1) Let $\boxed{u = \frac{\pi}{x}\ ,\ du = -\frac{\pi}{x^2}dx\ ,\ \text{and}\ -\frac{du}{\pi} = \frac{dx}{x^2}}$

$$\begin{aligned}\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}dx &= -\frac{1}{\pi}\int \cos(u)du \\ &= -\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C \\ &= -\frac{\sin\left(\frac{\pi}{x}\right)}{\pi} + C\end{aligned}$$

2)

Integral Calculus Examples You May Need:

'The Power Rule'

$$\frac{d}{dx}x^n = nx^{n-1},$$

where n is any real number. Reversing the power rule to find an antiderivative is equally straight forward, and we have that

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C,$$

Integration by Parts with the following choices of u and v

$$\begin{aligned} u &= 2x, & dv &= e^x, \\ du &= 2, & v &= e^x. \end{aligned}$$

This now gives

$$\begin{aligned}\int x^2e^xdx &= x^2e^x - \left(2xe^x - \int 2e^xdx\right) \\ &= x^2e^x - 2xe^x + 2e^x + C \\ &= (x^2 - 2x + 2)e^x + C.\end{aligned}$$

Evaluating definite integrals using integration by parts is done in the following way. We consider

$$\int_0^\pi x\sin(x)dx.$$

Then from our work earlier on this handout, we have that

$$\begin{aligned}\int_0^\pi x\sin(x)dx &= [-x\cos(x)]_0^\pi + \int_0^\pi \cos(x)dx \\ &= -\pi\cos(\pi) - (-0\cos(0)) + [\sin(x)]_0^\pi \\ &= -\pi(-1) - 0 + (0 - 0) \\ &= \pi\end{aligned}$$

Trigonometric functions

The following three identities (among others) are often useful when dealing with trigonometric functions

$$\begin{cases} \sin^2(\theta) + \cos^2(\theta) &= 1, \\ \sin(m\theta \pm n\theta) &= \sin(m\theta)\cos(n\theta) \pm \sin(n\theta)\cos(m\theta), \\ \cos(m\theta \pm n\theta) &= \cos(m\theta)\cos(n\theta) \mp \sin(m\theta)\sin(n\theta). \end{cases}$$