shown to the bombardier-gunners so if they see a fighter doing one of the manoeuvres, they would know where to aim.[7] This is a great example of how civilian companies in collaboration with mathematicians were used to help the war effort.

A third solution was being developed by the AMG-C that consisted of a mechanical sight that on some basic input would aim the gun so the bullet would hit a fighter in the crosshair. The required input was the speed of the bomber, which was calculated by one of the instruments in the cockpit of the bomber and relayed to the crew, and the axis of the bomber; axis refers to the orientation of the bomber in the sky. The axis was calculated mechanically in the linkage between the gun and the crosshairs whereas the speed was to be input on a dial on the side. The calculations that the linkage would do used the mathematic model of ballistics found by the AMG-C mentioned earlier. The mathematician Daniel Zelinsky was hired by the AMG-C to put together this linkage. As for its effectiveness, Zelinsky had finished his part and noted that some others near the end of the war were experimenting with using an early analog computer to move the reticule around. This made him believe that it never made it into the battle field in a significant number.[8]

The problem of accuracy was not limited to bombardier-gunners. Since air-to-air missiles were in their infancy, hitting the target with them was a difficult task, missing more times than not. Since mathematics is independent of the problem it is being used for, much of the

7 Rosser, "Mathematics and Mathematicians in World War II." 305.
8 Ibid.

mathematics used in air-to-air ballistics created for the bombardier-gunners in the linkage and zone system could also be applied in this situation. Hassler Whitney, who studied music and physics at Yale and got his PhD in mathematics at Harvard, was hired as a consultant for AMG-C to develop a fire control system for air-to-air missiles. The base mathematics for this was the same as the mathematics found by AMG-C originally for the bombardier-gunners, but more mathematics was still needed for this specific problem to get started. Most of the work after was to develop a mechanical sight that would aim the crosshair utilizing these mathematics so the missiles would hit the plane in the crosshair. This system was purposefully made in such a way that the pilot would need minimum user input in order for it to work. Whitney emphasized this by saying:

> An important consideration in assessing a tracking mechanism is the mental effort it requires on the part of the operator. He may have other matters to attend to at the same time, such as ranging and pressing a trigger. In the heat of combat, one cannot expect a gunner to go through mental gymnastics.[9]

This sight was ready for its testing and a variation of it was accepted to be used about a month before the end of the war, so unfortunately its effect on the war was ultimately not seen at the time.[10] The research, however, continued after the war and was influential in the mathematics incorporated by modern sights and used now with great effect.

Having a good sight is of no use in the absence of consistent missiles with a predictable flight path, as it would be impossible to model. This is why at the same time as the sight was being researched and developed, the flight characteristics of fin-stabilized missiles was also being modeled. *Mathematical Theory of Rocket Flight* written by J. Barkley Rosser, Robert R. Newton and George L. Gross, gives a compilation of all the work they and their colleagues have

9 McCleary, "Airborne Weapons Accuracy: Topologists and the Applied Mathematics Panel." 20.
10 Ibid., 21.

done on this topic. It goes into great depth on the mathematics of how a missile would fly in multiple conditions and the differences behind the model for how it behaves while it is burning compared to when it is finished with its burn.[11] This would be useful to the people working on the sight as it gave them the ability to predict where the missile would go so they could make the crosshairs point to the right spot. It was also influential in the design of the newer, more accurate missiles in production during the war, causing fewer missiles to be needed per target. Like the sight, this research was done at the end of the war and was not sufficiently tested in the battle field of World War II, but it did lead to the advances of missiles post-war, especially in the Cold War.

A problem with fully analyzing the effects of mathematics developed for the war is that the effects were not found *during* the war, as they finished the research at the *end* of the war, were classified, or were never tracked with detail. The effects of the mathematics behind the missile and its sight as stated, was not fully known till after the war but it had major impact on the continued post-war research, and now missiles are the primary weapon used for air-to-air combat. The mathematics behind the different aiming techniques was classified for some years and when it was finally declassified, the statistic behind its effectiveness was found to never be explicitly tracked. There are statistics for kills credited to bombardier-gunners which all show an increase in the ratio of fighters killed per bomber,[12] but these numbers may be inflated since gunners from different planes may all get credited for the same kill. This increase is also in general terms with other factors to keep in mind like an overall gain of experience or better radar, so the aiming strategies can't take all the credit for the increase. It is of note to the importance of

11 J. Barkley Rosser et al., *Mathematical Theory of Rocket Flight*. (New York: McGraw-Hill Book Co., 1947) 1-184.

12 Office of Statistical Control *Army Air Forces Statistical Digest World War II* (n.p.: n.p., 1945) 262. http://www.dtic.mil/dtic/tr/fulltext/u2/a542518.pdf

the work that the AMG-C was given the Naval Ordnance Development Award for all the work they did in the war,[13] which includes the bombardier-gunner and missile models, and the aiming systems mentioned. When Warren Weaver, Chief of the Applied Mathematics Panel which oversaw the AMG-C,[14] was asked about how the military saw the value of the work they did, he said “the Navy and what later became the Air Force were among the first believers.”[15] This meant that the Air Force saw what the mathematicians were doing for them and believed they made a significant difference.

The results found by the mathematicians of World War II provided the needed information for the USAAF to give the troops the upper hand in air-to-air combat by giving them the theory and equipment they needed. Even though this work is not necessarily known by the general public, it was still very important in the war effort. The work not only saved more lives of the crews that used it, but it also paved the way for some of the next big inventions in the Air Force after the war. Most notable is the mathematics used in the software of modern aircraft computer-generated sights and of drone targeting systems. Just thinking about where the world would be technically if mathematicians didn’t do the work they did for the war is enough to see how deep of an impact they made.

13 McCleary, “Airborne Weapons Accuracy: Topologists and the Applied Mathematics Panel.” 21.
14 Rees, “The Mathematical Sciences and World War II.” 609.
15 Ibid., 616.

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