Statistical Signal Processing: Homework 1

This is the first out of three homework sheets for the 2019 Statistical Signal Processing problem class. Each homework sheet includes three different examples resulting in a total number of nine homework examples over progress of the course. The presented homework examples are mostly MATLAB examples with a strong real world focus.

The grading of the problem class is based on student presentations (Power Point or Latex presentation) and exercise interviews for five out of the nine examples. For each of the three homework sheets, students have to present one example, which will be selected by the lecturers (*must show*). Students will be informed three days prior to the deadline of the problem sheet, about the number of the example, which has to be presented. The remaining two examples can be selected by the students itself (*will show*). Hereby only one of the remaining two examples per problem sheet can be selected. Note, that the grading is an individual grading.

All presentations and necessary MATLAB-files have to be provided by the deadline to ssp@emt.tugraz.at. This holds for all presentations (*must show* and selected *will show*). Please follow the guidelines for the upload, which are provided by an additional sheet on the server.

The presentation slides have to consider all necessary derivations, algorithmic considerations, diagrams, conclusions, etc.. A pure presentation of the final results will lead to reduction of your points. Also take care to the representation of necessary diagrams. Slides can be in German of English. Note, that your slides have to include all graphics. A reference to a generating MATLAB-script is not accepted.

Deadline: May 10, 2019

Example 1

Let X be a random variable with the distribution $X \propto \mathcal{U}(0,2)$. Consider the systems

$$
\begin{aligned}
y_1 = g_1(x) &= e^x && (1)\\
y_2 = g_2(x) &= x + x^2 && (2)\\
y_3 = g_3(x) &= \sqrt{x} && (3)
\end{aligned}
$$

Please provide answers to the following questions:

1. Derive analytical expressions for the pdfs $\pi_Y(y)$.
2. Plot your analytical results and verify them by means of MATLABs `hist` command (provide one plot for each function, which includes the analytical result and the numerical validation).
3. Estimate the mean μ and the variance σ^2 of the random variable Y by a) using samples, b) linearization, c) the unscented transformation. Provide meaningful plots, which illustrate the behavior of the approximated estimation techniques for μ and the variance σ^2 with respect to the sample based technique.

Example 2

In this example you are provided with the personalized MATLAB function

```
[n,dtilde] = func_sigGEN_HW2(ID),
```

which generates the signal

$$\tilde{d}[n] = s[n] + v[n]. \tag{4}$$

$s[n]$ is the signal of interest and $v[n]$ is additive measurement noise. For $s[n]$ we have the signal model

$$s[n] = A + B \cdot n + C \cdot \sin(0.4n + 0.1) + D \cdot \cos(0.1n + 0.8) + E \cdot \exp(-0.1n). \tag{5}$$

Your task is to estimate the signal parameters $\boldsymbol{x} = \begin{bmatrix} A & B & C & D & E \end{bmatrix}^T$ from the signal $\tilde{d}$. Note, that the vector $\boldsymbol{x}$ is personalized by your ID and that one element of $\boldsymbol{x}$ is zero.

Please provide answers to the following questions:

- Estimate the noise model from the signal $s[n]$. Hint: for this you have to use several realizations of $s[n]$.
- Use an appropriate technique and estimate the vector $\boldsymbol{x}$ from the data.
- Apply a Monte Carlo analysis and estimate the variance of the estimation error. Compare the result against the theoretical result.
- Which element of $\boldsymbol{x}$ is zero for your personalized problem set?
- Repeat the estimation task for the reduced vector $\boldsymbol{x}$ (you can skip the variable, which you believe is zero).
- Analyze the impact on the variance of the estimation result when estimating a smaller set of variables.

Example 3

This example is about the localization of a sound source in a domain Ω. To locate the position

$$\boldsymbol{x}_\mathrm{s} = \begin{bmatrix} x_\mathrm{s} & y_\mathrm{s} \end{bmatrix}^T,$$

of a sound source, where x_s and y_s are cartesian coordinates, a set of N_mic microphones are placed on encircling positions around the source.

Let $\boldsymbol{x}_{\mathrm{m},i}$ denote the cartesian position of the i^th microphone. Then the time of flight (ToF) of the sound signal to the i^th microphone is given by

$$t_i = \frac{1}{c}||\boldsymbol{x}_{\mathrm{m},i} - \boldsymbol{x}_\mathrm{s}||_2$$

where $c = 343\,\mathrm{m/s}$ is the speed of sound.

In this example you are provided by the MATLAB function

```
[S,t] = func_soundlocalization(XYmic,ID),
```

which simulates the measurement process for the source localization problem. The parameter `ID` is your student ID (personalized problem). The source position is known to fulfill the condition $||\boldsymbol{x}_\mathrm{s}||_2 \leq 100\,\mathrm{m}$ (source is within a centered circle with a radius of $100\,\mathrm{m}$). The source signal is a burst signal. You can place an arbitrary number of microphones by means of the $N_\mathrm{mic} \times 2$ matrix `XYmic`. The function provides the N_mic microphone signals by means of the columns of `S`. The vector `t` is a time vector. For simplicity, no attenuation of the signal is assumed and microphones with an omnidirectional characteristic are used. Note, that the microphones and the transmitter are not synchronized. You have to take care about this circumstance in your approach. The measurement frame contains all receiver signals.

Run the `m`-file `Demo_EX3.m` for a demo implementation of the measurement process.

Your task in this example is to estimate the position $\boldsymbol{x}_\mathrm{s}$ from the signals of the N_mic microphones using a model based approach!

Please provide answers to the following questions:

1. Develop an estimation strategy to estimate $\boldsymbol{x}_\mathrm{s}$ from the time signal. Summarize your result by means of a mathematical formulation or some pseudo code.
2. Estimate $\boldsymbol{x}_\mathrm{s}$ (personalized) from the data provided by the MATLAB function. Note: you do not have to implement a sophisticated solution algorithm. A grid search and a surface plot are sufficient.
3. Analyze the behavior of your method for a different number of microphones. What is the minimum number of microphones, which is required?

4. Why is it not convenient to solve the localization problem without a model, e.g. by means of a bearing approach using triangulation?